6.77 For the frame and loading shown, determine the force acting on member *ABC* (*a*) at *B*, (*b*) at *C*.

$\Sigma M_C = 0$

$90 \cdot 200 - 240 \cdot F_{BDy} = 0$

$F_{BDy} = 75\ N$

$F_{BD} = \frac{F_{BDy}}{\lambda_y} = \frac{75}{-90/150} = \boxed{-125\ N}$

$\lambda_{BD} = \frac{120}{150} i - \frac{90}{150} j$

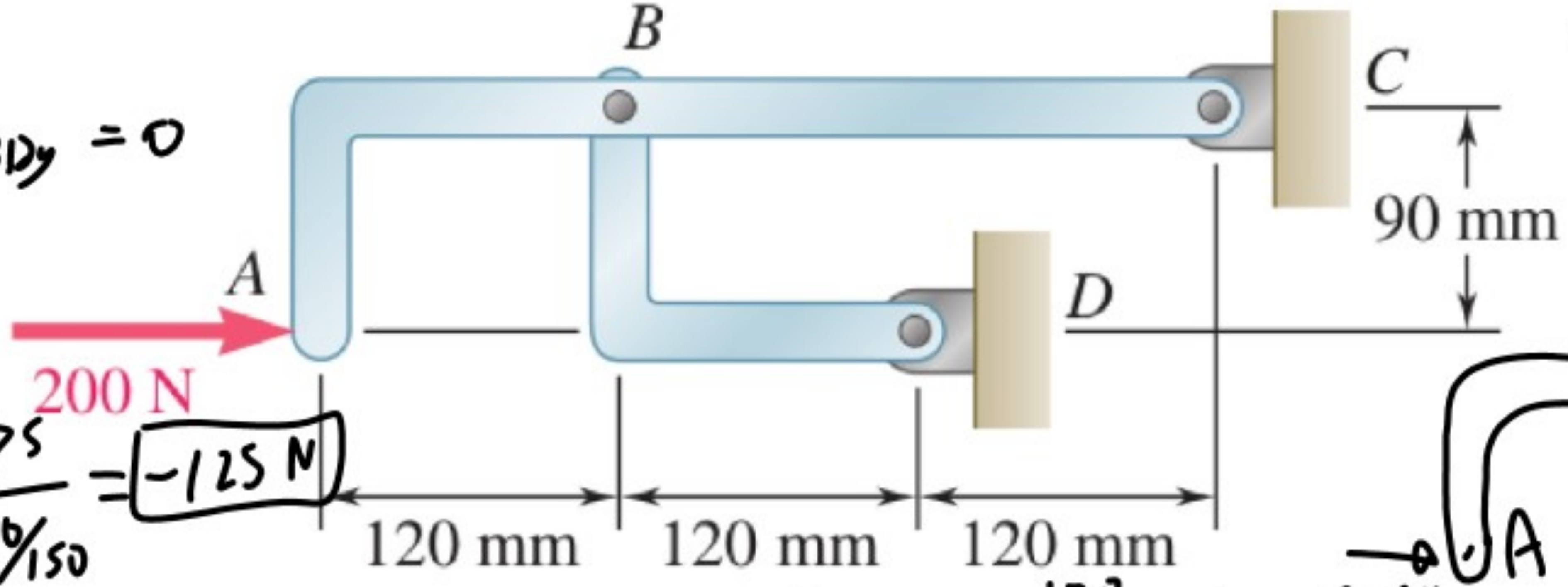
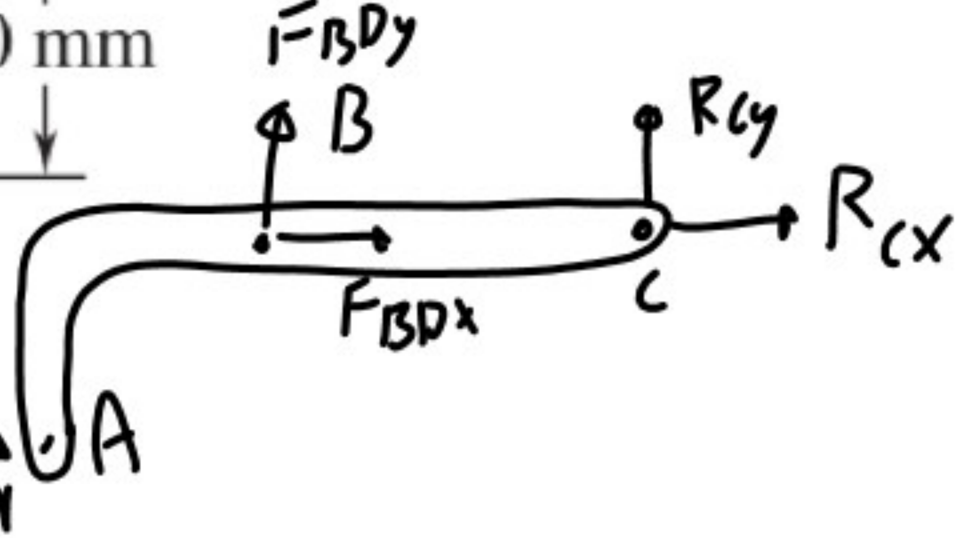


Fig. P6.77



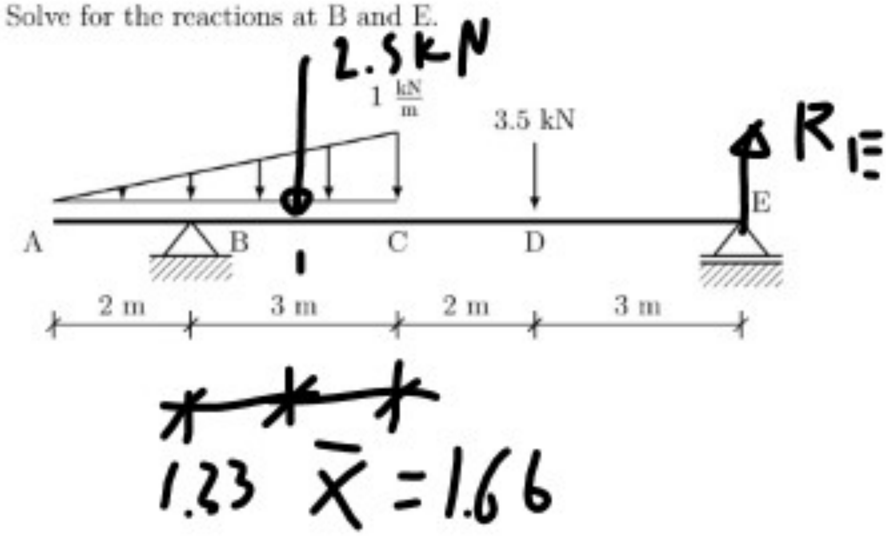
$F_{BDx} = F_{BD} \lambda_x = -125 \frac{120}{150} = -100$

Open notes, calculators allowed, centroid table provided on the last page. Work neatly and clearly mark your answers, partial credit may be given. If you run out of room for an answer, continue on the back of the page.

Name: ______________________

Question:	1	2	3	Total
Points:	30	25	45	100
Score:				

1. (30 points) Solve for the reactions at B and E.



$\sum M_B = 0$

$-1.33 \cdot 2.5 - 5 \cdot 3.5 + 8 R_E = 0$

$R_E = 2.6$ kN

$\sum F_y = 0$

$R_{By} + R_E - 2.5 - 3.5 = 0$

$R_{By} = 5 - R_E = 5 - 2.6 = 3.4$ kN

$R_{Bx} = 0$

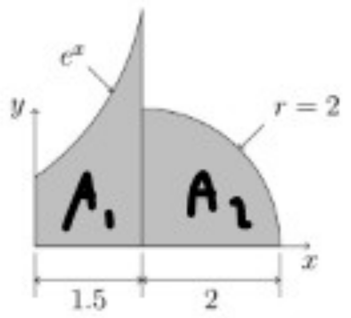
$1 \frac{kN}{m} \, 5m \, \frac{1}{2} = 2.5$ kN

$\bar{x} = 5/3 = 1\,2/3 = 1.66$

$3 - \bar{x} = 3 - 5/3 = 9/3 - 5/3 = 4/3$

$= 1.33$

2. (25 points) Find the x location of the centroid, $\bar{x}$, of the shape show below.
Hint: $\frac{d}{dx}(x-1)e^x = xe^x$.



$$dA = e^x dx$$

$$A_1 = \int_0^{1.5} dA = \int_0^{1.5} e^x dx$$

$$= e^x \Big|_0^{1.5} = e^{1.5} - e^0 = 4.48 - 1 = 3.48$$

$$\bar{x}_1 = \frac{\int x\,dx}{A_1} = \frac{1}{3.48}\int_0^{1.5} xe^x dx = \frac{1}{3.48}(x-1)e^x \Big|_0^{1.5}$$

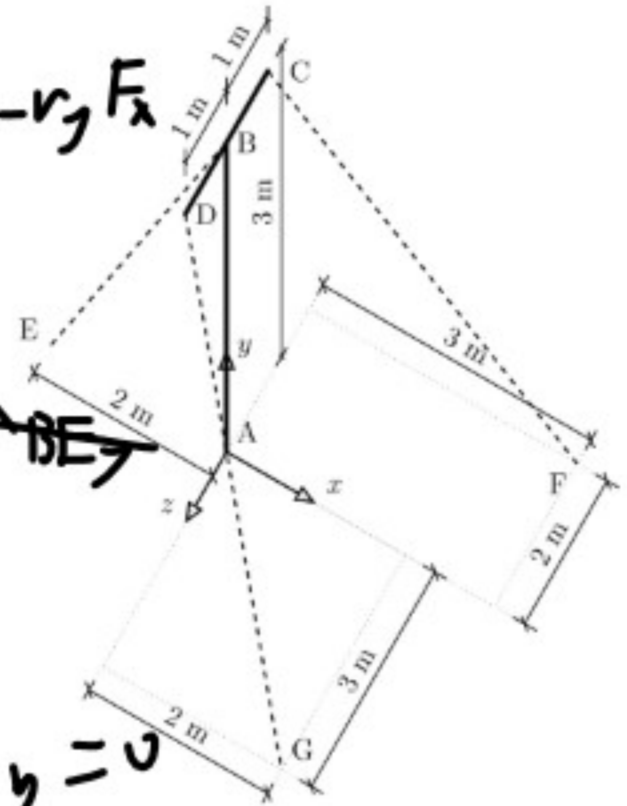
$$= \frac{1}{3.48}\left((1.5-1)e^{1.5} - (0-1)e^0\right) = 0.93$$

$$A_2 = \frac{\pi r^2}{4} = \frac{\pi 2^2}{4} = \pi$$

$$\bar{x}_2 = \frac{4r}{3\pi} + 1.5 = \frac{4 \cdot 2}{3\pi} + 1.5$$
$$= 2.35$$

$$\bar{x} = \frac{A_1\bar{x}_1 + A_2\bar{x}_2}{A_1 + A_2} = \frac{3.48 \cdot 0.93 + \pi\, 2.35}{3.48 + \pi} = \boxed{1.6}$$

3. (45 points) Given the T shaped structure show below, find the tension in cables BE and DG if the tension in cable CF is 120 N.



$\Sigma M_A^x = 0$

$M_x = r_y F_z - r_z F_y$ $\quad M_z = r_x F_y - r_y F_x$

$\vec{AB} = 3j$

$\vec{AC} = 3j - 1k$

$\vec{AD} = 3j + 1k$

$\lambda_{BE} = \frac{-2}{3.6} i + \frac{3}{3.6} j$

$\lambda_{CF} = \frac{3}{4.7} i - \frac{3}{4.7} j - \frac{2}{4.7} k$

$\lambda_{DG} = \frac{2}{4.7} i - \frac{3}{4.7} j + \frac{3}{4.7} k$

$\Sigma M_A^x =$ ~~$AB_y T_{BE} \lambda_{BE_z} - AB_z T_{BE} \lambda_{BE_y}$~~

$+ AC_y T_{CF} \lambda_{CF_z} - AC_z T_{CF} \lambda_{CF_y}$

$+ AD_y T_{DG} \lambda_{DG_z} - AD_z T_{DG} \lambda_{DG_y} = 0$

$3(120)\left(\frac{-2}{4.7}\right) + 1(120)\left(\frac{-3}{4.7}\right) + 3 T_{DG} \frac{3}{4.7} - 1 T_{DG}\left(\frac{-3}{4.7}\right) = 0$

$T_{DG} = 180\ N$

$\Sigma M_A^z =$ ~~$AB_x T_{BE} \lambda_{BE_y}$~~ $- AB_y T_{BE} \lambda_{BE_x}$

$+$ ~~$AC_x T_{CF} \lambda_{CF_y}$~~ $- AC_y T_{CF} \lambda_{CF_x}$

$+$ ~~$AD_x T_{DG}$~~ $\lambda_{DG_y} - AD_y T_{DG} \lambda_{DG_x} = 0$

$-3 T_{BE}\left(\frac{-2}{3.6}\right) - 3(120)\frac{3}{4.7} - 3(180)\frac{2}{4.7} = 0$

$T_{BE} = 230\ N$

of symmetry does not necessarily possess a center of symmetry (Fig. 5.6*a*). However, if a figure possesses two axes of symmetry at right angles to each other, the point of intersection of these axes is a center of symmetry (Fig. 5.6*b*).

Determining the centroids of unsymmetrical areas and lines and of areas and lines possessing only one axis of symmetry will be discussed in the next section. Centroids of common shapes of areas and lines are shown in Fig. 5.8A and B.

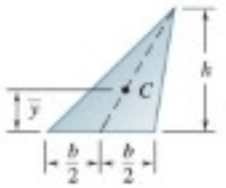
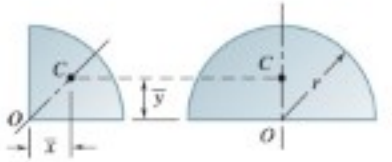
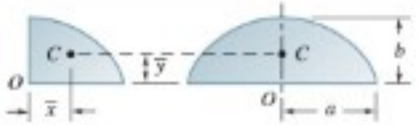
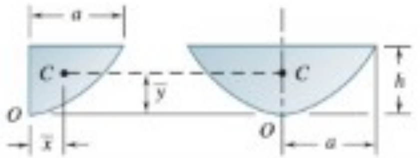
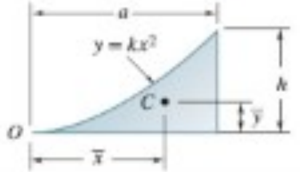
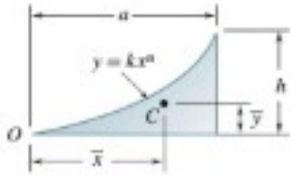
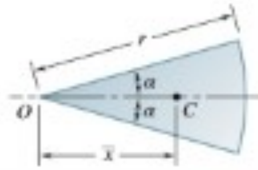
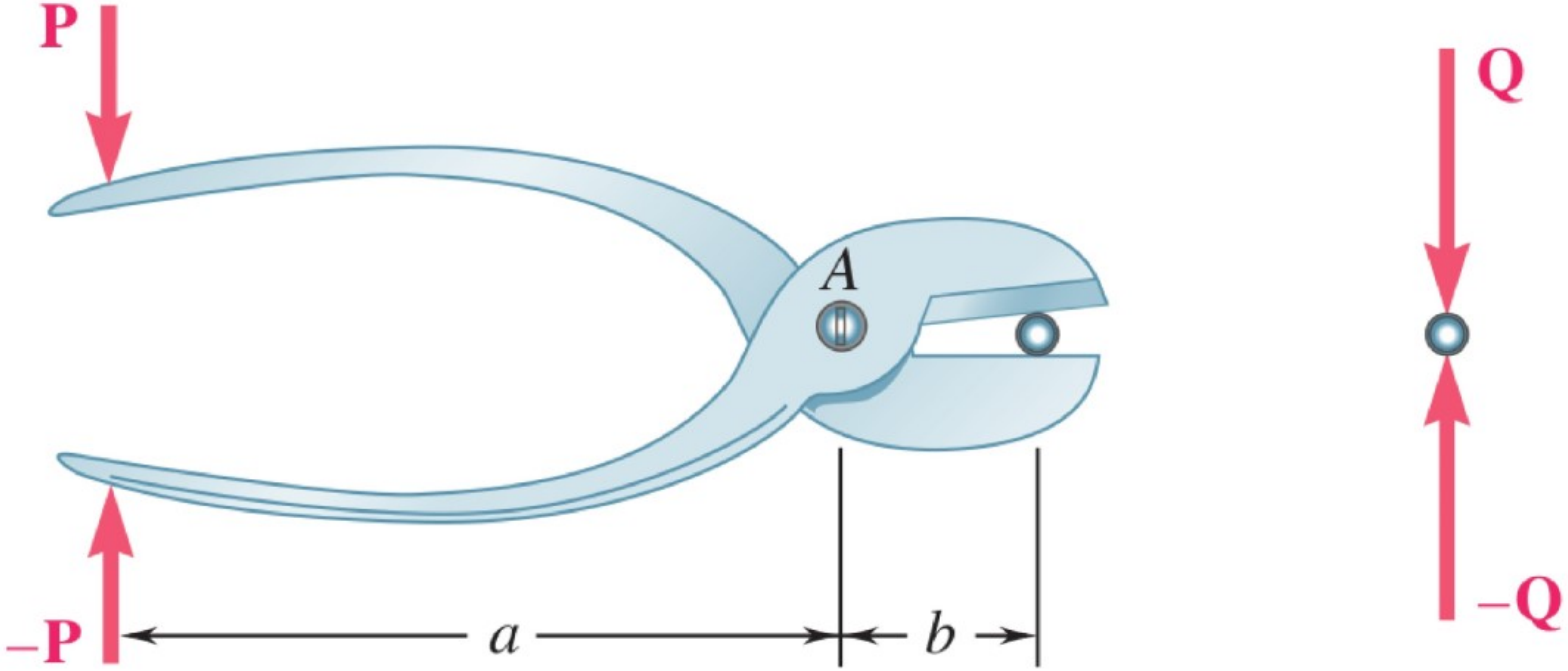
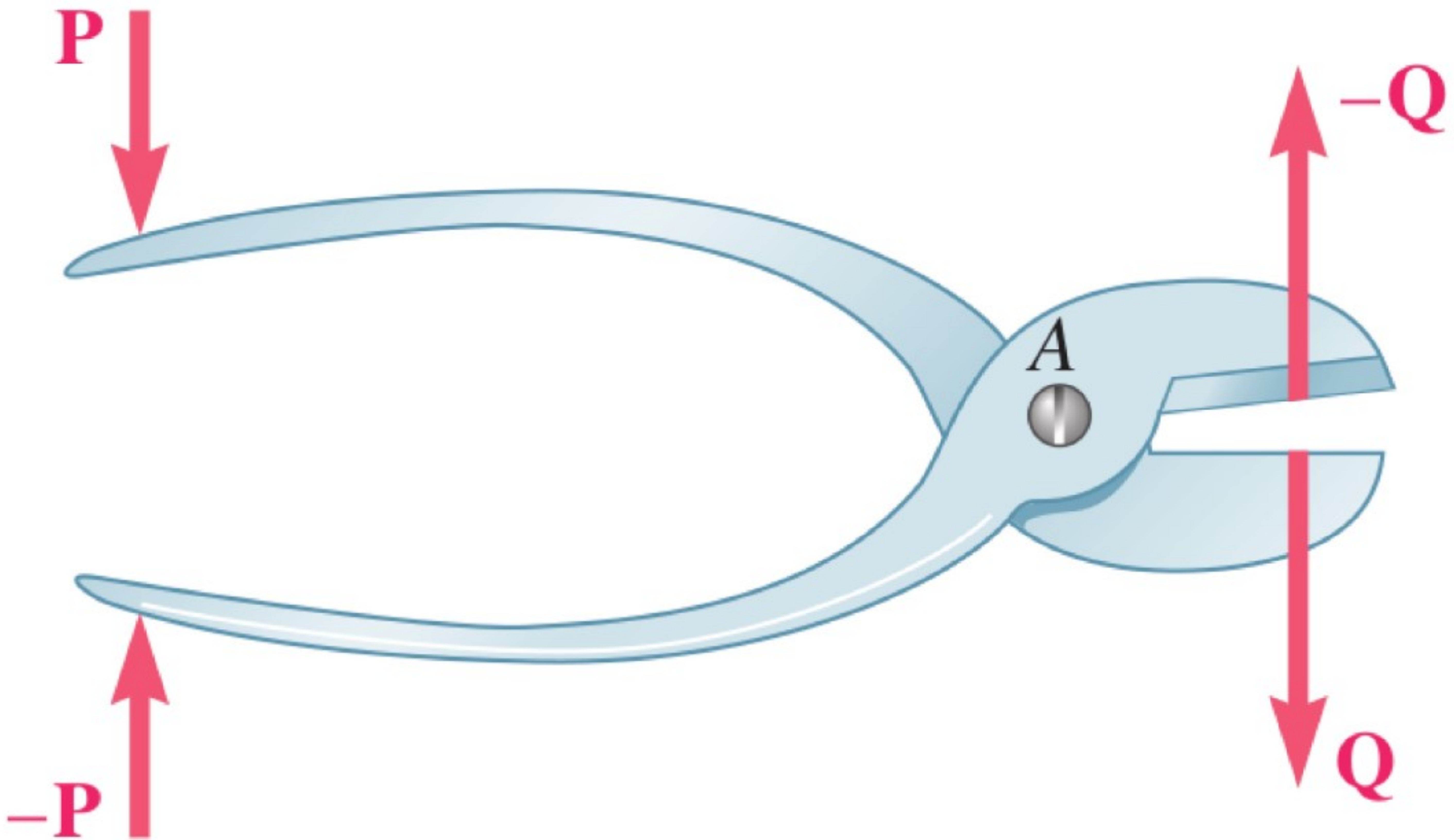
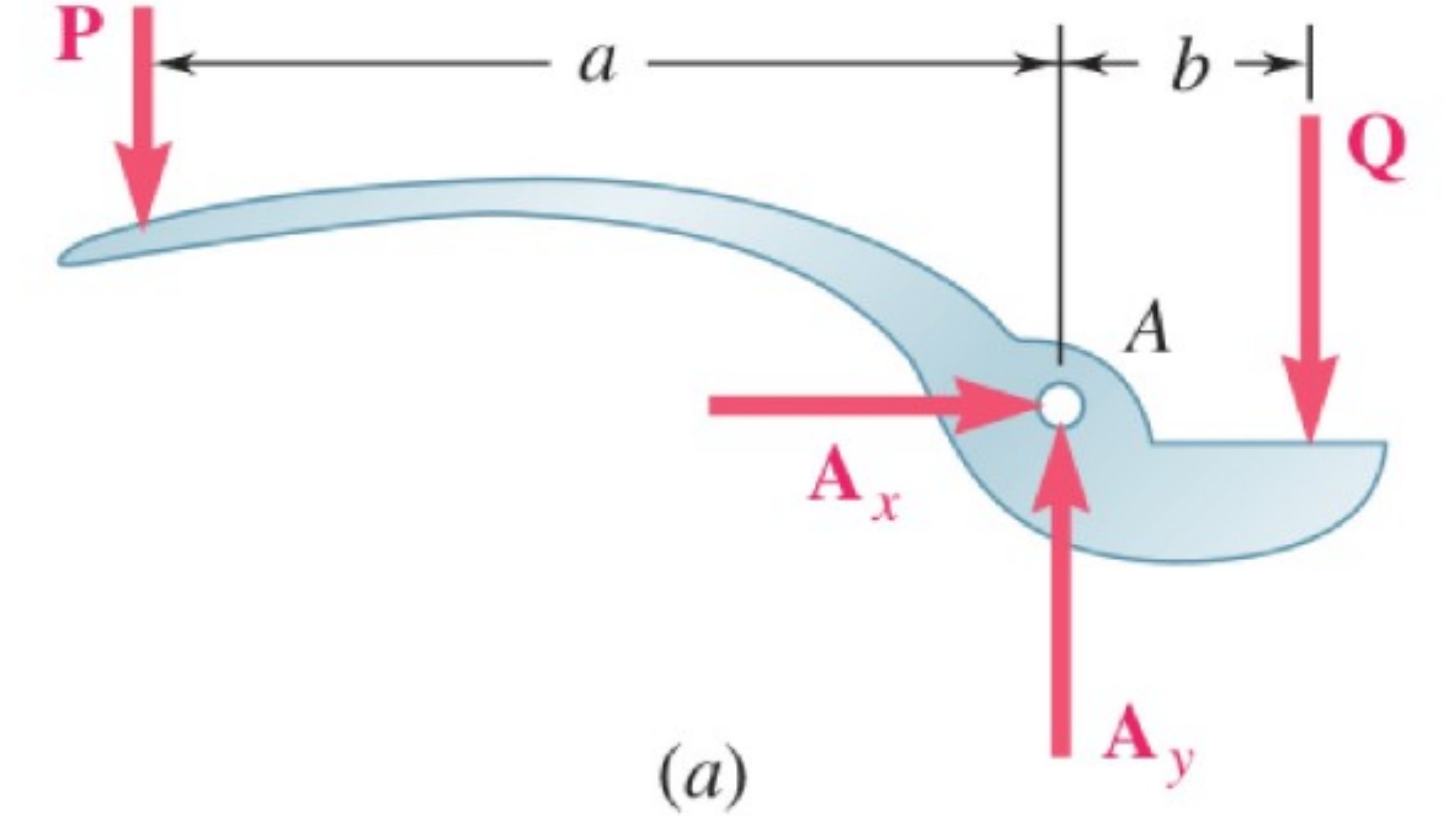
Shape		$\bar{x}$	$\bar{y}$	Area
Triangular area			$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter-elliptical area		$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
Semielliptical area		0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic area		0	$\frac{3h}{5}$	$\frac{4ah}{3}$
Parabolic spandrel		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
General spandrel		$\frac{n+1}{n+2}a$	$\frac{n+1}{4n+2}h$	$\frac{ah}{n+1}$
Circular sector		$\frac{2r \sin \alpha}{3\alpha}$	0	αr^2

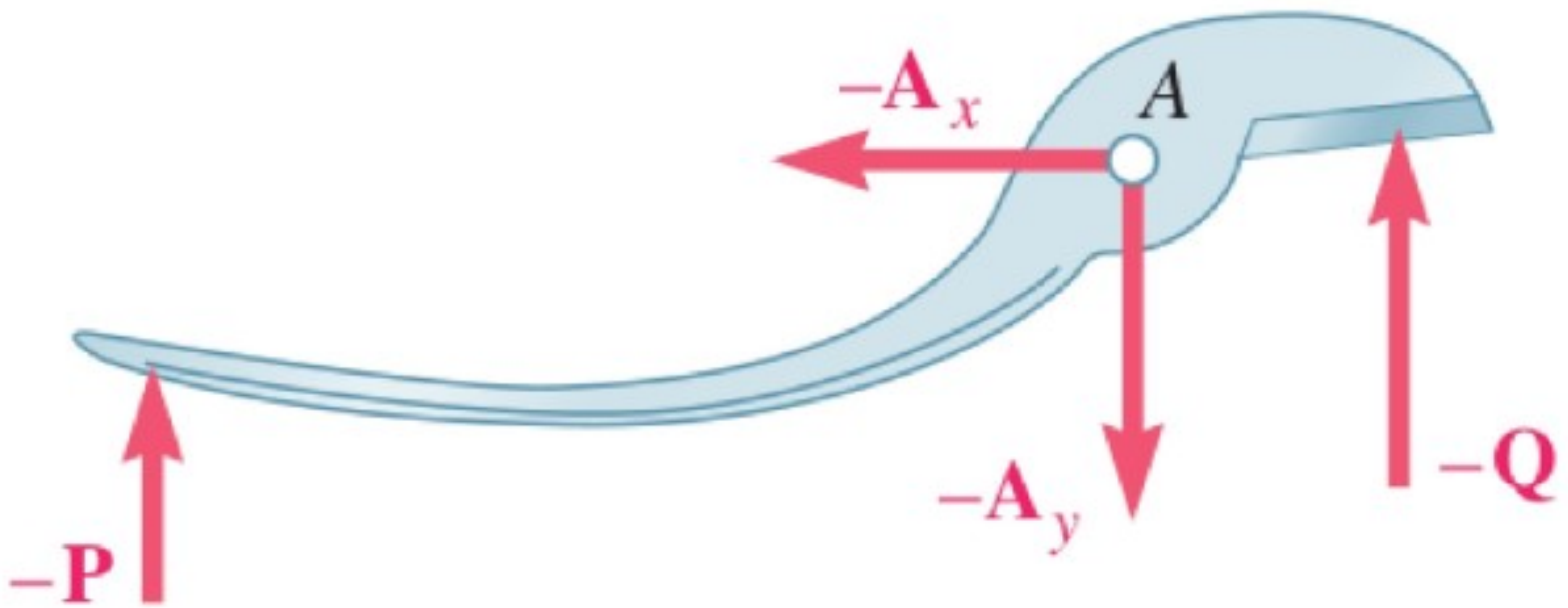
Fig. 5.8A Centroids of common shapes of areas.


P
–P
A
a
b
Q
–Q


P
−Q
A
−P
Q

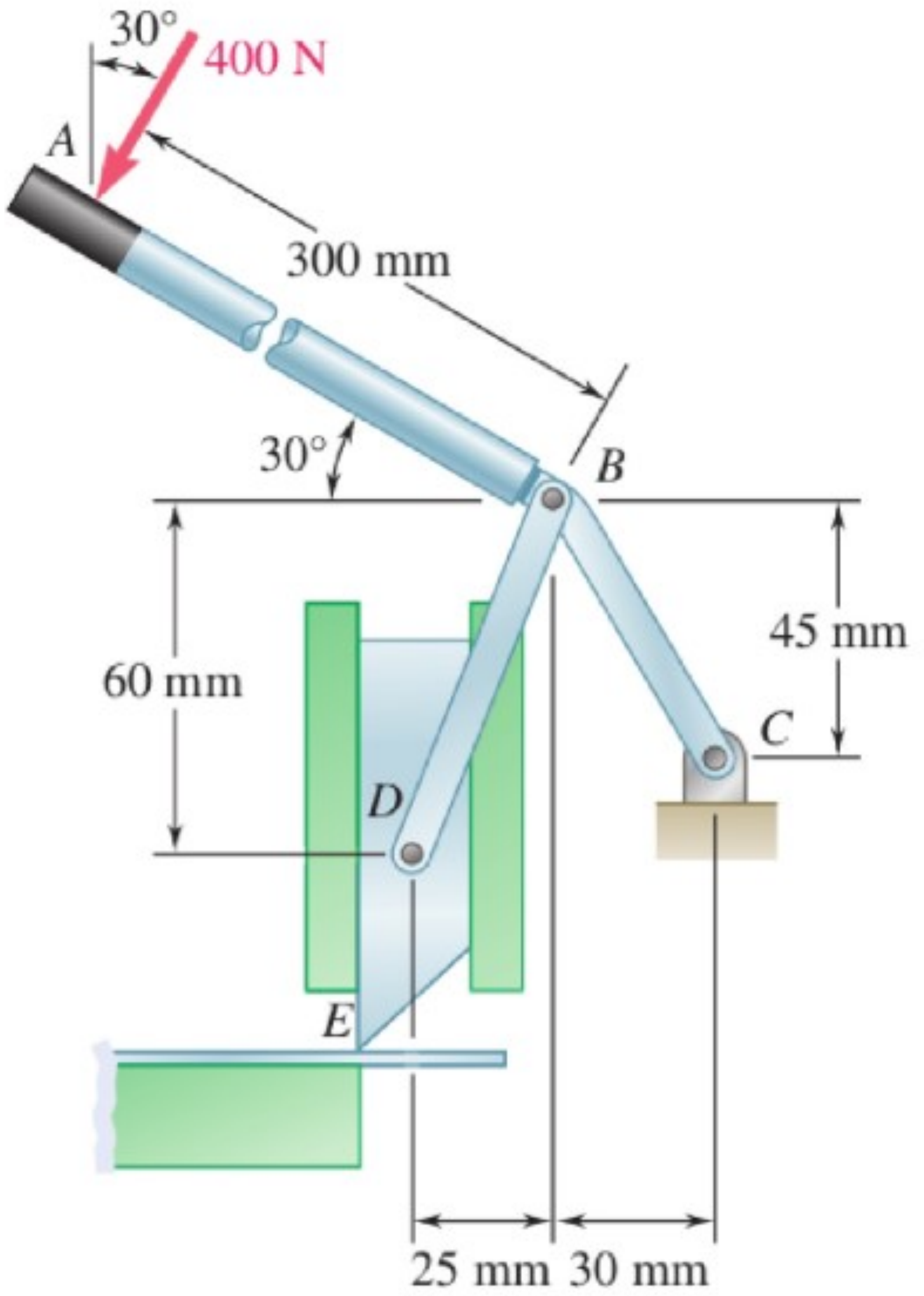


(a)



(b)

6.122 The shear shown is used to cut and trim electronic-circuit-board laminates. For the position shown, determine (*a*) the vertical component of the force exerted on the shearing blade at *D*, (*b*) the reaction at *C*.



6.123 A 100-lb force directed vertically downward is applied to the toggle vise at C. Knowing that link BD is 6 in. long and that $a = 4$ in., determine the horizontal force exerted on block E.