

Centroids

2D \rightarrow 1D \rightarrow 3D

$$W = \int dw$$

$$\bar{x} = \frac{\int x dw}{W}$$

$$\bar{y} = \frac{\int y dw}{W}$$

$$\bar{z} = \frac{\int z dw}{W}$$

$$dw = \gamma dv$$

$$W = \gamma V$$

$$\bar{x} = \frac{\int x dv}{V}$$

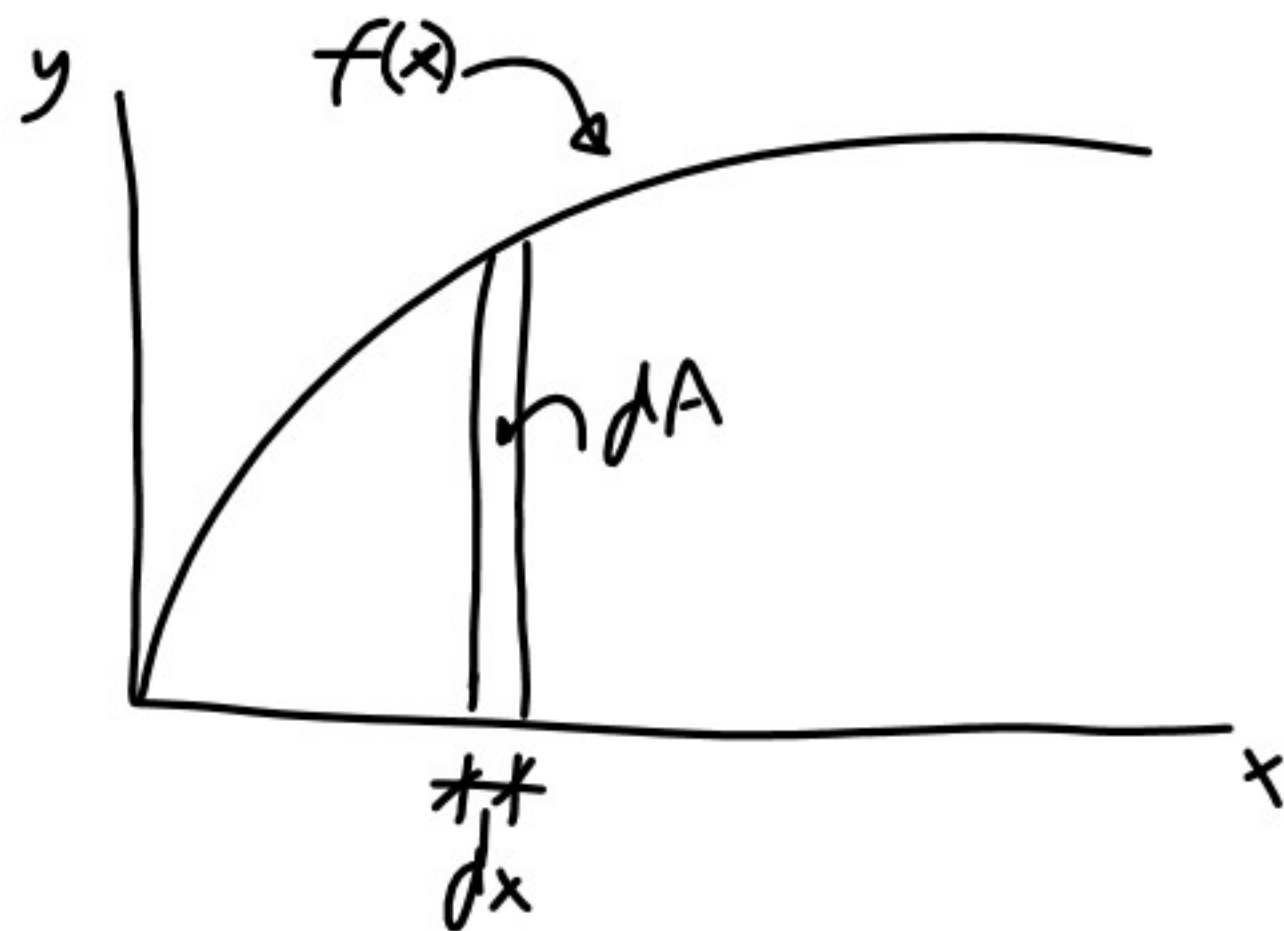
$$\bar{y} = \frac{\int y dv}{V}$$

$$\bar{z} = \frac{\int z dv}{V}$$

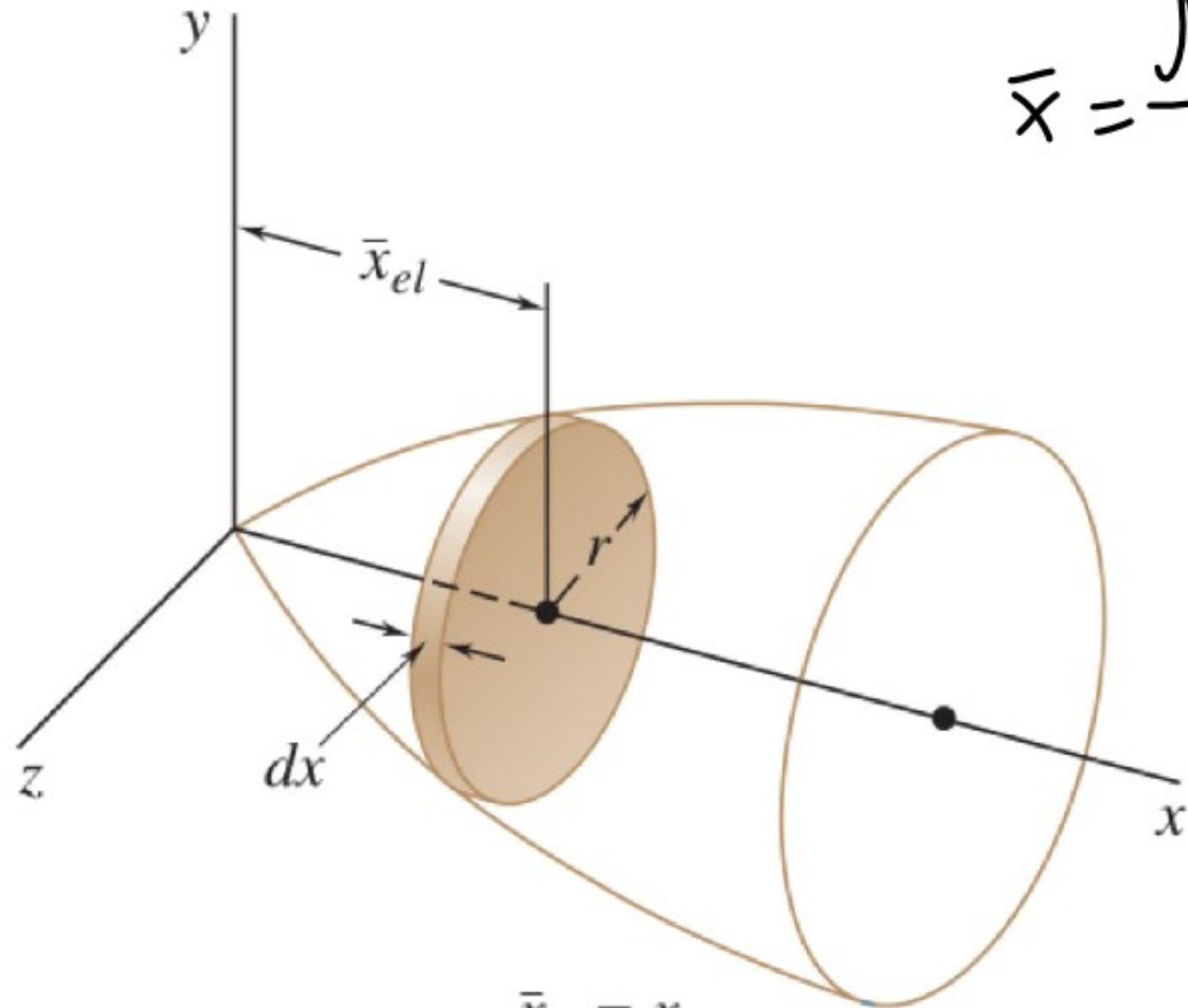
2D

$$A = \int dA$$

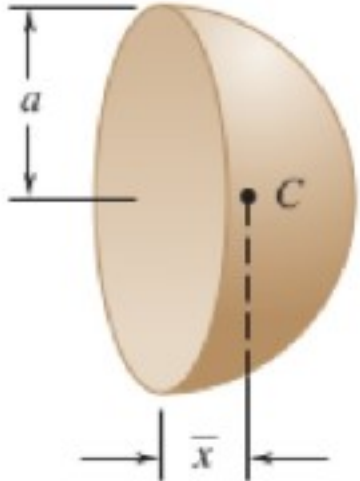
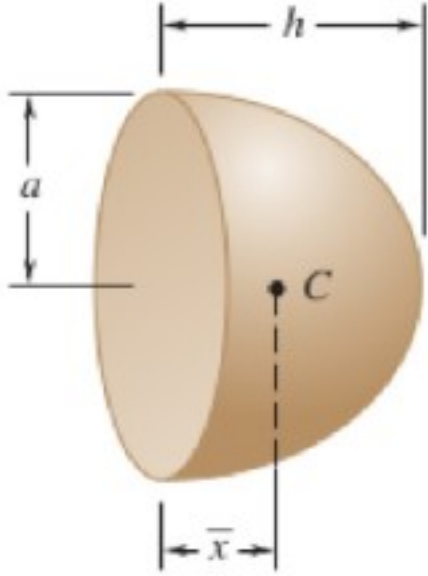
$$\bar{x} = \frac{\int x dA}{A}$$

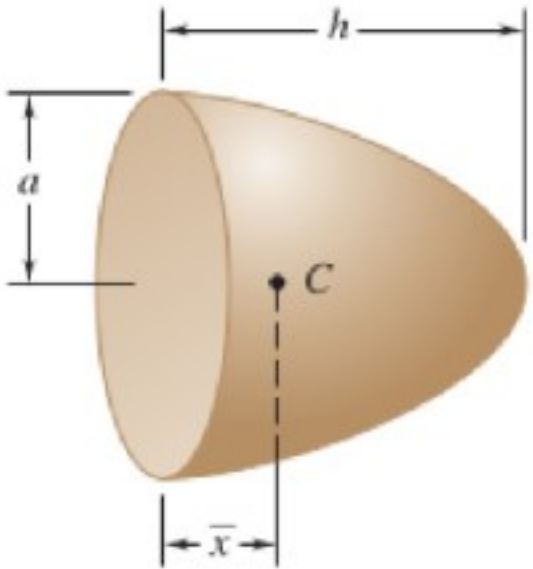
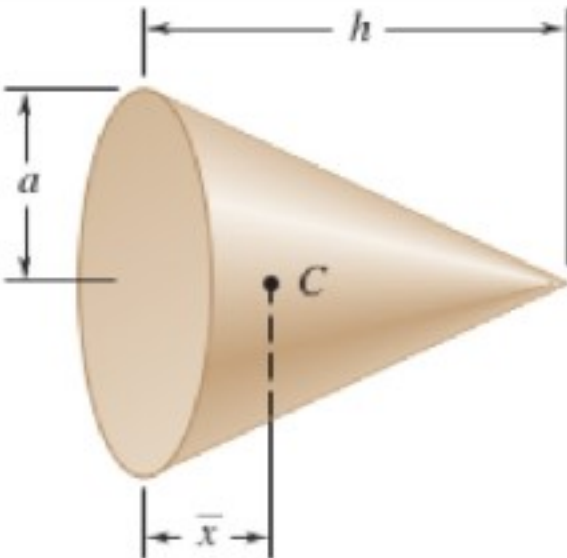
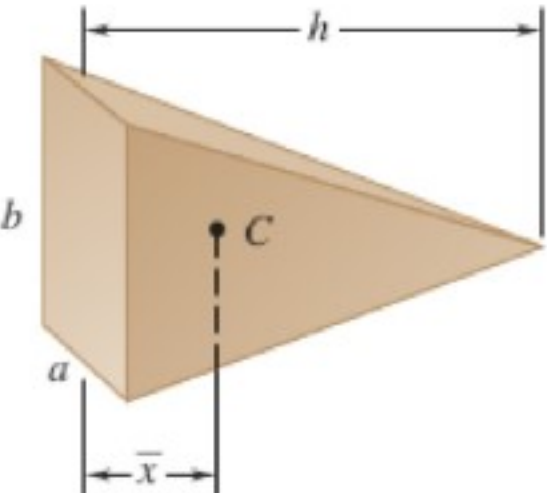


$$\bar{x} = \frac{\int x dV}{V}$$



$$\bar{x}_{el} = x$$
$$\underline{dV = \pi r^2 dx}$$

Shape		\bar{x}	Volume
Hemisphere	 <p>A 3D diagram of a hemisphere. The radius of the flat circular face is labeled as a. The center of mass is marked with a dot and labeled C. A dashed vertical line extends from C to the center of the flat face. The distance from the center of the flat face to C is labeled as \bar{x}.</p>	$\frac{3a}{8}$	$\frac{2}{3}\pi a^3$
Semiellipsoid of revolution	 <p>A 3D diagram of a semiellipsoid of revolution. The radius of the flat circular face is labeled as a. The height of the ellipsoid is labeled as h. The center of mass is marked with a dot and labeled C. A dashed vertical line extends from C to the center of the flat face. The distance from the center of the flat face to C is labeled as \bar{x}.</p>	$\frac{3h}{8}$	$\frac{2}{3}\pi a^2 h$

<p>Paraboloid of revolution</p>		$\frac{h}{3}$	$\frac{1}{2} \pi a^2 h$
<p>Cone</p>		$\frac{h}{4}$	$\frac{1}{3} \pi a^2 h$
<p>Pyramid</p>		$\frac{h}{4}$	$\frac{1}{3} abh$

$$\bar{X} = \frac{\sum_i x_i w_i}{w}$$

$$\bar{y} = \frac{\sum_i y_i w_i}{w}$$

$$\bar{z} = \frac{\sum_i z_i w_i}{w}$$

$$\bar{x} = \frac{\sum_i \bar{x}_i v_i}{v}$$

$$\bar{y} = \frac{\sum_i y_i v_i}{v}$$

$$\bar{z} = \frac{\sum_i \bar{z}_i v_i}{v}$$

For the machine element shown, locate the x coordinate of the center of gravity.

$$\bar{x}_1 = \frac{1}{2} \text{ in}$$

$$\bar{x}_2 = 1 + \frac{3}{2} = 2.5 \text{ in}$$

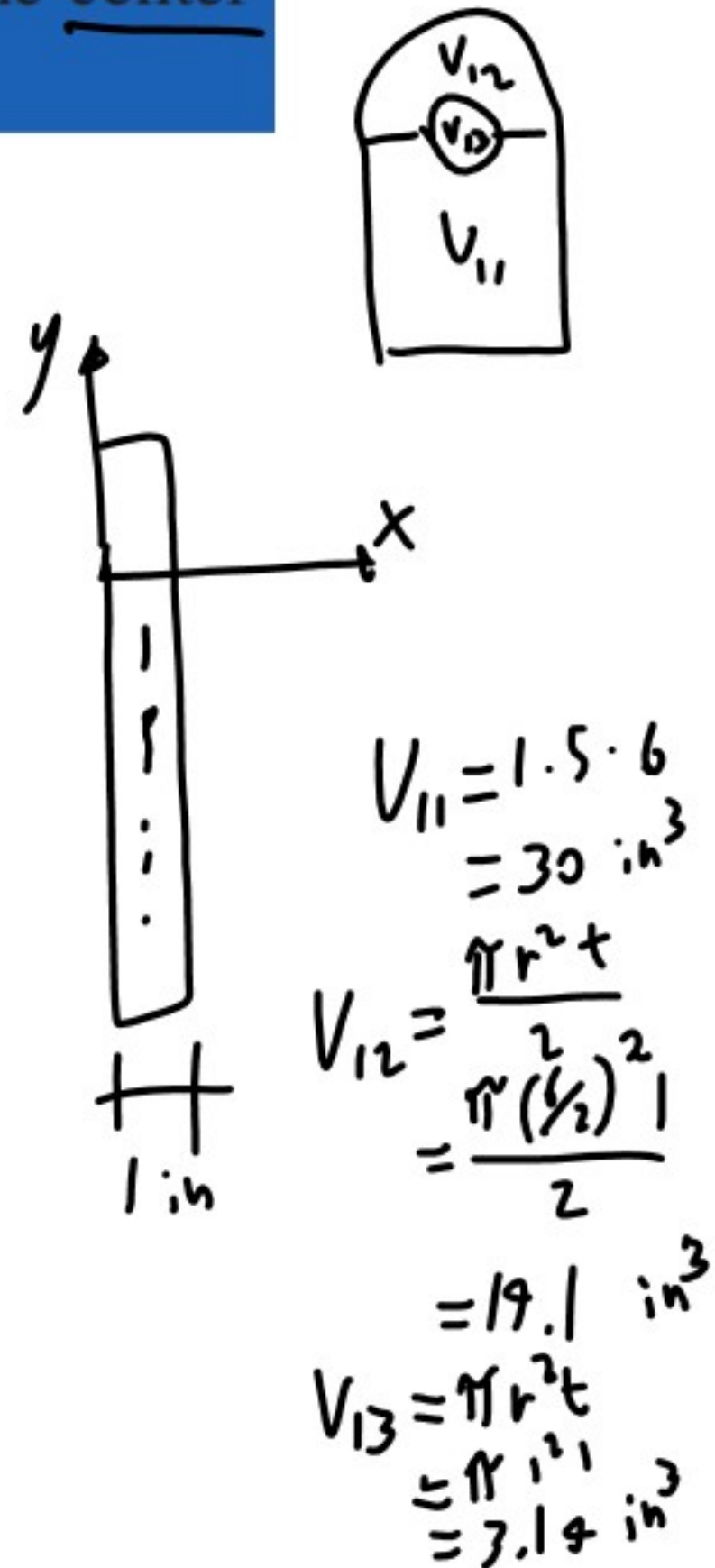
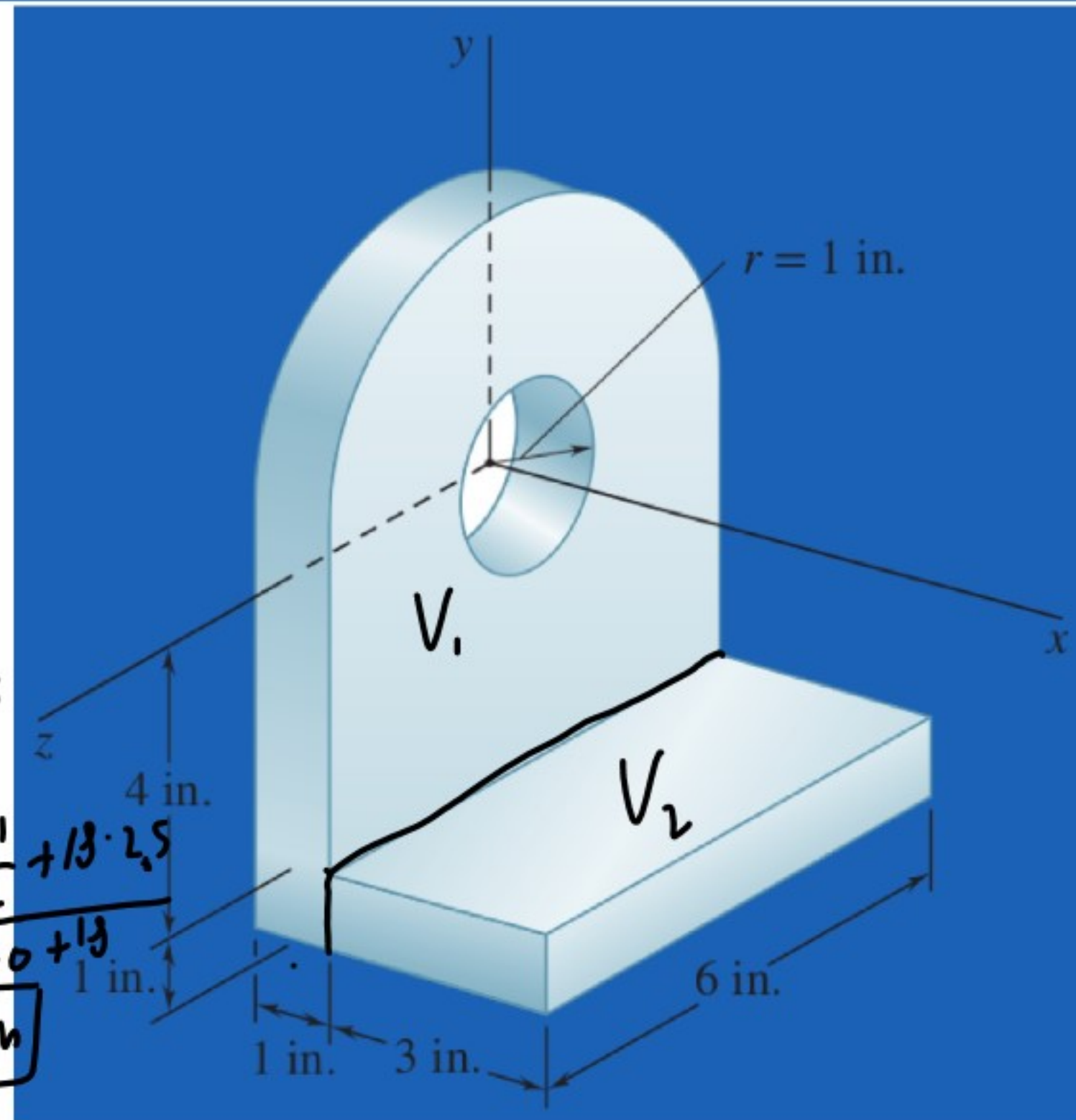
$$V_2 = 1 \cdot 3 \cdot 6 = 18 \text{ in}^3$$

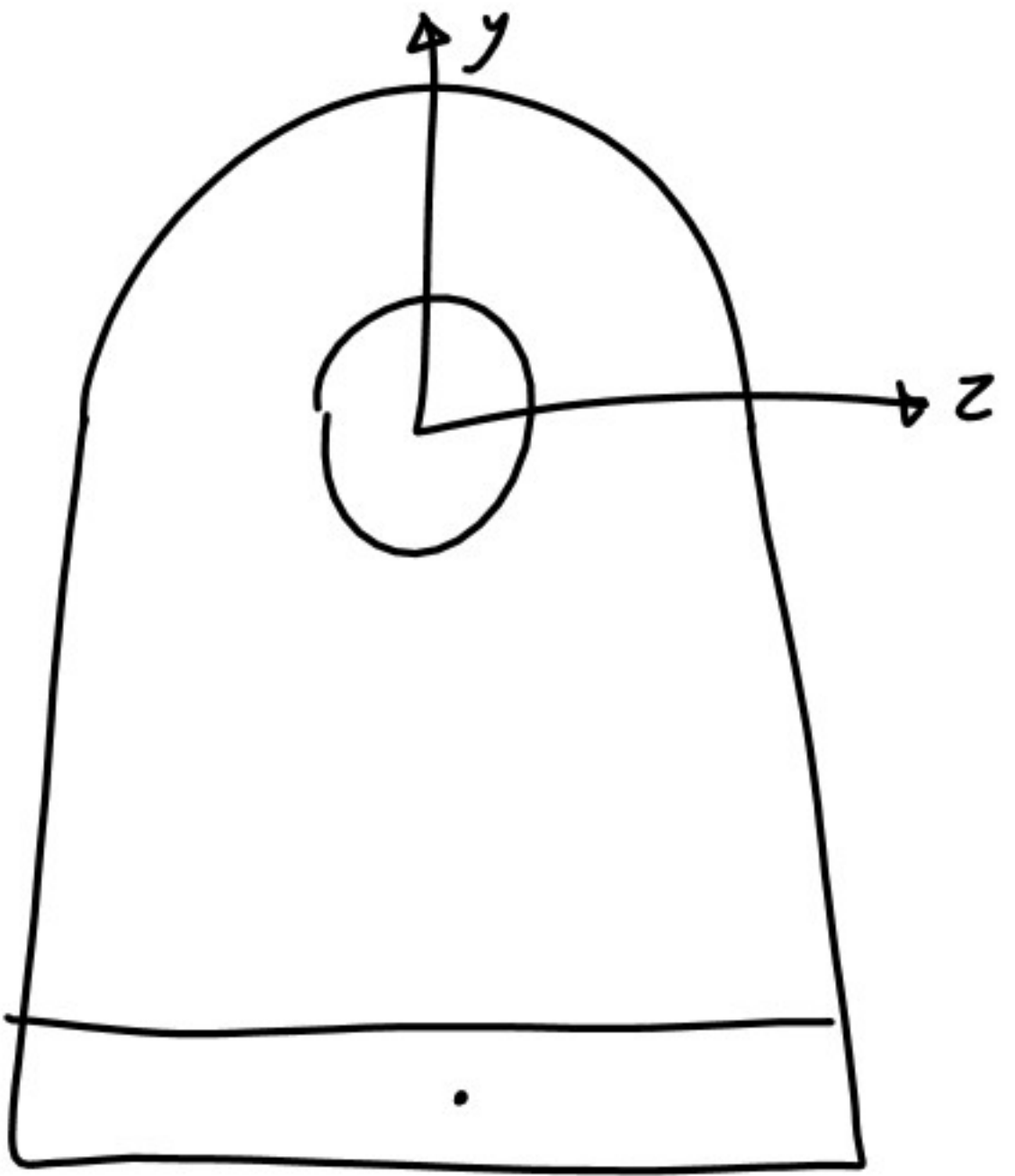
$$V_1 = V_{11} + V_{12} - V_{13}$$

$$= 30 + 19.1 - 3.14 = 40 \text{ in}^3$$

$$\bar{x} = \frac{\bar{x}_1 V_1 + \bar{x}_2 V_2}{V_1 + V_2} = \frac{40 \cdot \frac{1}{2} + 18 \cdot 2.5}{40 + 18}$$

$$= 1.12 \text{ in}$$





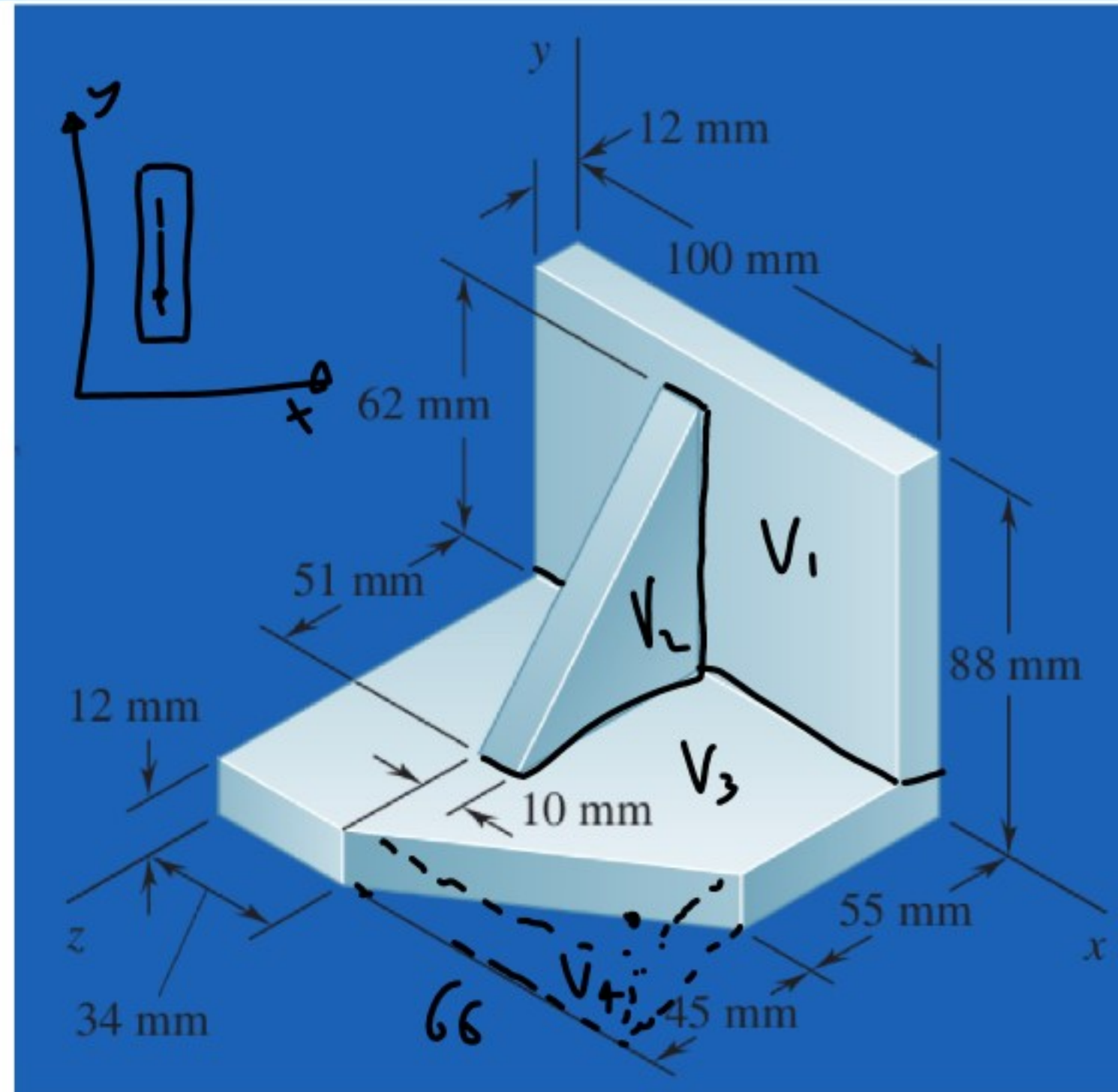
For the stop bracket shown, locate the x coordinate of the center of gravity.

$$V_1 = 91200 \text{ mm}^3$$

$$V_2 = 15310 \text{ mm}^3$$

$$V_3 = 120000 \text{ mm}^3$$

$$V_4 = 17320 \text{ mm}^3$$



$$\bar{x}_1 = 50 \text{ mm}$$

$$\bar{x}_2 = 39 \text{ mm} = 37 + 2$$

$$\bar{x}_3 = 50 \text{ mm}$$

$$\bar{x}_4 = 100 - 22 = 78 \text{ mm}$$

$$\bar{x} = \frac{V_1 \bar{x}_1 + V_2 \bar{x}_2 + V_3 \bar{x}_3 - V_4 \bar{x}_4}{V_1 + V_2 + V_3 - V_4}$$

