



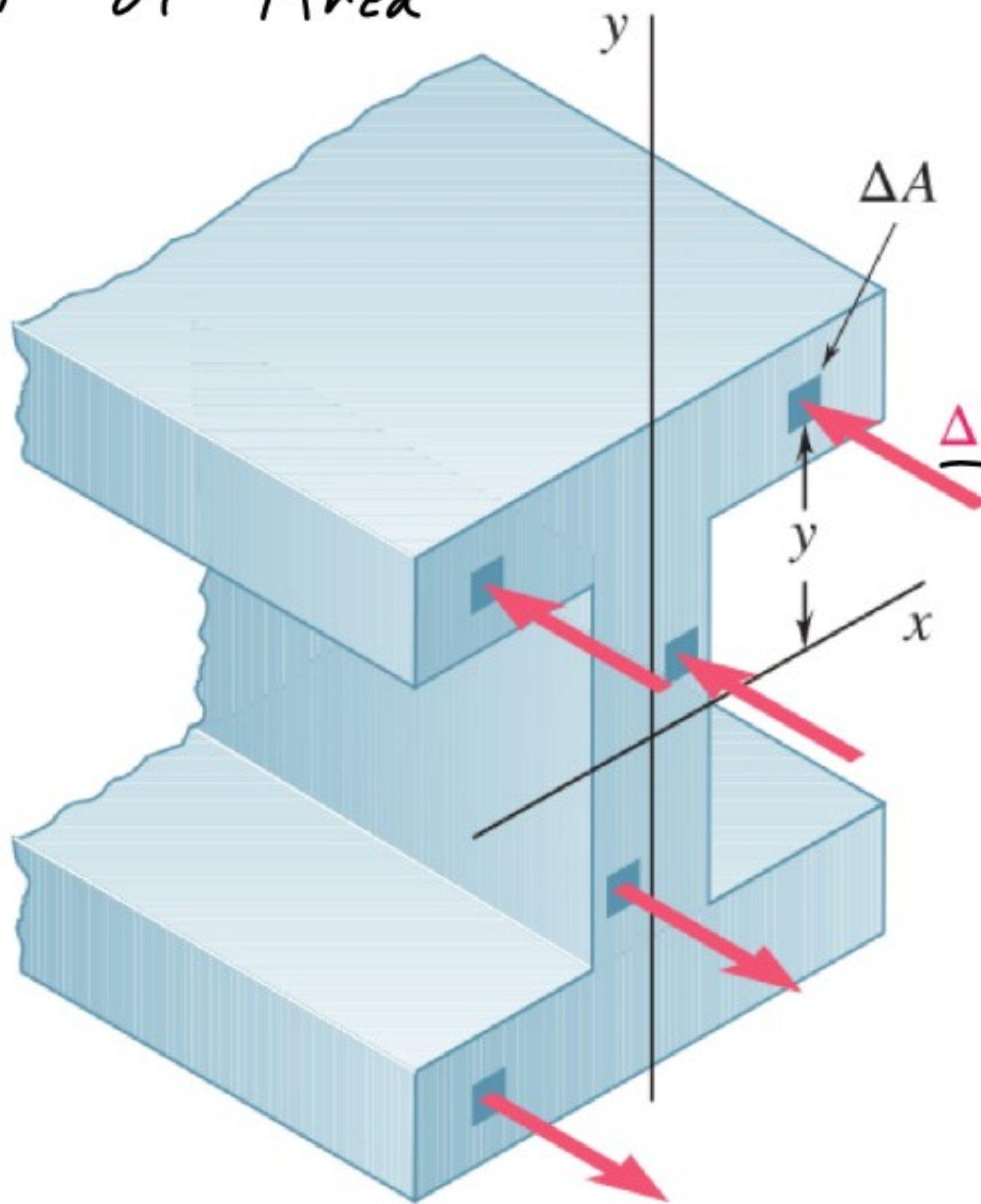
# Second Moment of Area

$$M_x = \int y dF$$

$$= \int y k y dA$$

$$= k \int y^2 dA$$

Second moment of Area



$$\Delta F = k y \Delta A$$

$$\Rightarrow dF = k y dA$$

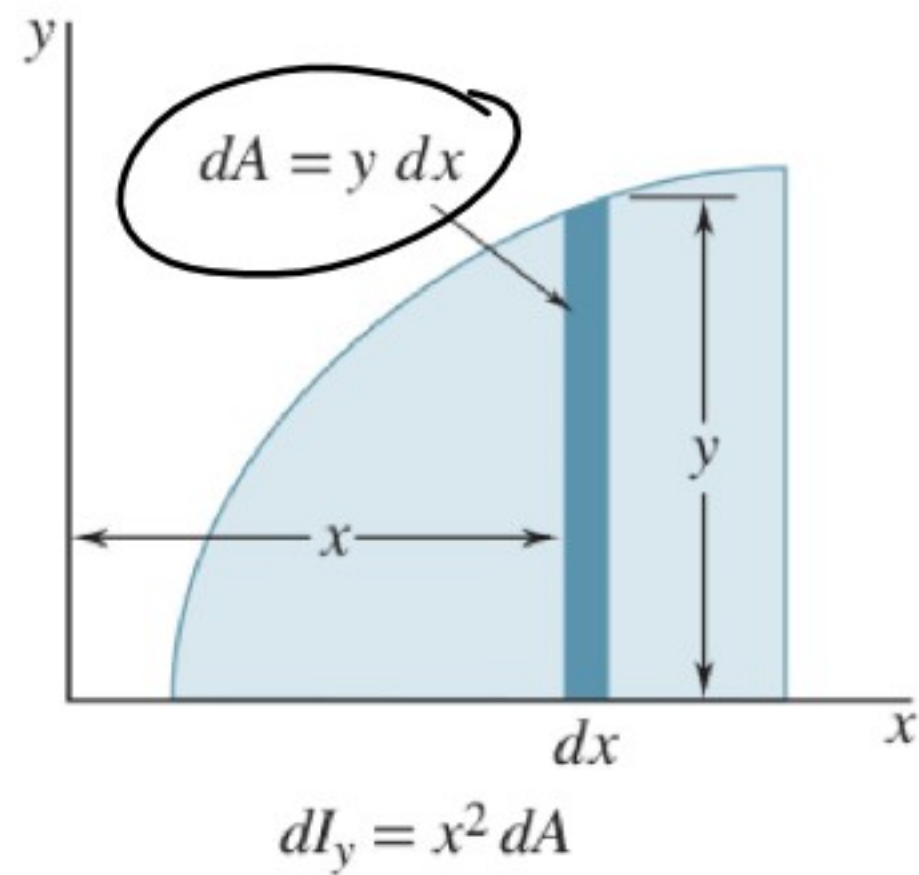
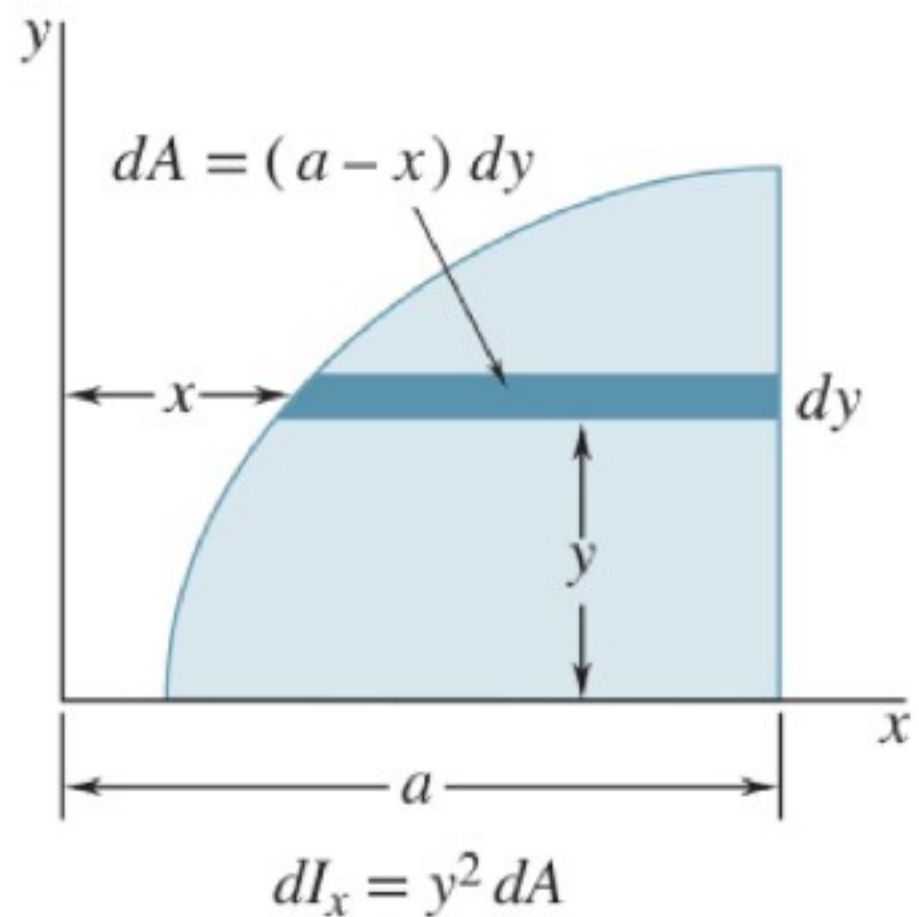
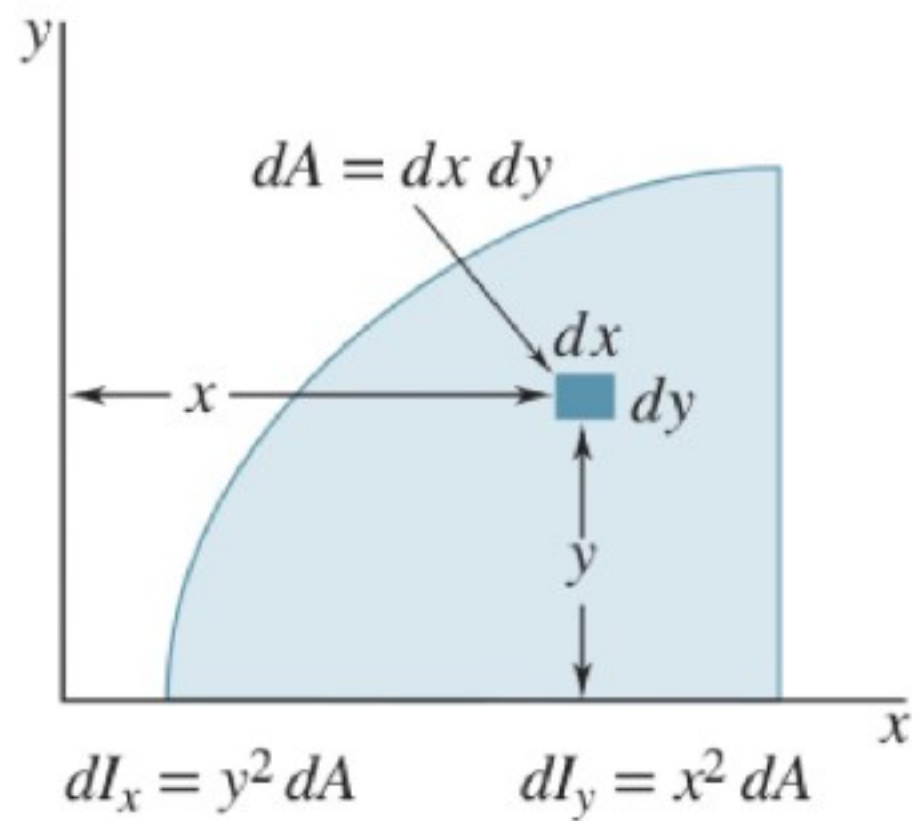
$$R = \int dF = \int k y dA$$

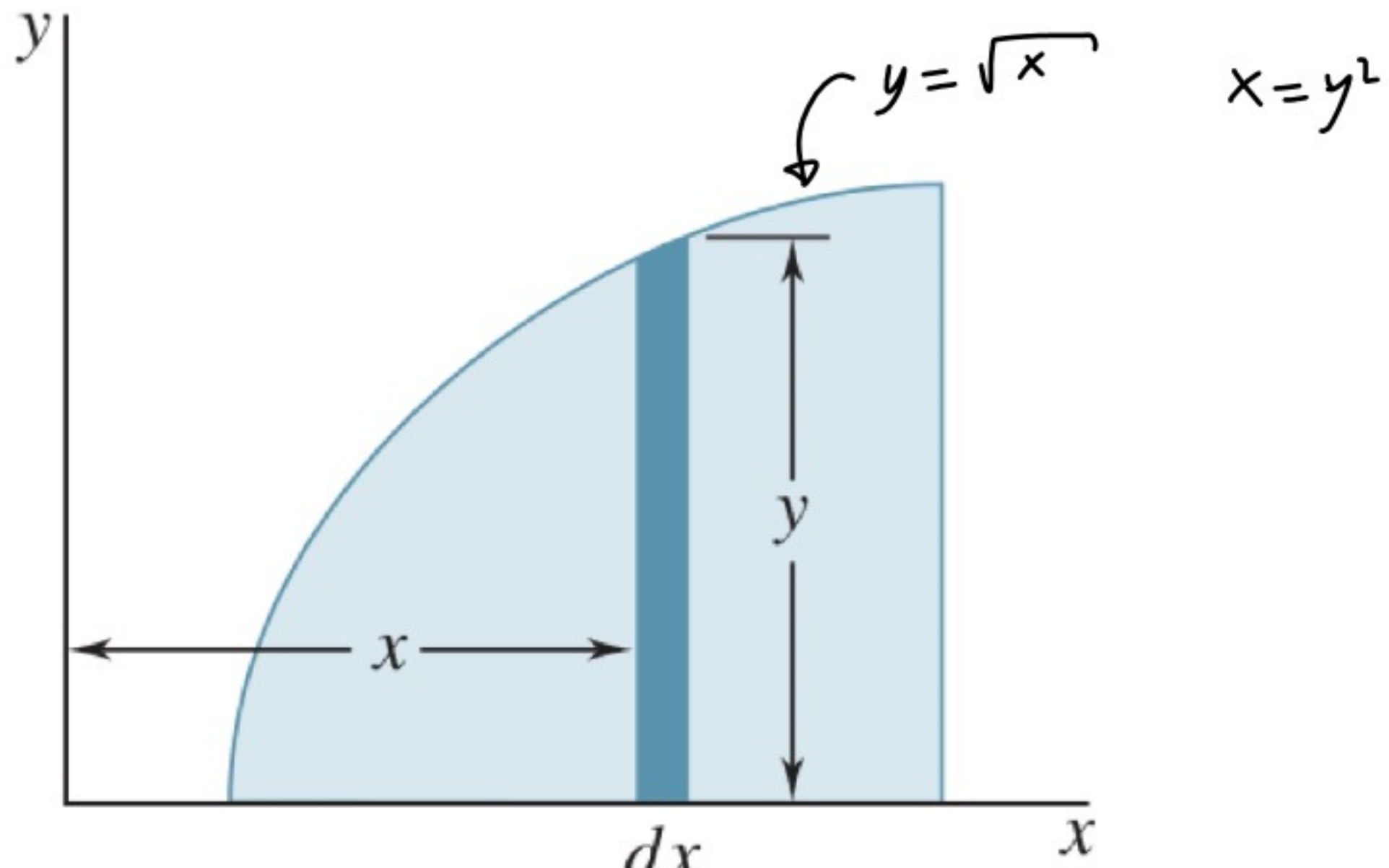
$$= k \int y dA$$

first Moment of Area

$$I_x = \int y^2 dA$$

$$I_y = \int x^2 dA$$





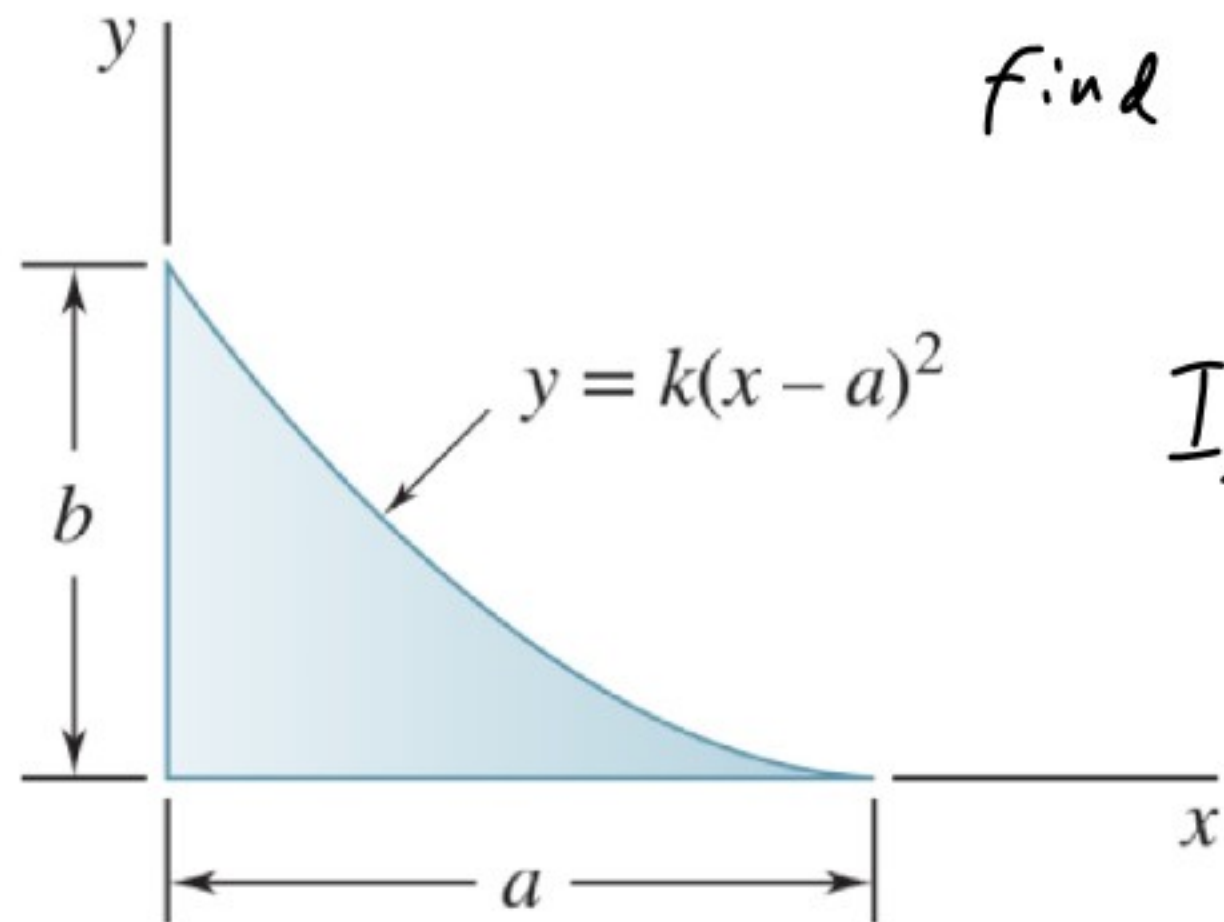
$$dI_x = \frac{1}{3} y^3 dx$$

$$dI_y = x^2 y dx$$

Polar second Moment of area

$$J_0 = \int r^2 dA$$

$$= \int (x^2 + y^2) dA = \int y^2 dA + \int x^2 dA = \underline{I_x} + \underline{I_y}$$



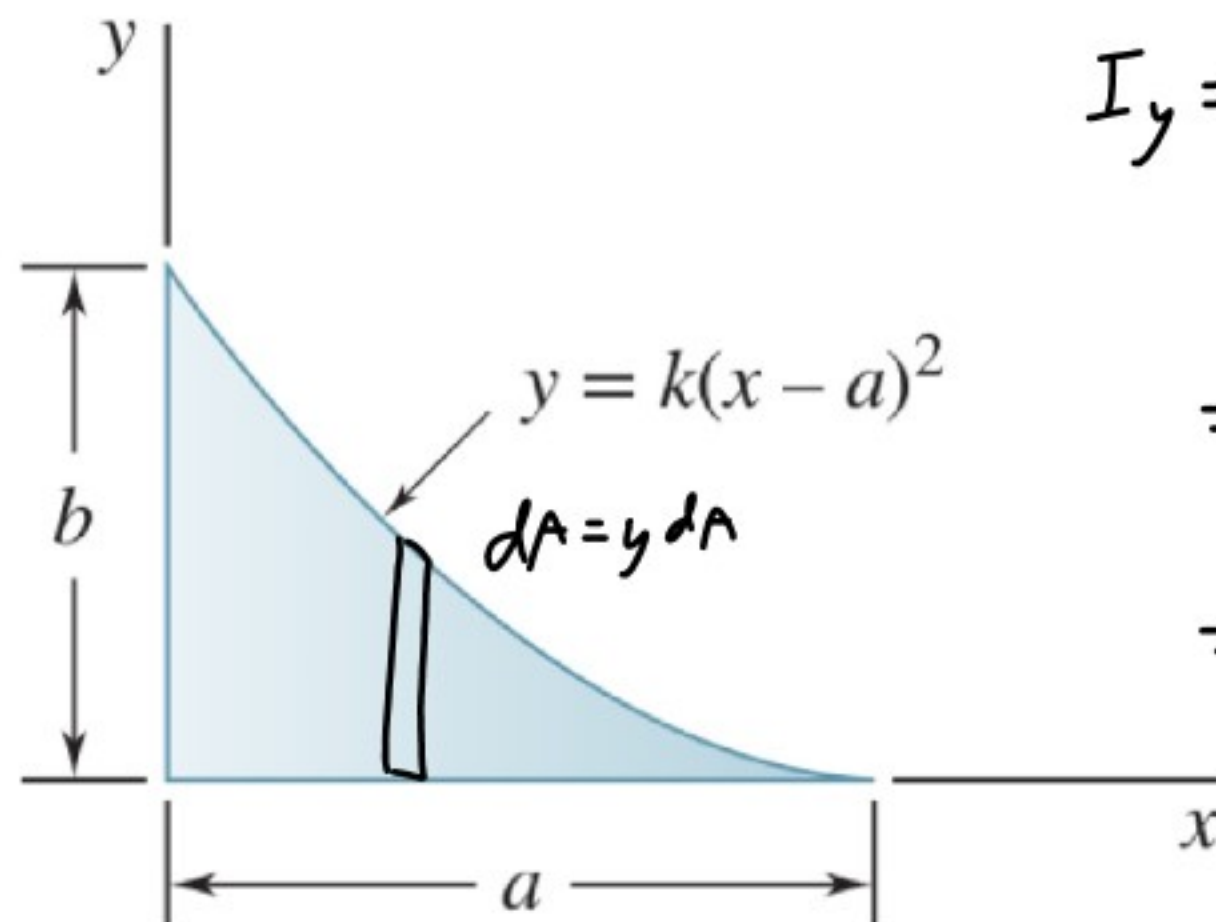
find  $I_x$  and  $I_y$

$$I_x = \int y^2 dA = \int \frac{1}{3} y^3 dx = \frac{1}{3} \int_0^a (k(x-a)^2)^3 dx$$

$$= \frac{k^3}{3} \int_0^a (x-a)^6 dx = \frac{k^3}{3} \int_{-a}^0 u^6 du = \frac{k^3}{3} \left. \frac{u^7}{7} \right|_{-a}^0$$

$$= -\frac{k^3}{3} \frac{(-a)^7}{7} = \boxed{\frac{k^3 a^7}{21}}$$

$u = x - a$   
 $du = dx$



$$I_y = \int x^2 dA = \int x^2 y dx = \int_0^a x^2 k(x-a)^2 dx$$

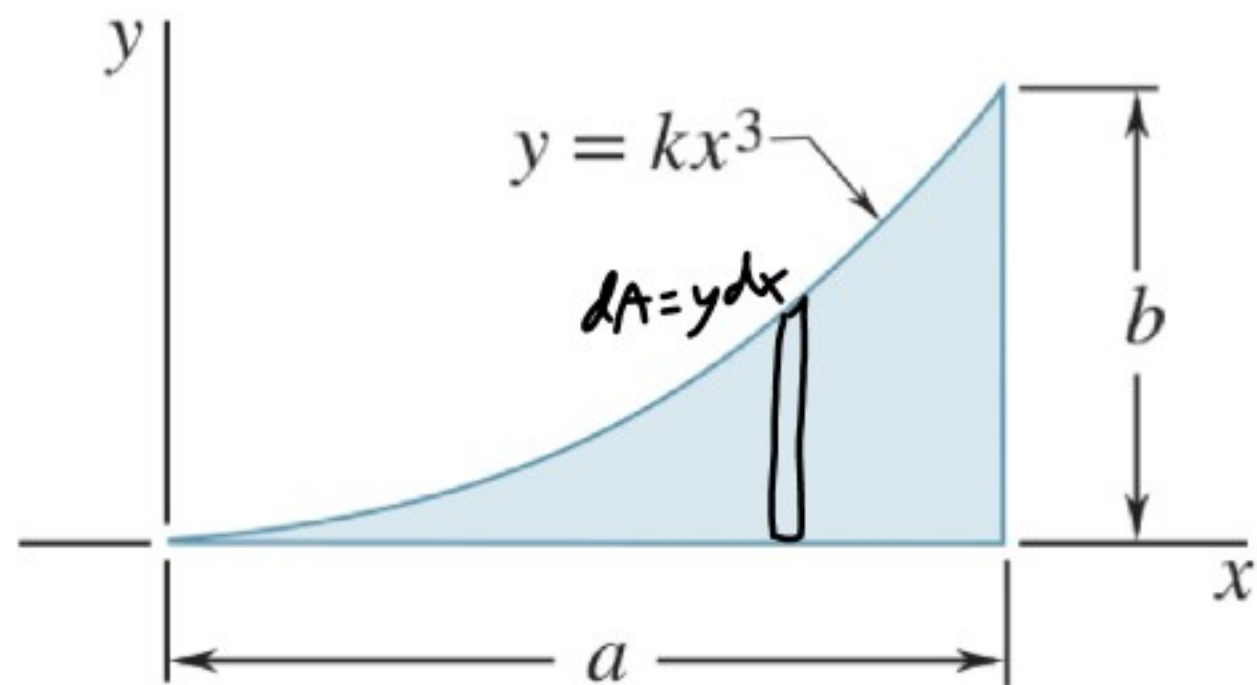
$$= k \int_0^a x^2 (x^2 - 2ax + a^2) dx = k \int_0^a x^4 - 2ax^3 + a^2x^2 dx$$

$$= k \left( \frac{x^5}{5} - \frac{2ax^4}{4} + \frac{a^2x^3}{3} \right) \Big|_0^a$$

$$= k \left( \frac{a^5}{5} - \frac{a^5}{2} + \frac{a^5}{3} \right) = k \left( \frac{6a^5}{30} - \frac{15a^5}{30} + \frac{10a^5}{30} \right)$$

$$= k \frac{a^5}{30}$$





$$I_x = \int y^2 dA$$

$$\text{Find } I_y = \int x^2 dA = \int x^2 y dx$$

$$= \int_0^a x^2 k x^3 dx$$

$$= k \int_0^a x^5 dx = k \left. \frac{x^6}{6} \right|_0^a = \boxed{\frac{ka^6}{6}}$$

$$\text{Find } I_x = \int \frac{1}{3} y^3 dx = \frac{1}{3} \int_0^a (kx^3)^3 dx$$

$$= \frac{k^3}{3} \int_0^a x^9 dx = \frac{k^3}{3} \left. \frac{x^{10}}{10} \right|_0^a$$

$$= \boxed{\frac{k^3 a^{10}}{30}}$$