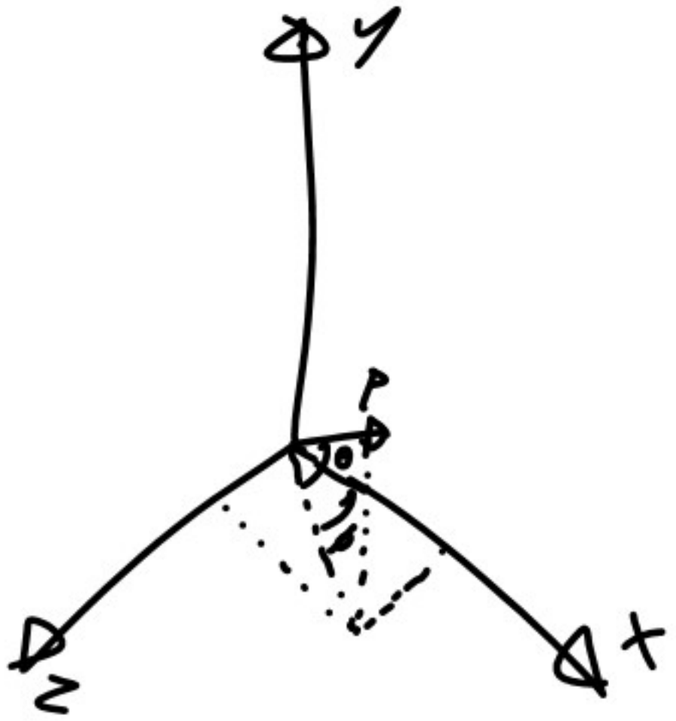


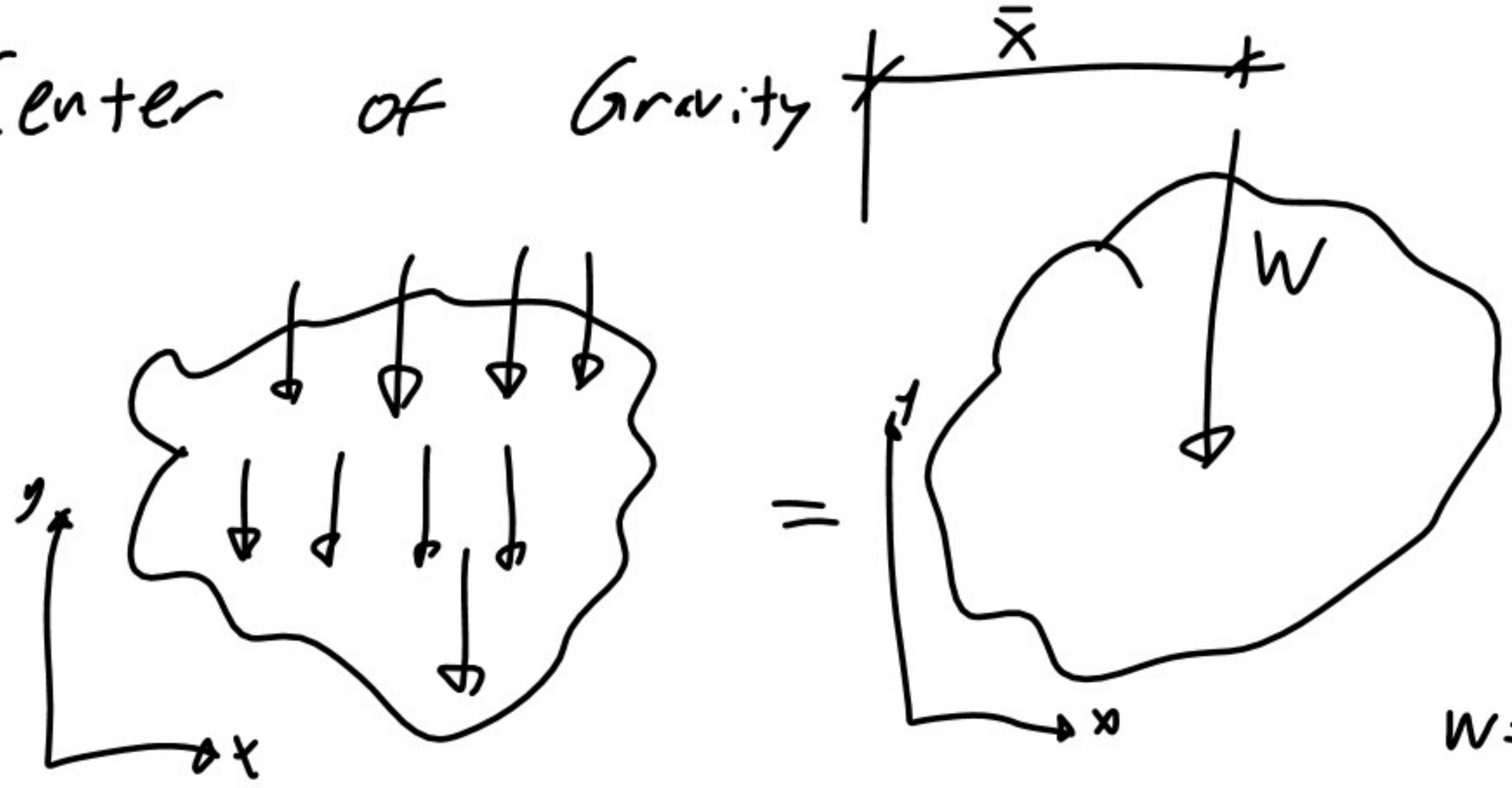
$$P_y = P \sin\theta$$

$$P_{xz} = P \cos\theta$$

$$P_x = P_{xz} \cos\phi$$

$$P_z = P_{xz} \sin\phi$$

Center of Gravity



$\leq M_y \quad \bar{x}W = x_1 \Delta W_1 + x_2 \Delta W_2 + \cdots x_n \Delta W_n$

$W = \Delta W_1 + \Delta W_2 + \cdots + \Delta W_n$

$$\bar{x} \leq \frac{x_1 \Delta W_1 + x_2 \Delta W_2 + \cdots + x_n \Delta W_n}{W}$$

$$\lim_{\Delta \to 0} \bar{x} \quad \Longrightarrow \quad \bar{x} = \frac{\int x \, dw}{w} \qquad \bar{y} = \frac{\int y \, dw}{w}$$

$\Delta w = \gamma t \, \Delta A$

$w = \gamma t A$

$dw = \gamma t \, dA$

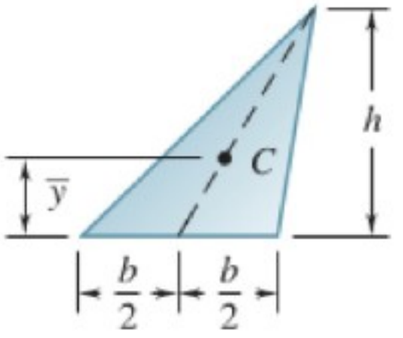
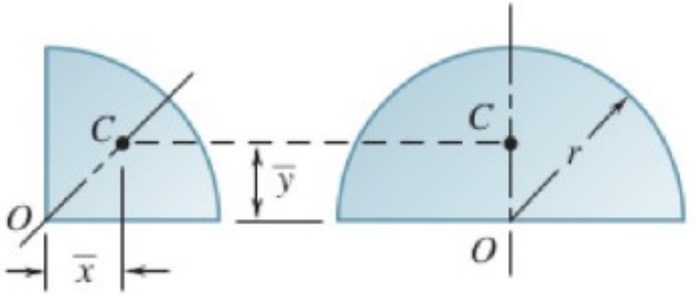
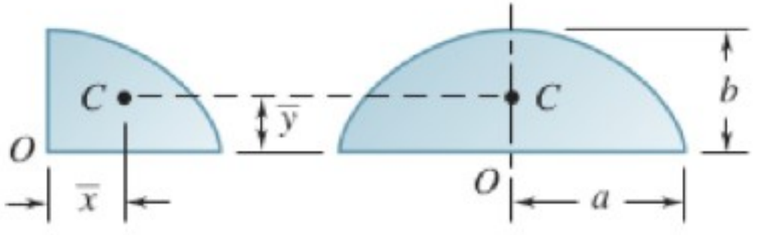
ΔA surface area

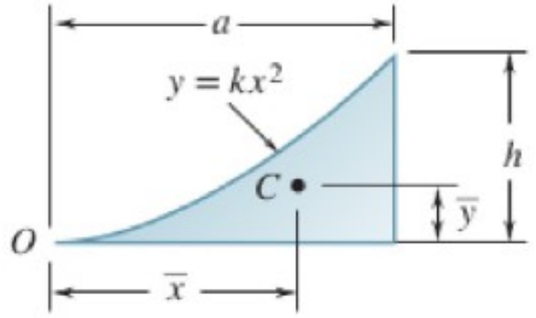
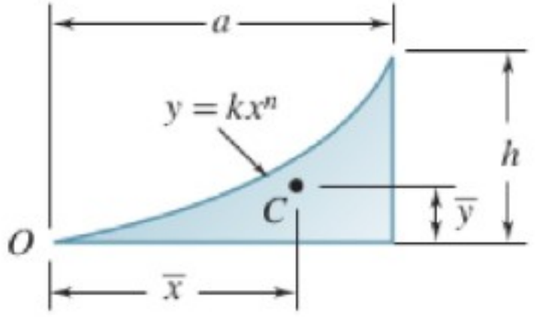
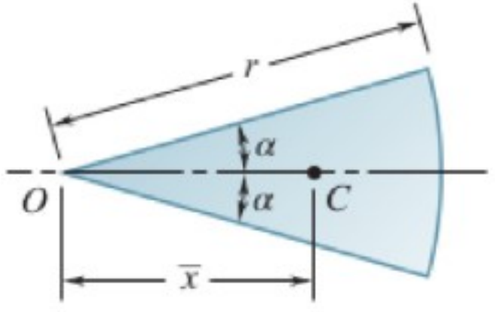
γ specific weight (density)

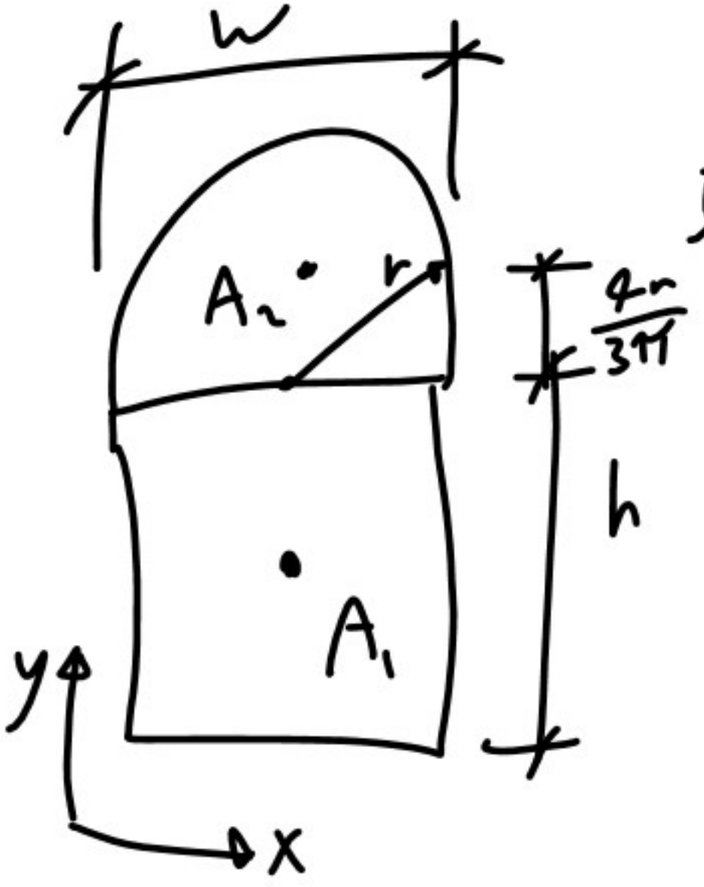
t thickness

(centroid

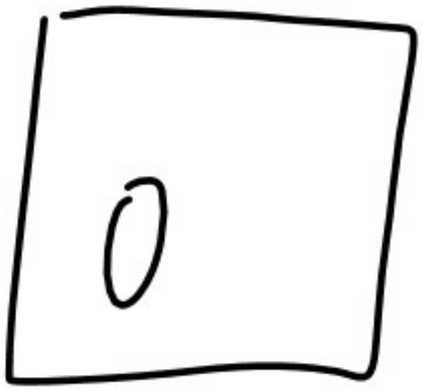
$$\bar{x} = \frac{\int x \gamma t \, dA}{\gamma t A} = \frac{\cancel{\gamma t}}{\cancel{\gamma t}_1} \frac{\int x \, dA}{A} \qquad \bar{y} = \frac{\int y \, dA}{A}$$

Shape		$\bar{x}$	$\bar{y}$	Area
Triangular area			$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter-elliptical area		$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
Semielliptical area		0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$

Parabolic spandrel		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
General spandrel		$\frac{n+1}{n+2}a$	$\frac{n+1}{4n+2}h$	$\frac{ah}{n+1}$
Circular sector		$\frac{2r \sin \alpha}{3\alpha}$	0	αr^2



$$\bar{y} = \frac{\sum \bar{y}_i A_i}{A} = \frac{\bar{y}_1 A_1 + \bar{y}_2 A_2}{A}$$



$A_1 = wh$

$A_2 = \pi r^2/2$

$A = A_1 + A_2$

$\bar{y}_1 = h/2$

$\bar{y}_2 = \frac{4r}{3\pi} + h$

$$\bar{y} = \frac{\int y\,dA}{A} = \frac{\int_0^{15} y\,(33 - y\,\frac{12}{15})\,dy}{705} = \frac{\int_0^{15} 33y - \frac{4}{5}y^2\,dy}{705} = \frac{33y^2/2 - \frac{4}{5}y^3/3\Big|_0^{15}}{705}$$

$$dA = (33 - y\,\tfrac{12}{15})\,dy$$

$$A = \frac{21+33}{2}\,15 = 705$$

$$= \frac{33(15)^2/2 - \frac{4}{5}\,15^3/3}{705}$$

$$= 6.944 \text{ in}$$

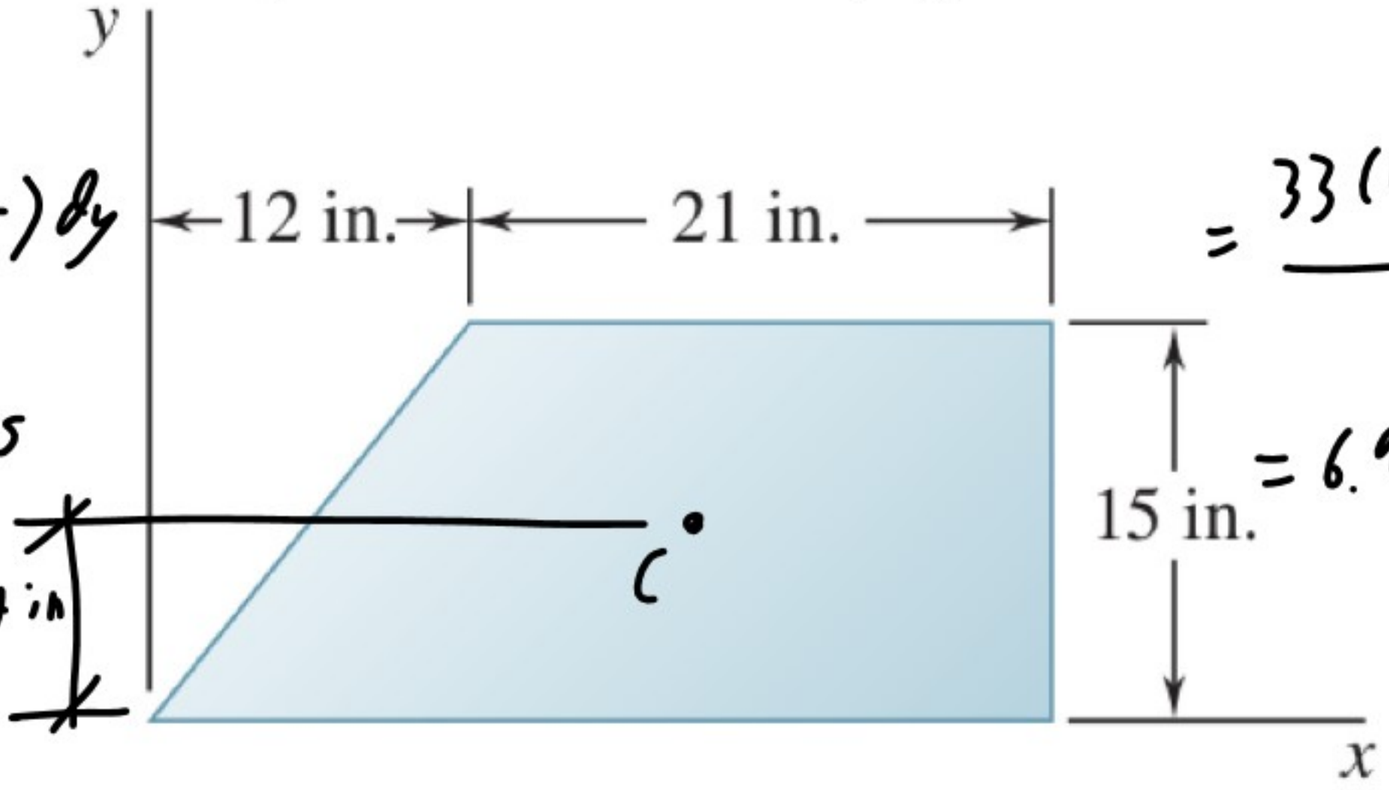


Fig. P5.2

$$\bar{x} = \frac{\bar{x}_1 A_1 + \bar{x}_2 A_2}{A}$$

$A_1 = 21 \cdot 15 = 315$

$A_2 = \frac{12 \cdot 15}{2} = 90$

$\bar{x}_1 = 12 + \frac{21}{2} = 22.5$

$\bar{x}_2 = 8$

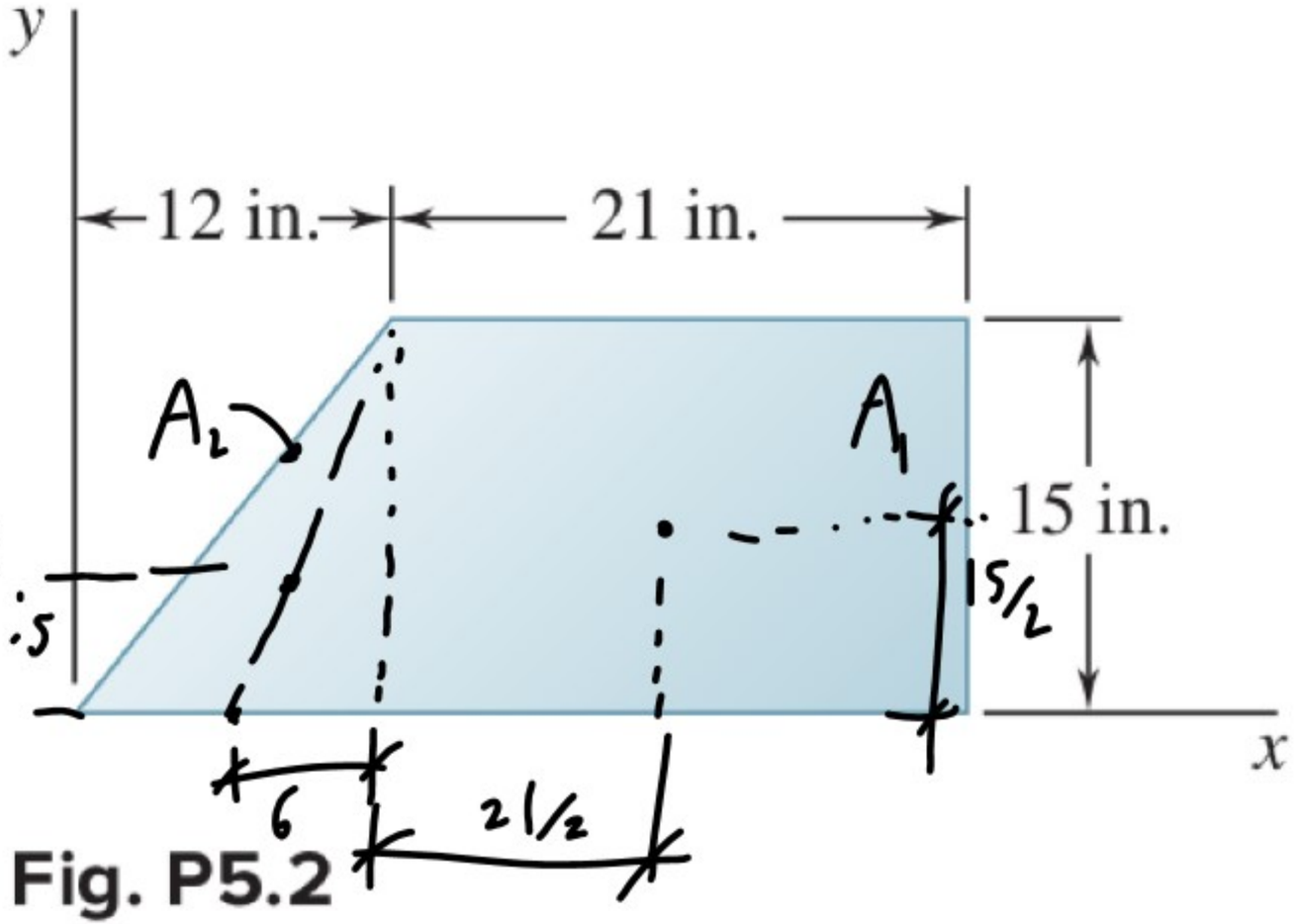


Fig. P5.2