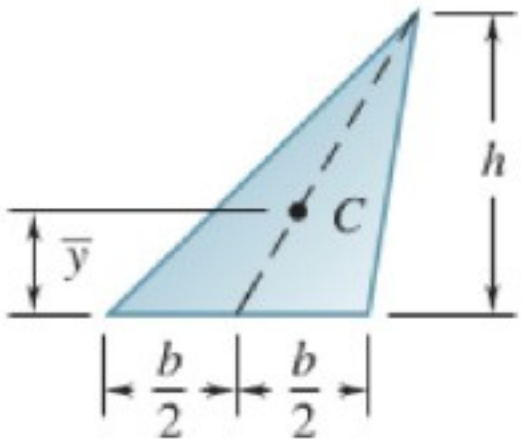
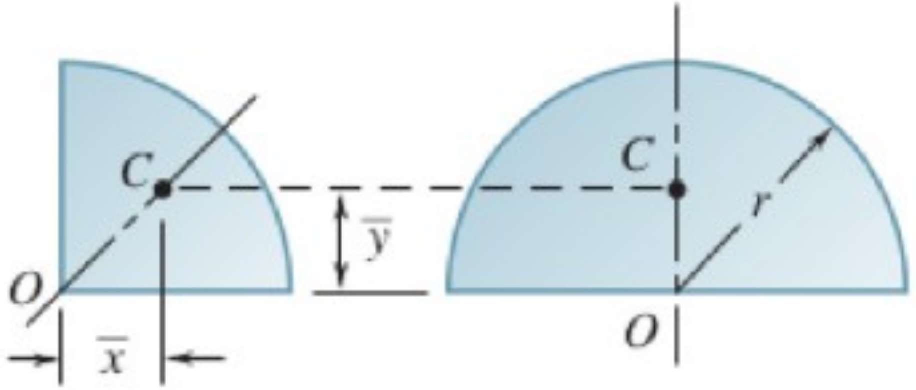
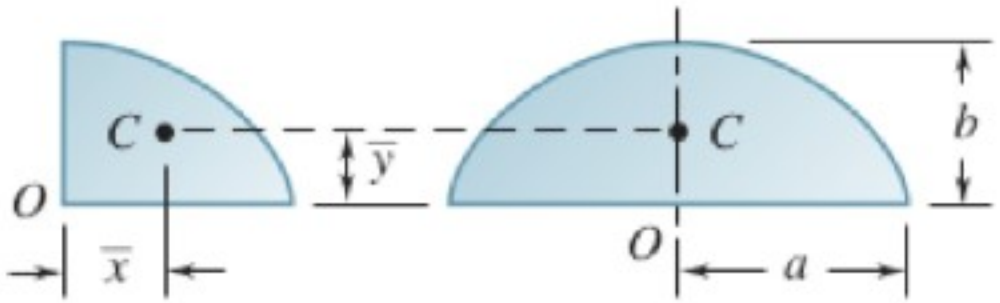
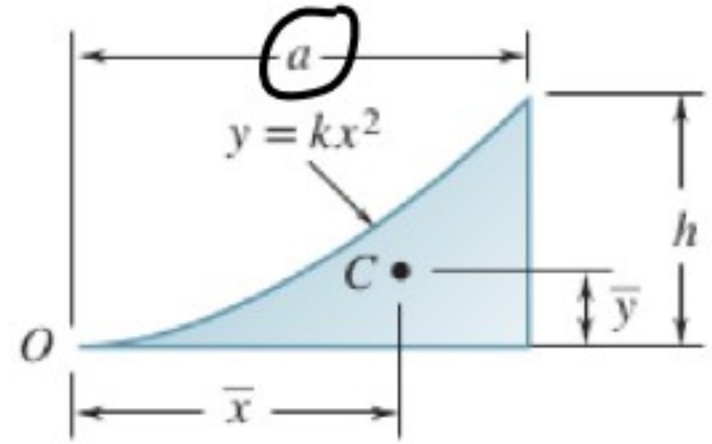
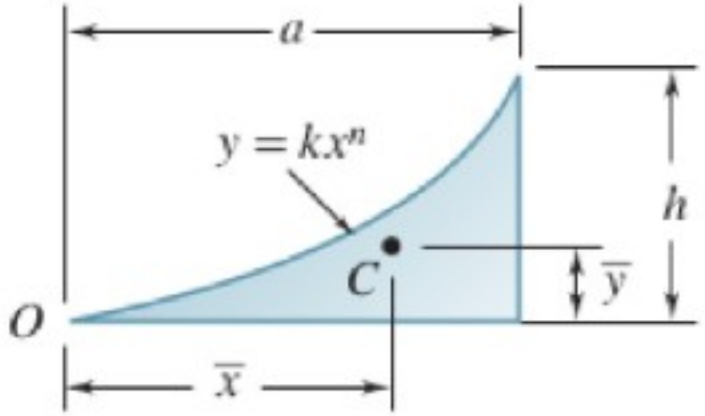
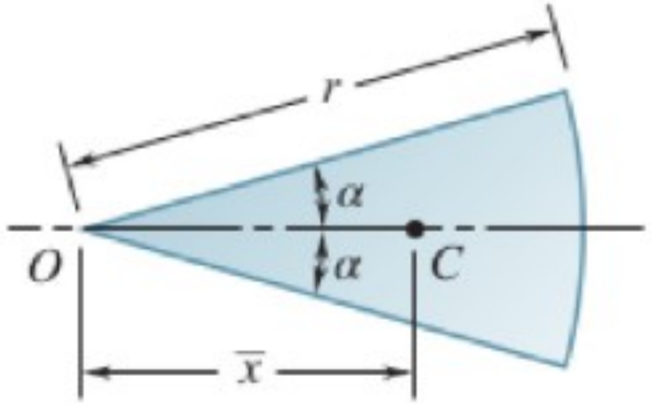
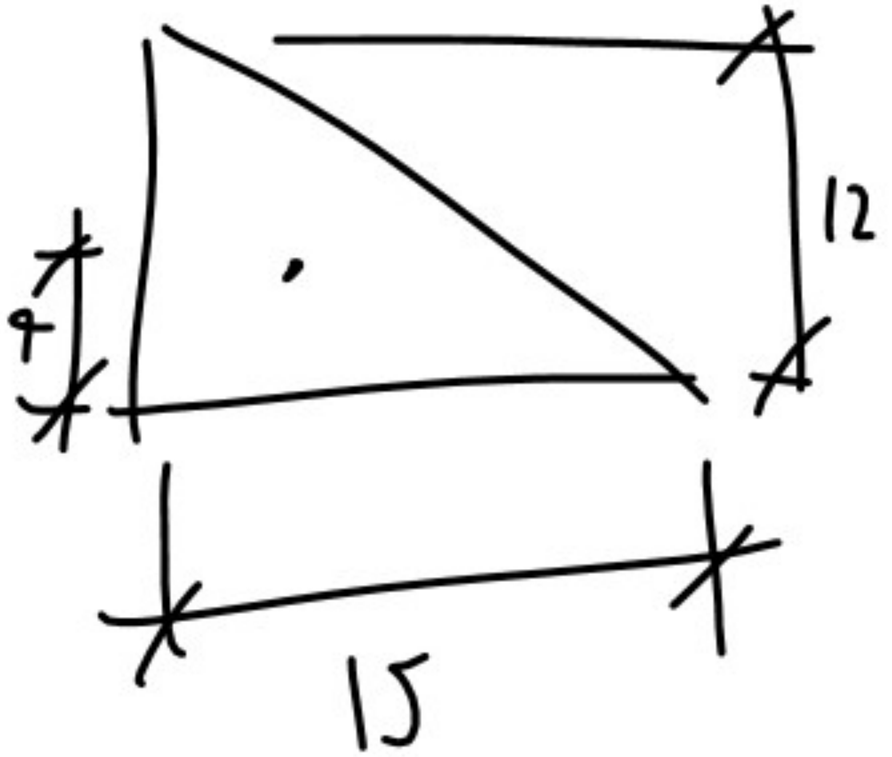
<table>
<tr><th>Shape</th><th></th><th>$\bar{x}$</th><th>$\bar{y}$</th><th>Area</th></tr>
<tr><td>Triangular area</td><td>
</td><td></td><td>$\frac{h}{3}$</td><td>$\frac{bh}{2}$</td></tr>
<tr><td>Quarter-circular area</td><td rowspan="2">
</td><td>$\frac{4r}{3\pi}$</td><td>$\frac{4r}{3\pi}$</td><td>$\frac{\pi r^2}{4}$</td></tr>
<tr><td>Semicircular area</td><td>0</td><td>$\frac{4r}{3\pi}$</td><td>$\frac{\pi r^2}{2}$</td></tr>
<tr><td>Quarter-elliptical area</td><td rowspan="2">
</td><td>$\frac{4a}{3\pi}$</td><td>$\frac{4b}{3\pi}$</td><td>$\frac{\pi ab}{4}$</td></tr>
<tr><td>Semielliptical area</td><td>0</td><td>$\frac{4b}{3\pi}$</td><td>$\frac{\pi ab}{2}$</td></tr>
</table>

		$\bar{x}$	$\bar{y}$	A
Parabolic spandrel		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
General spandrel		$\frac{n+1}{n+2}a$	$\frac{n+1}{4n+2}h$	$\frac{ah}{n+1}$
Circular sector		$\frac{2r \sin \alpha}{3\alpha}$	0	αr^2

$h = 12$

$\bar{y} = 12/3 = 4$



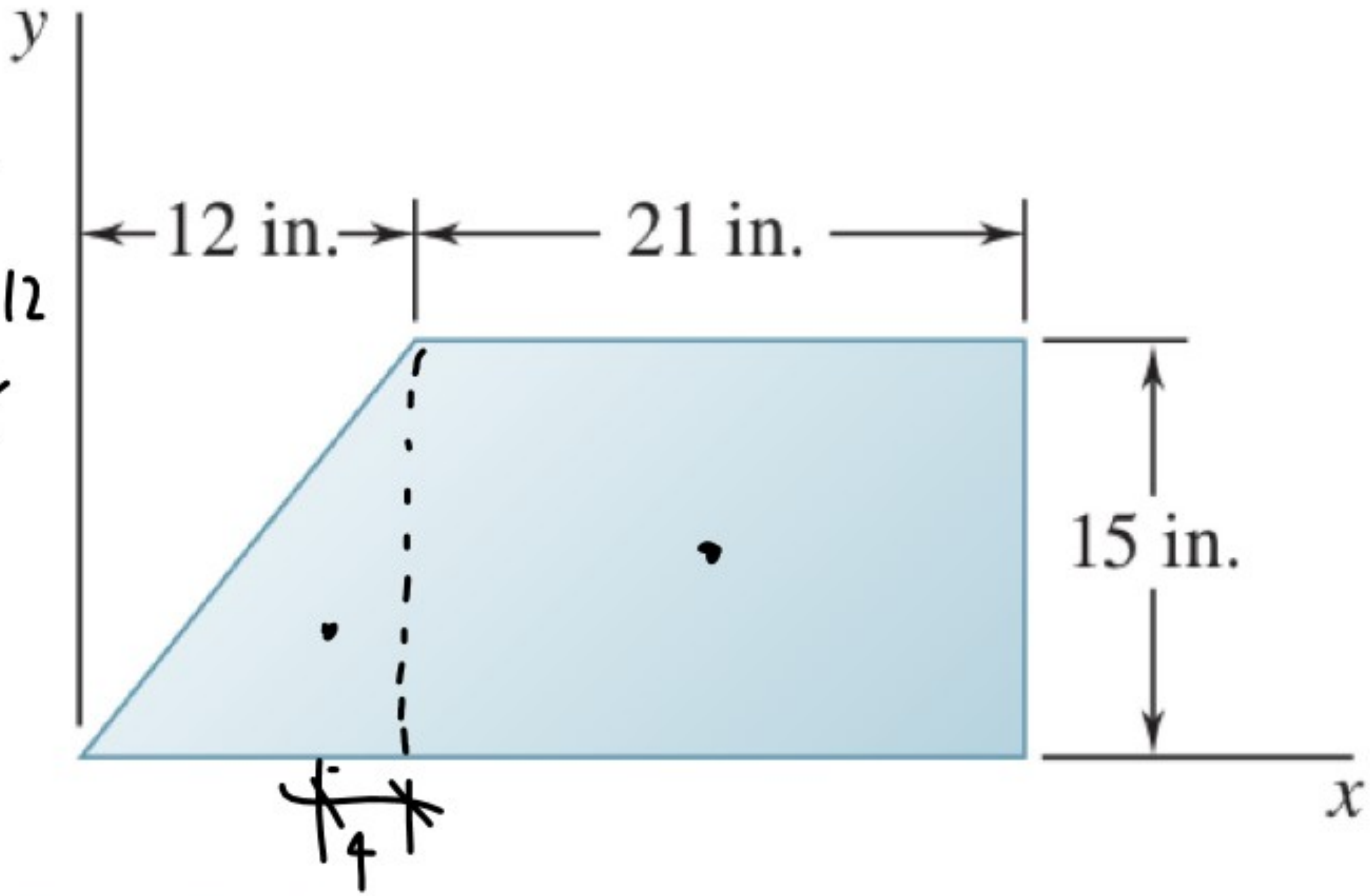


Fig. P5.2

$$\bar{x} = \frac{\int x\,dA}{A} = \frac{\int_0^{16} x(0.039x^2+3)\,dx}{101} = \frac{1}{101}\int_0^{16} 0.039x^3+3x\,dx = \frac{1}{101}\left(\frac{0.039x^4}{4}+\frac{3x^2}{2}\right)$$

$$= \frac{1}{101}\left(\frac{0.039(16)^4}{4}+\frac{3(16)^2}{2}\right) = \boxed{10.1}$$

$$A = \int dA$$

$$dA = f(x)\,dx$$
$$= 0.039x+3\,dx$$

$$A = \int_0^{16} 0.039x^2+3\,dx$$

$$= \frac{0.039x^3}{3} + 3x \Big|_0^{16} = \frac{0.039(16)^3}{3} + 3(16) = 101$$

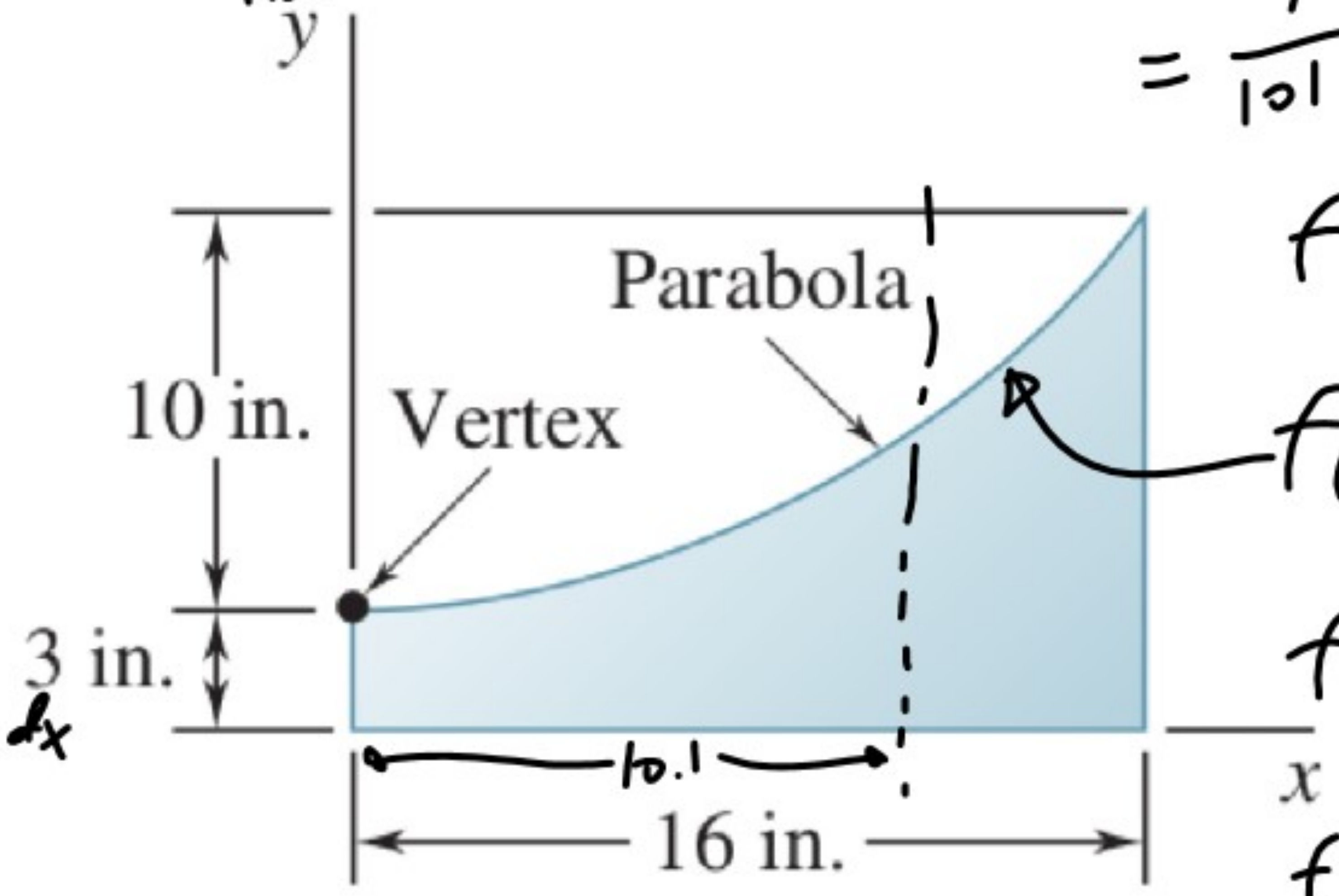


Fig. P5.10

$$f(x) = 0.039x^2+3$$

$$f(x) = ax^2+b$$

$$f(0) = 3 \Rightarrow b = 3$$

$$f(16) = 13$$

$$a\,16^2 + 3 = 13$$

$$a = \frac{13-3}{16^2} = 0.039$$

$\bar{x}_1 = 16/2 = 8$

$\bar{x}_2 = \frac{3a}{4} = \frac{3(16)}{4} = 12$

$A_1 = 16 \cdot 3 = 48$

$A_2 = \frac{ah}{3} = \frac{16(10)}{3} = 53.3$

$A = 48 + 53.3 = 101$

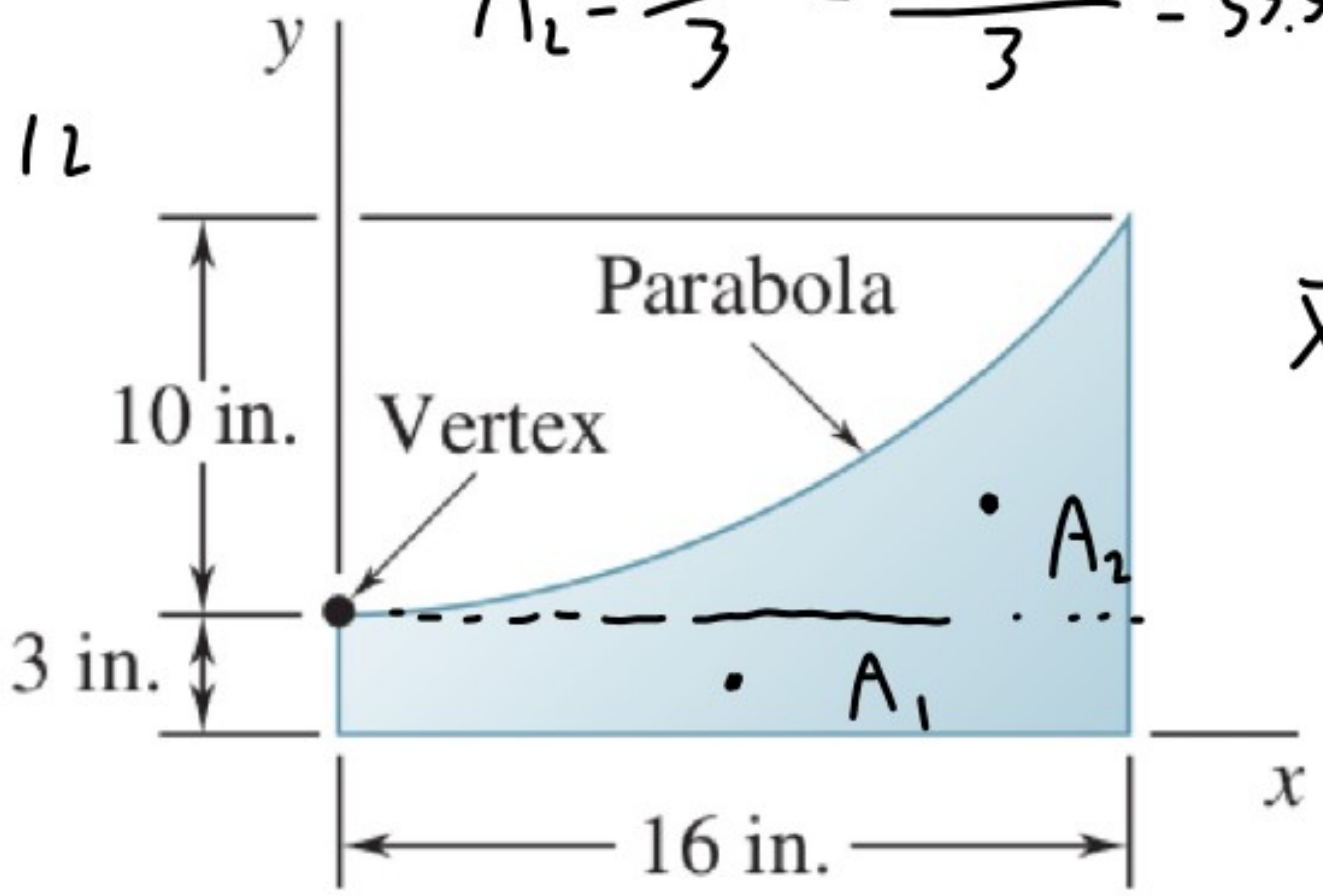


Fig. P5.10

$\bar{x} = \frac{\bar{x}_1 A_1 + \bar{x}_2 A_2}{A}$

$= \frac{48(8) + 53.3(12)}{101}$

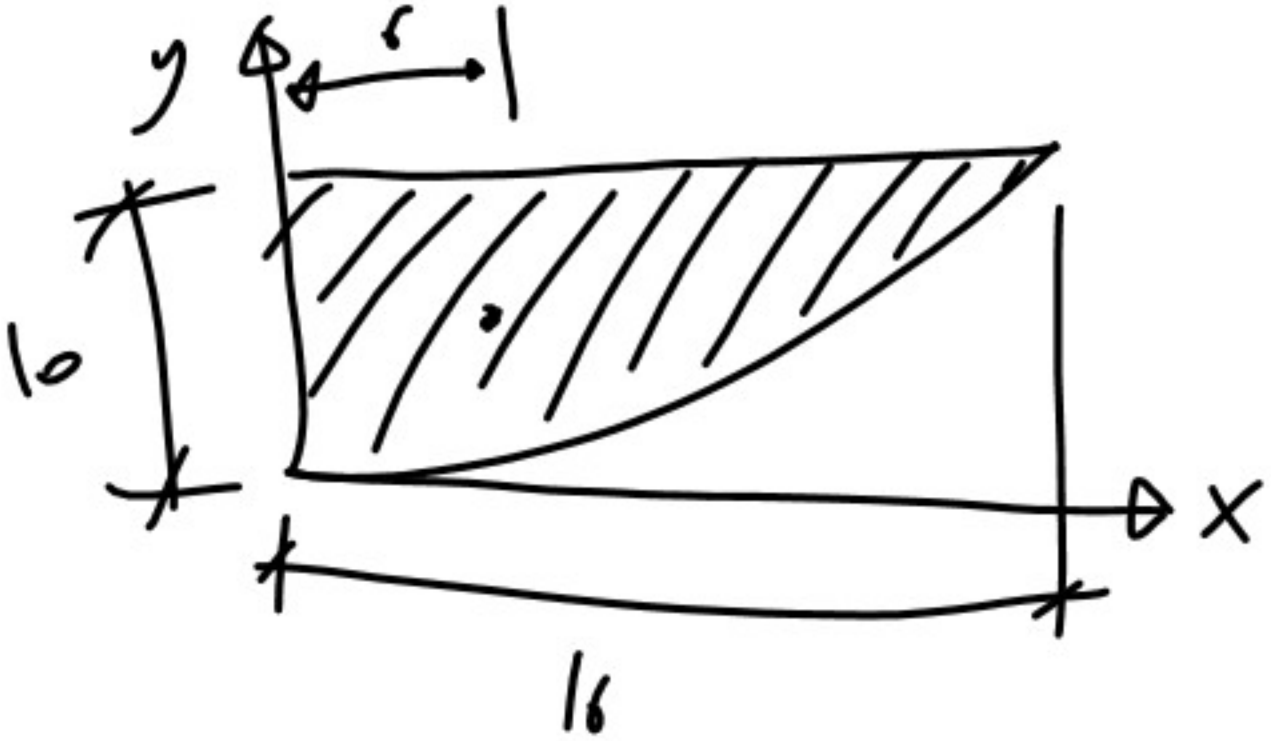
$\boxed{\bar{x} = 10.1}$

A_1 rectangle

A_2 parabola (from table)

$$\bar{x} = \frac{A_1 \bar{x}_1 - A_2 \bar{x}_2}{A}$$

$$= \frac{160 \cdot 8 - 53.3 \cdot 12}{106.7} = 6$$



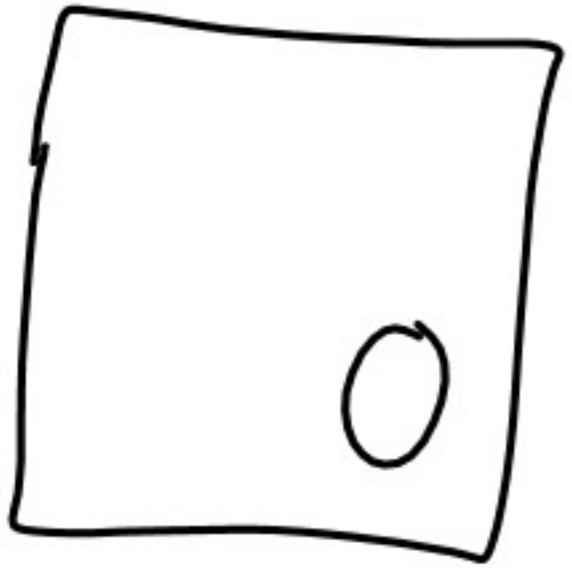
$A_1 = 10 \cdot 16 = 160$

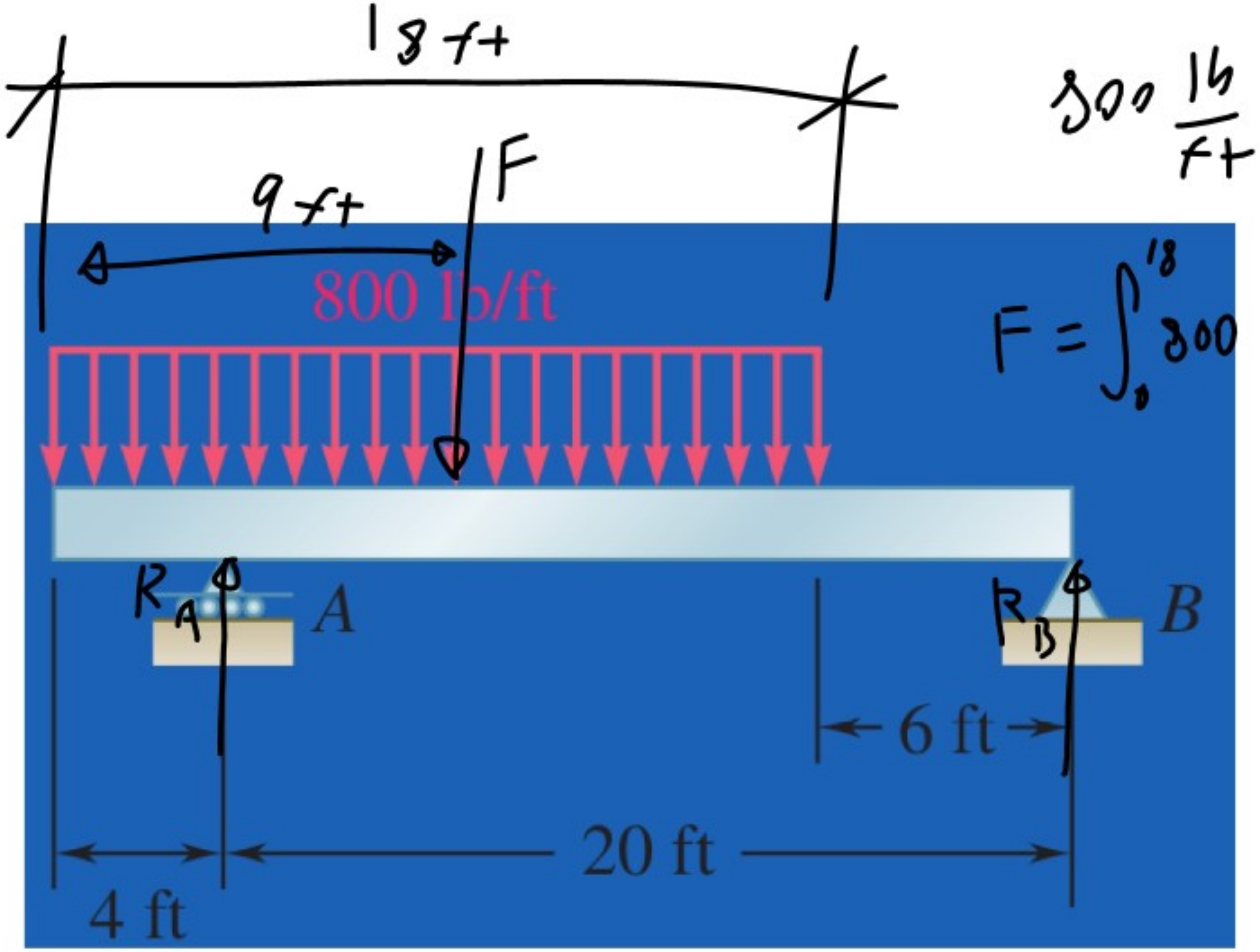
$A_2 = 53.3$

$\bar{x}_1 = 8$

$\bar{x}_2 = 12$

$A = A_1 - A_2 = 160 - 53.3 = 106.7$





$$800 \frac{lb}{ft} \quad 18\,ft = \boxed{14{,}400\ lb}$$

$$F = \int_0^{18} 800\, dx = 800x \Big|_0^{18} = 800(18)$$

$$\bar{x} = \frac{\int_0^{18} 800\, x\, dx}{F}$$

$$= \frac{1}{F} \frac{800x^2}{2} \Big|_0^{18}$$

$$= 9\,ft$$