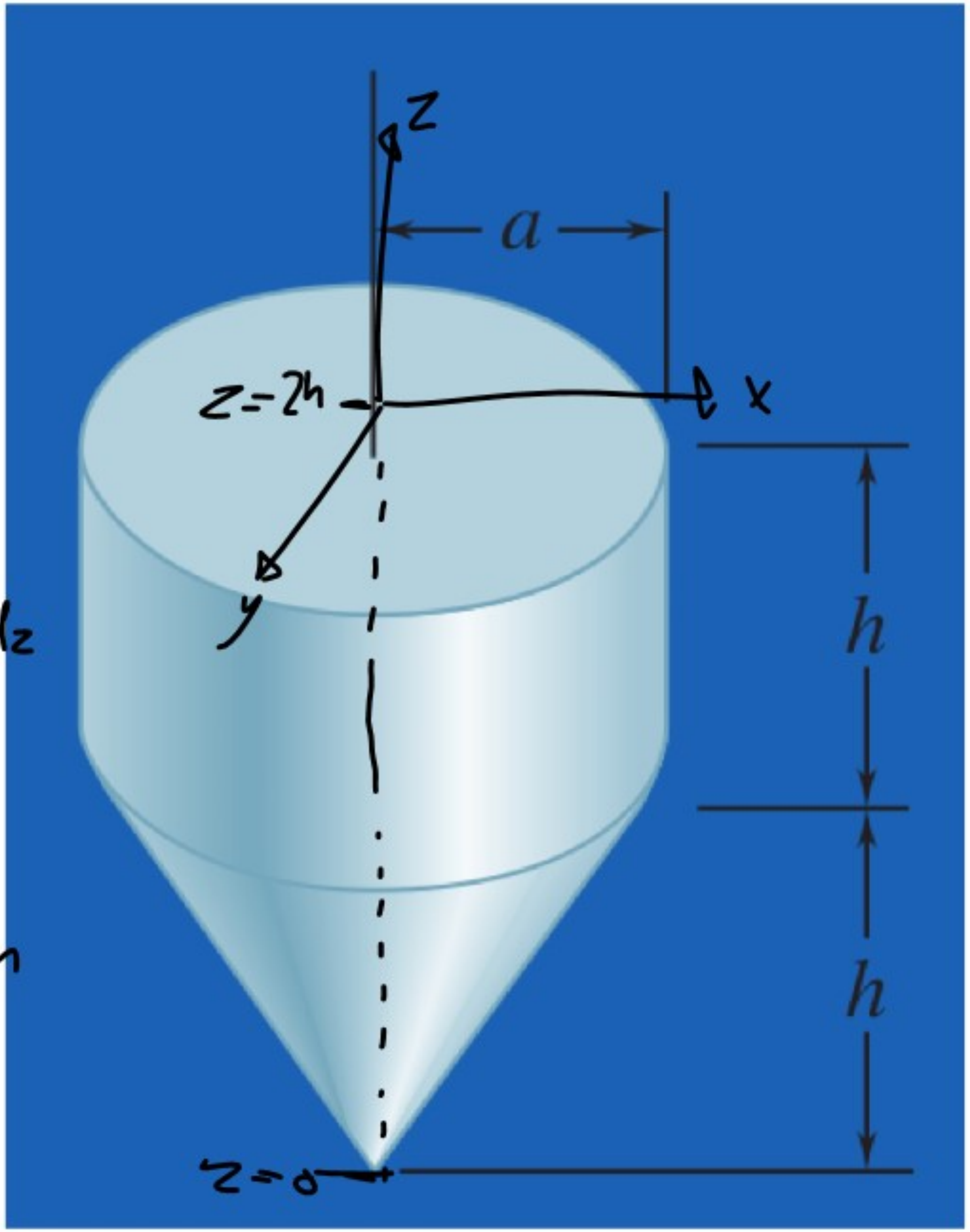
$$V = \int_0^{2h} dV$$

$$= \int_0^{2h} \pi r^2(z)\, dz$$

$$= \int_0^{h} \pi \left(\frac{az}{h}\right)^2 dz + \int_h^{2h} \pi a^2 dz$$

$$= \frac{\pi a^2}{h^2}\int_0^{h} z^2 dz + \pi a^2 \int_h^{2h} dz$$

$$= \frac{\pi a^2}{h^2}\frac{z^3}{3}\Big|_0^h + \pi a^2 z\Big|_h^{2h}$$



$$\bar{x} = \bar{y} = 0$$

$$dV = \pi r^2(z)\, dz$$

$$r(z) = \begin{cases} a\,z/h & z < h \\ a & \text{otherwise} \end{cases}$$

$$V = \frac{\pi a^2 \cancel{h^3}}{3\cancel{h^2}} + \pi a^2 2h - \pi a^2 h$$

$$= \frac{\pi a^3 h}{3} + \pi a^2 h$$

$$\bar{z} = \frac{\int z\,dV}{V} = \frac{1}{V}\int_0^{2h} z\pi r^2(z)\,dz = \frac{1}{V}\Bigg(\int_0^h z\pi\left(\frac{az}{h}\right)^2 dz + \int_h^{2h} z\pi a^2\,dz$$

$$= \frac{1}{V}\left(\frac{\pi a^2}{h^2}\int_0^h z^3\,dz + \pi a^2\int_h^{2h} z\,dz\right) = \frac{1}{V}\left(\frac{\pi a^2}{h^2}\,\frac{z^4}{4}\Bigg|_0^h + \pi a^2\,\frac{z^2}{2}\Bigg|_h^{2h}\right)$$

$$= \frac{1}{V}\left(\frac{\pi a^2 h^4}{4h^2} + \frac{\pi a^2 4h^2}{2} - \frac{\pi a^2 h^2}{2}\right) = \frac{\pi a^2}{V}\left(\cancel{\frac{h^2}{2}} + 2h^2 - \cancel{\frac{h^2}{2}}\right) = \frac{2\pi h^2 a^2}{V}$$

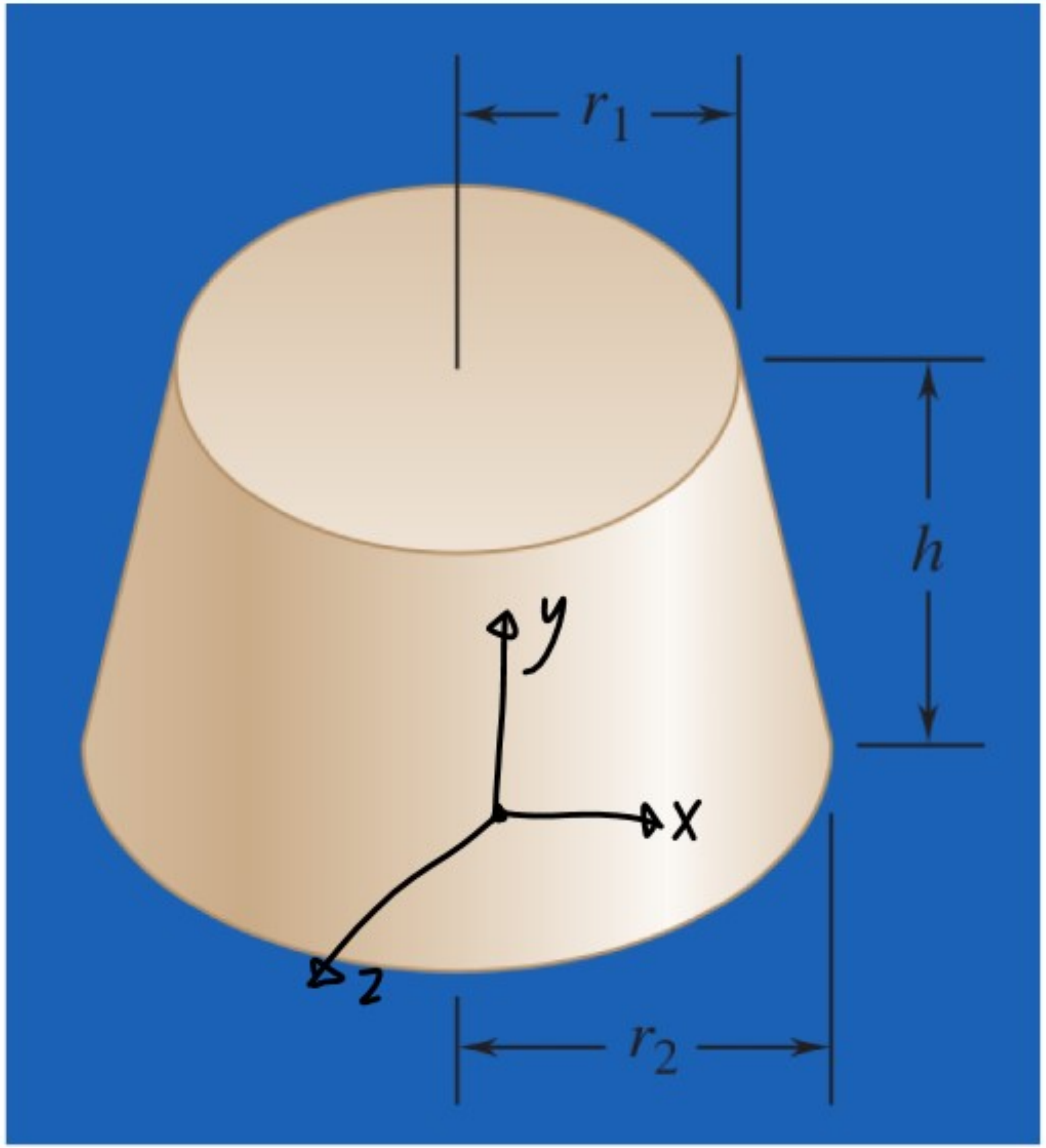
$$\bar{z} = \frac{2\pi a^2 h^2}{V} = \frac{2\cancel{\pi}\cancel{a^2} h^{\cancel{2}}}{\frac{\cancel{\pi} a^{3}\cancel{h}}{3} + \cancel{\pi}\cancel{a^2}\cancel{h}} = \frac{2h}{a/3 + 1} = \boxed{\frac{6h}{a+3}}$$

$dV = \pi r^2(y)\,dy$

Locate the centroid of the frustum of a right circular cone when $r_1 = 40$ mm, $r_2 = 50$ mm, and $h = 60$ mm.

$$V = \int dV$$

$$\bar{y} = \frac{\int y\,dV}{V}$$



$\bar{z} = \bar{x} = 0$

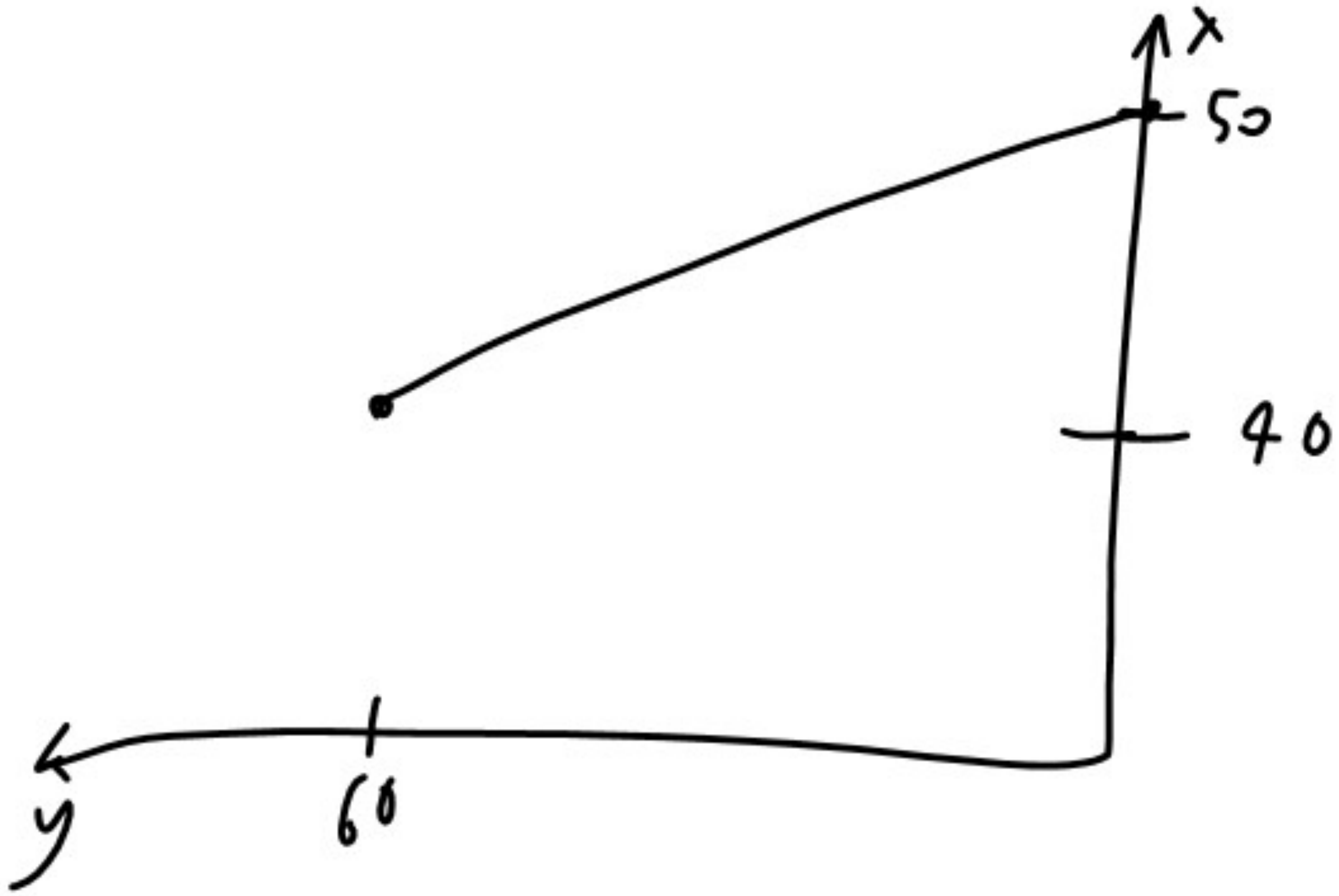
$$r(y) = \frac{-1}{6}y + 50$$

$r(0) = 50$

$$r(60) = \frac{-1}{6}60 + 50 = 40$$

$$V = \int_0^{60} \pi \left(\frac{-1}{6}y + 50\right)^2 dy$$

$$V = \int_0^{60} \pi \left(\frac{y^2}{36} - \frac{50}{3}y + 2500\right) d$$



$x = my + b$

$y = 0 \quad x = 50$

$y = 60 \quad x = 40$

$50 = 0m + b$

$b = 50$

$x = my + 50$

$40 = 60m + 50$

$-10 = 60m$

$\frac{-10}{60} = m$

$-1/6 = m$

$x = -1/6\, y + 50$