Centroids

2D $\longrightarrow$ 1D $\longrightarrow$ 3D

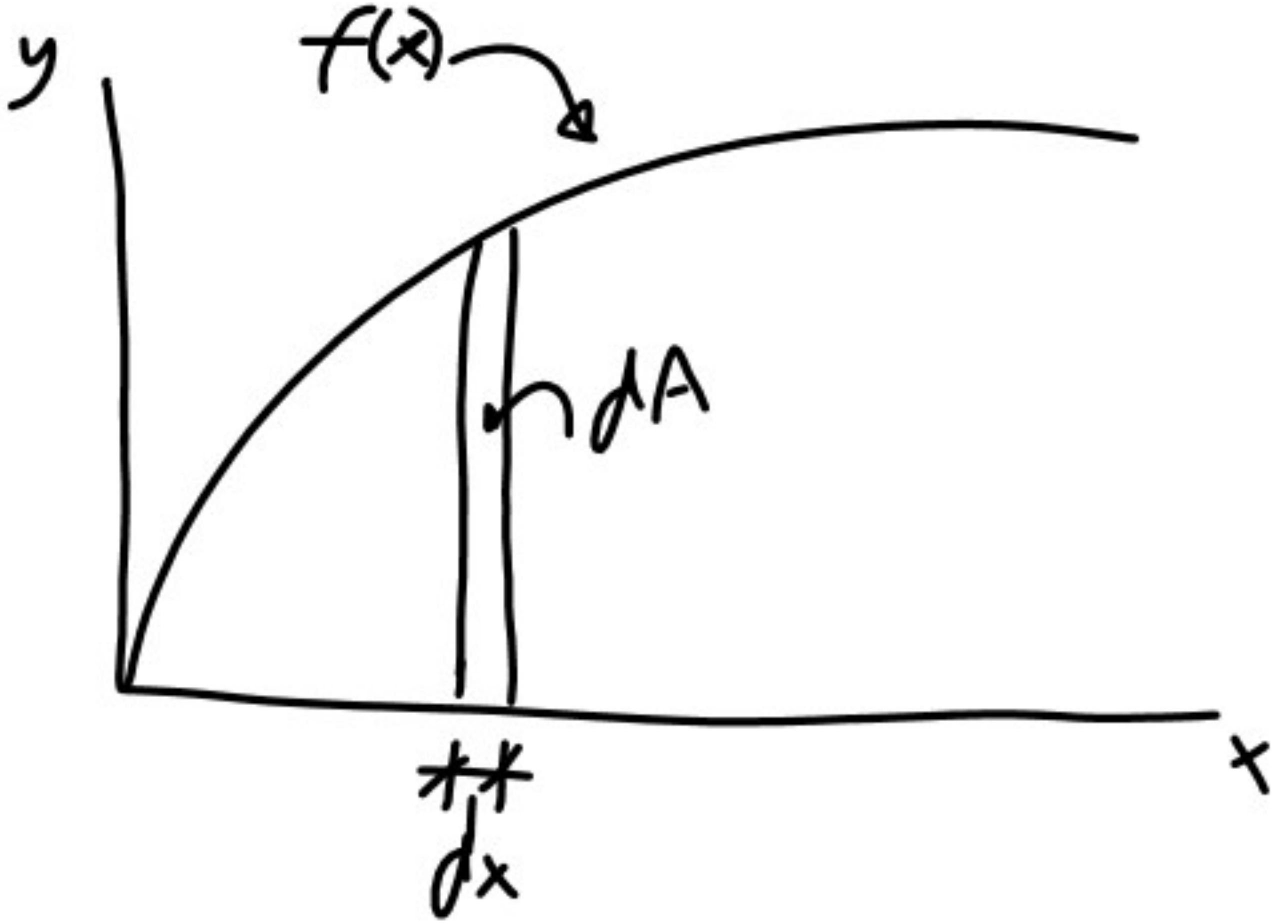
$$W = \int dW \qquad \bar{x} = \frac{\int x\,dW}{W} \qquad \bar{y} = \frac{\int y\,dW}{W} \qquad \bar{z} = \frac{\int z\,dW}{W}$$

$$dW = \gamma\,dV \qquad W = \gamma V$$

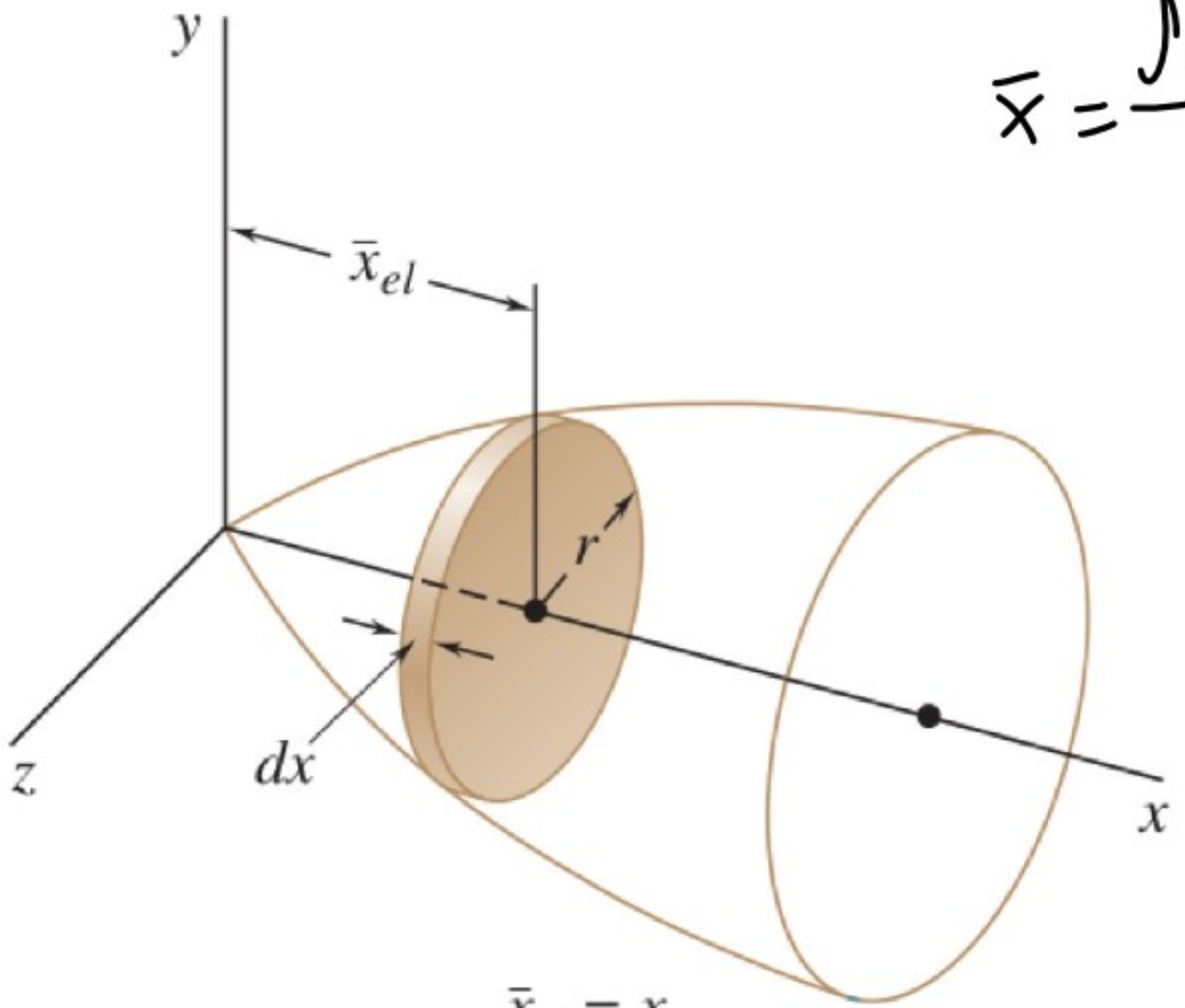
$$\bar{x} = \frac{\int x\,dV}{V} \qquad \bar{y} = \frac{\int y\,dV}{V} \qquad \bar{z} = \frac{\int z\,dV}{V}$$

2D

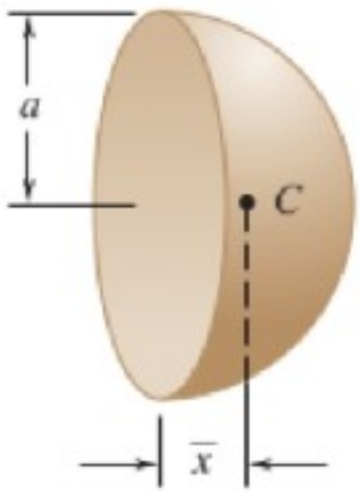
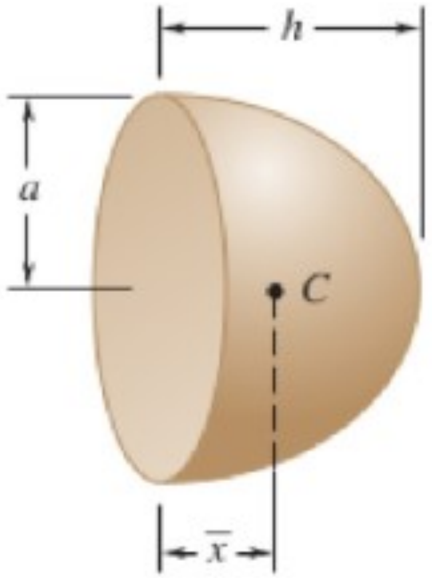
$$A = \int dA \qquad \bar{x} = \frac{\int x\,dA}{A}$$

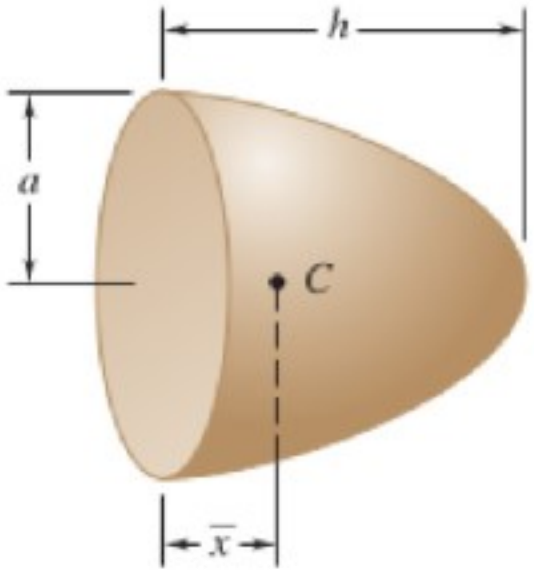
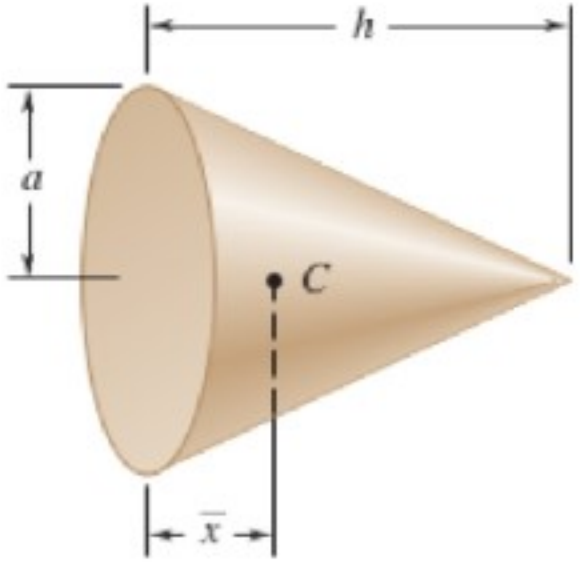
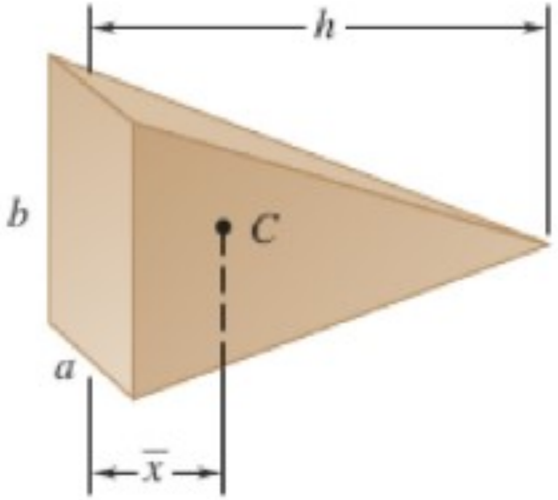


$$\bar{x} = \frac{\int x\,dV}{V}$$



$$\bar{x}_{el} = x$$
$$dV = \pi r^2\,dx$$

Shape		$\bar{x}$	Volume
Hemisphere		$\frac{3a}{8}$	$\frac{2}{3}\pi a^3$
Semiellipsoid of revolution		$\frac{3h}{8}$	$\frac{2}{3}\pi a^2 h$

Paraboloid of revolution		$\frac{h}{3}$	$\frac{1}{2}\pi a^2 h$
Cone		$\frac{h}{4}$	$\frac{1}{3}\pi a^2 h$
Pyramid		$\frac{h}{4}$	$\frac{1}{3} abh$

$$\bar{x} = \frac{\sum_i \bar{x}_i w_i}{W} \qquad \bar{y} = \frac{\sum_i \bar{y}_i w_i}{W} \qquad \bar{z} = \frac{\sum_i \bar{z}_i w_i}{W}$$

$$\bar{x} = \frac{\sum_i \bar{x}_i V_i}{V} \qquad \bar{y} = \frac{\sum_i \bar{y}_i V_i}{V} \qquad \bar{z} = \frac{\sum_i \bar{z}_i V_i}{V}$$

For the machine element shown, locate the *x coordinate* of the *center of gravity*.

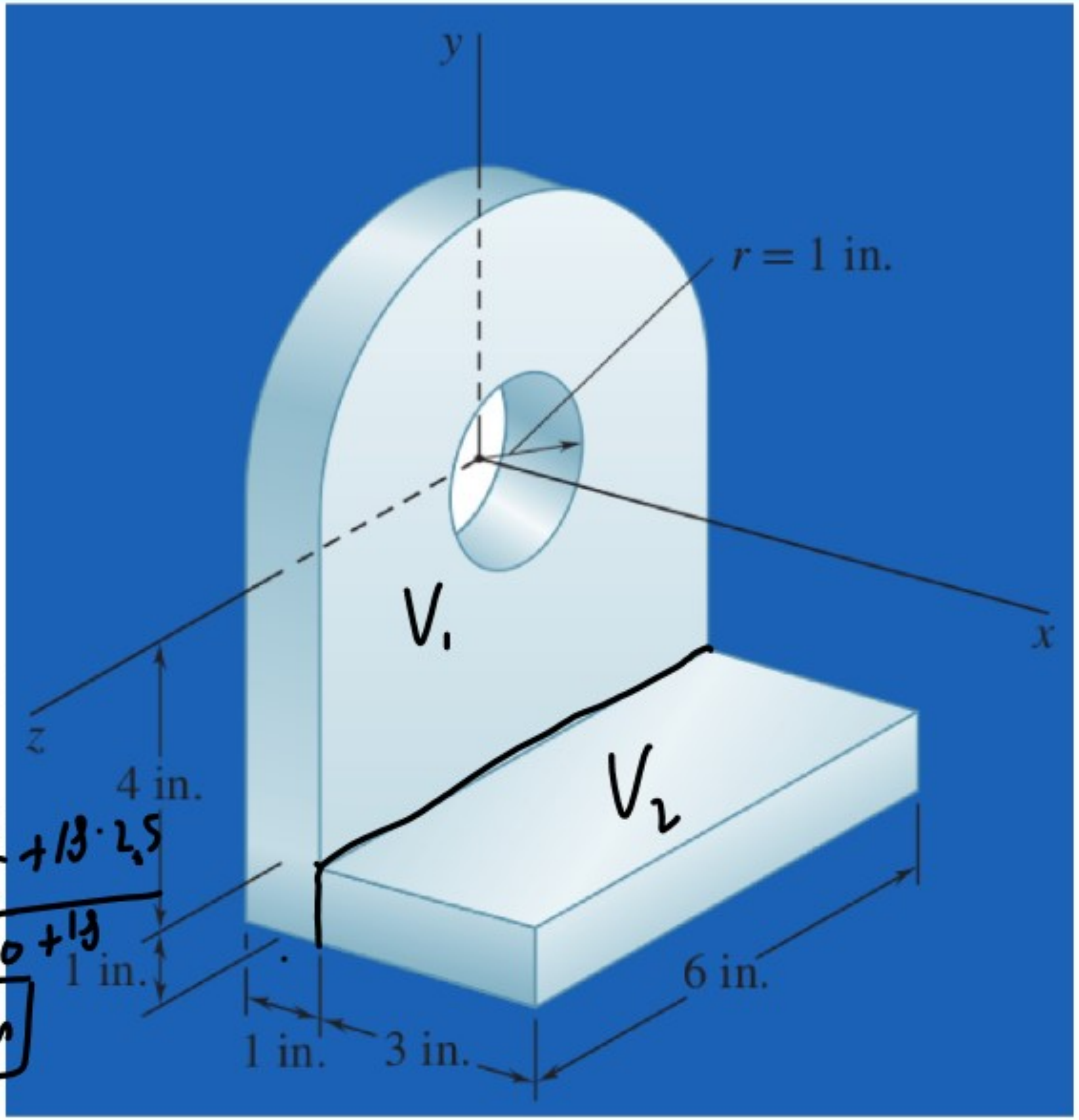
$$\bar{x}_1 = \tfrac{1}{2}\ \text{in}$$

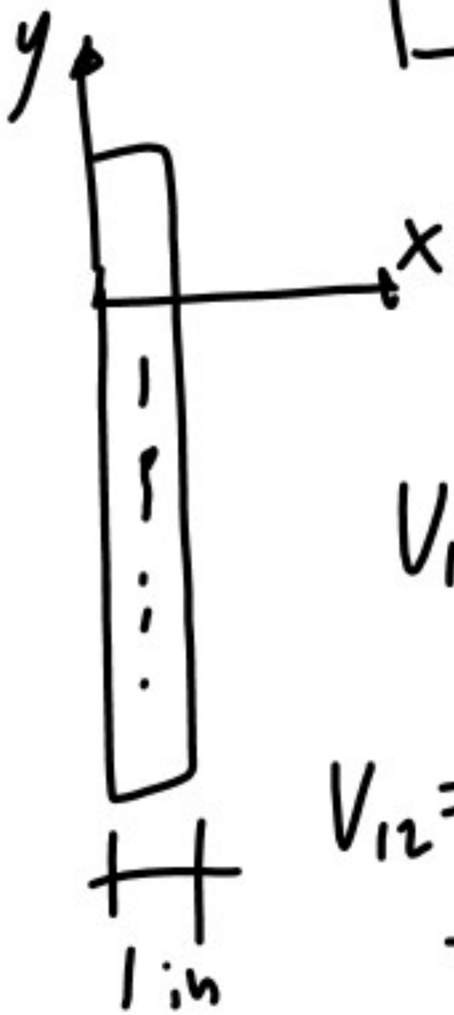
$$\bar{x}_2 = 1 + \tfrac{3}{2} = 2.5\ \text{in}$$

$$V_2 = 1 \cdot 3 \cdot 6 = 18\ \text{in}^3$$

$$V_1 = V_{11} + V_{12} - V_{13} = 30 + 19.1 - 3.14 = 40\ \text{in}^3$$

$$\bar{x} = \frac{\bar{x}_1 V_1 + \bar{x}_2 V_2}{V_1 + V_2} = \frac{40 \cdot \frac{1}{2} + 18 \cdot 2.5}{40 + 18} = 1.12\ \text{in}$$

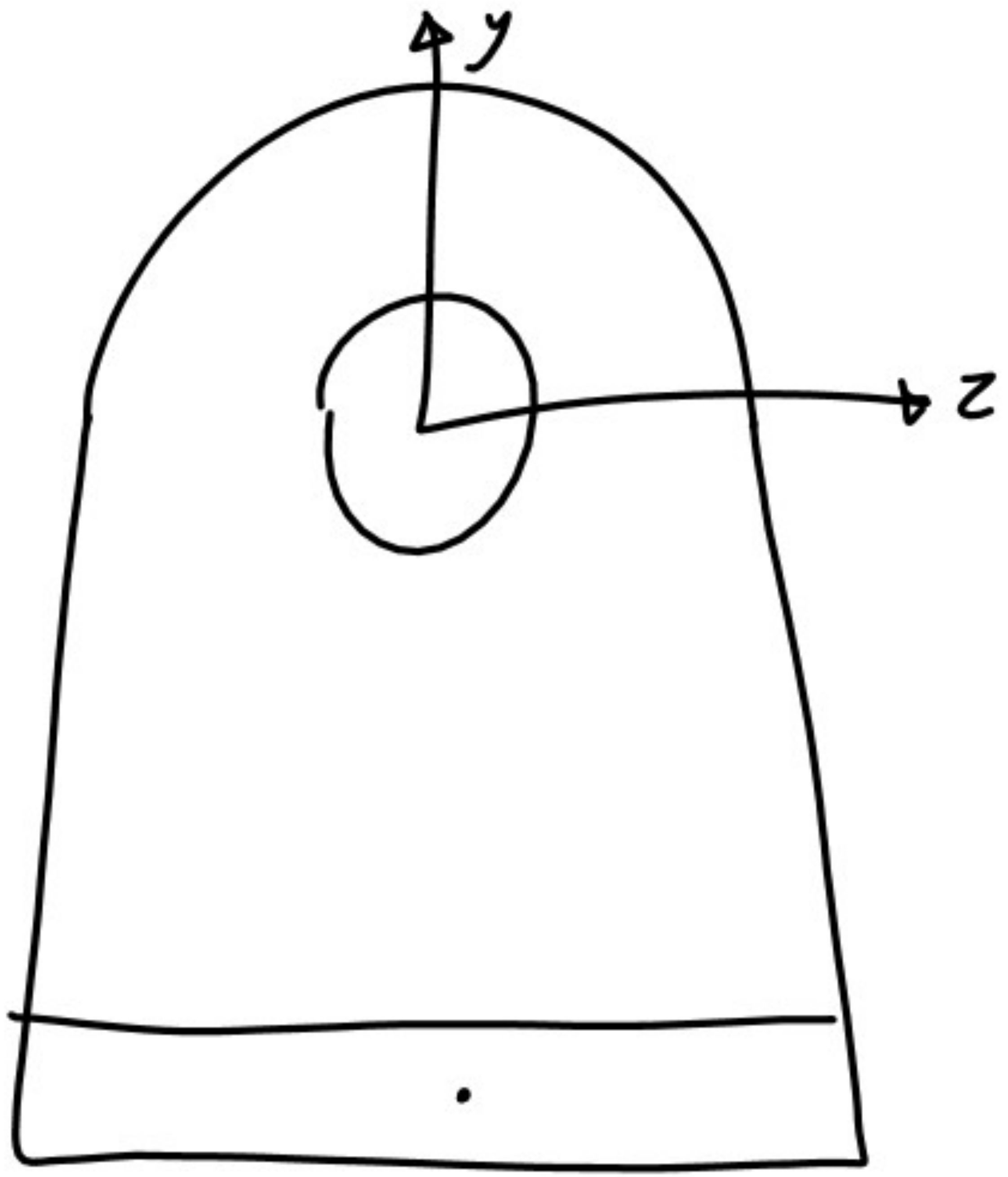




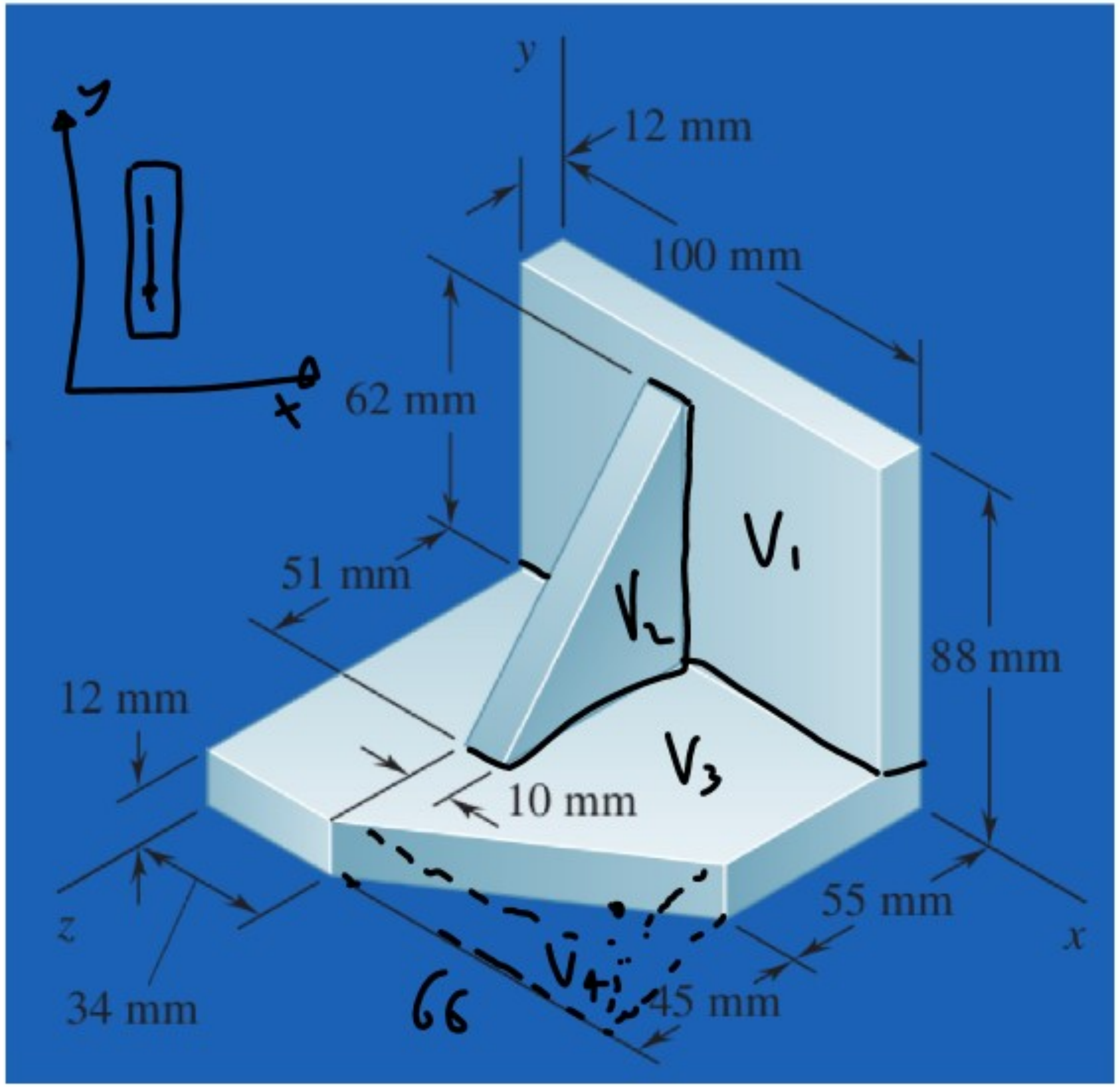
$$V_{11} = 1 \cdot 5 \cdot 6 = 30\ \text{in}^3$$

$$V_{12} = \frac{\pi r^2 t}{2} = \frac{\pi (5/2)^2\, 1}{2} = 19.1\ \text{in}^3$$

$$V_{13} = \pi r^2 t = \pi\, 1^2\, 1 = 3.14\ \text{in}^3$$


y
z

For the stop bracket shown, locate the *x* coordinate of the center of gravity.



$V_1 = 91200 \text{ mm}^3$

$V_2 = 15810 \text{ mm}^3$

$V_3 = 120000 \text{ mm}^3$

$V_4 = 17820 \text{ mm}^3$

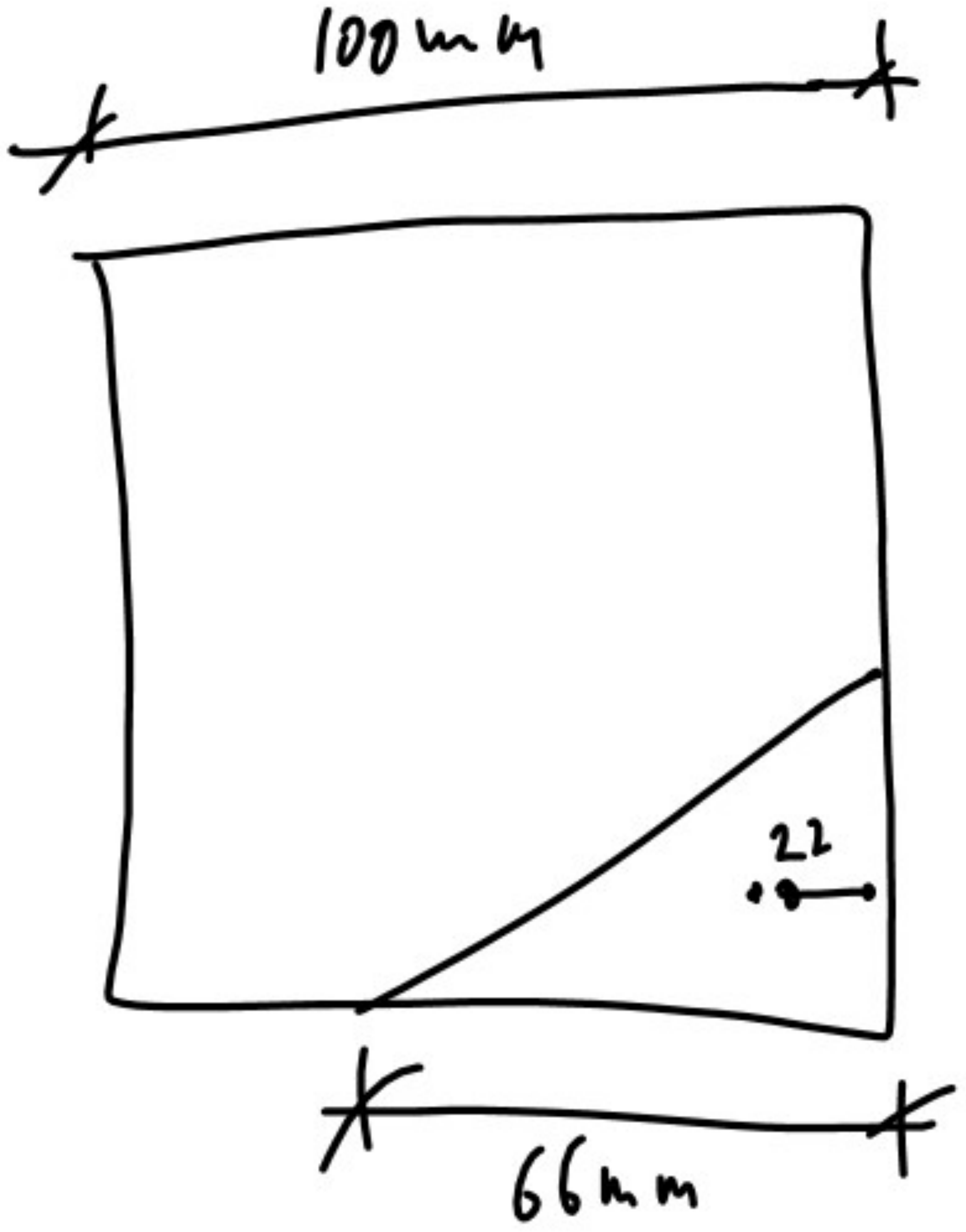
$\bar{x}_1 = 50 \text{ mm}$

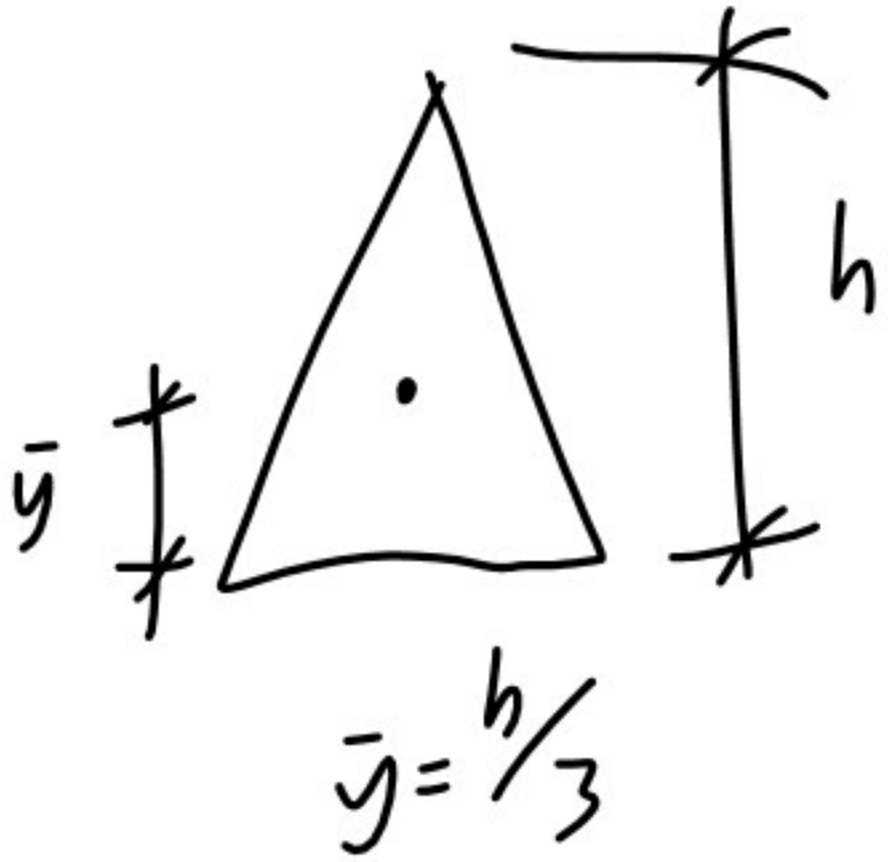
$\bar{x}_2 = 39 \text{ mm} = 34 + 5$

$\bar{x}_3 = 50 \text{ mm}$

$\bar{x}_4 = 100 - 22 = 78 \text{ mm}$

$$\bar{x} = \frac{V_1\bar{x}_1 + V_2\bar{x}_2 + V_3\bar{x}_3 - V_4\bar{x}_4}{V_1 + V_2 + V_3 - V_4}$$


100mm
22
66mm


h
ȳ
ȳ = h/3