$$I_x = \int y^2\, dA \qquad dA = y\, dx$$

$$= \int y^2 x\, dy$$

$$= \int \frac{y^3}{3}\, dx$$

$$I_x = \iint y^2\, dA \qquad dA = dy\, dx$$

$$= \iint y^2\, dy\, dx$$

$$= \int \frac{y^3}{3}\, dx$$

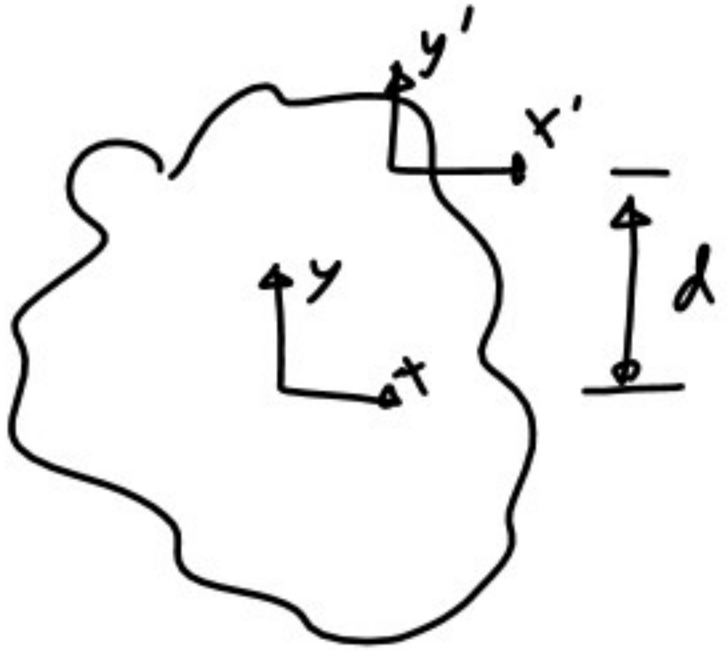
$$\iint f(x,y)\, dx\, dy$$

$$\int \left(\int f(x,y)\, dx \right) dy$$

$$I_x = \iint y^2\, dx\, dy$$

$$= \int y^2 x\, dy$$

Parallel axis Theorem



$$I_x = \iint y^2 \, dA$$

$$I_{x'} = \iint (y+d)^2 \, dA$$

$$= \iint (y^2 + 2dy + d^2) \, dA$$

$$= \iint y^2 \, dA + \iint 2dy \, dA + \iint d^2 \, dA$$

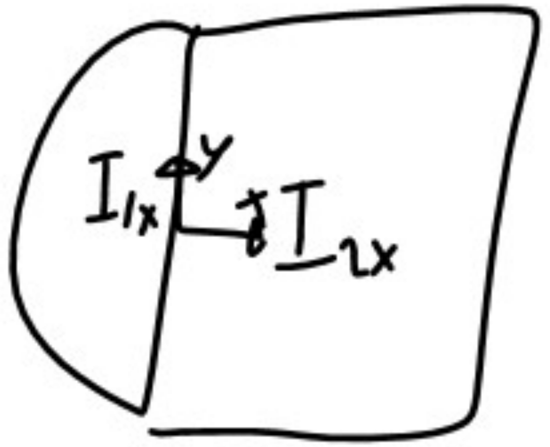
$$= I_x + 2d \iint y \, dA + d^2 \iint dA$$

$$= I_x + d^2 A$$

$$I' = \bar{I} + Ad^2$$

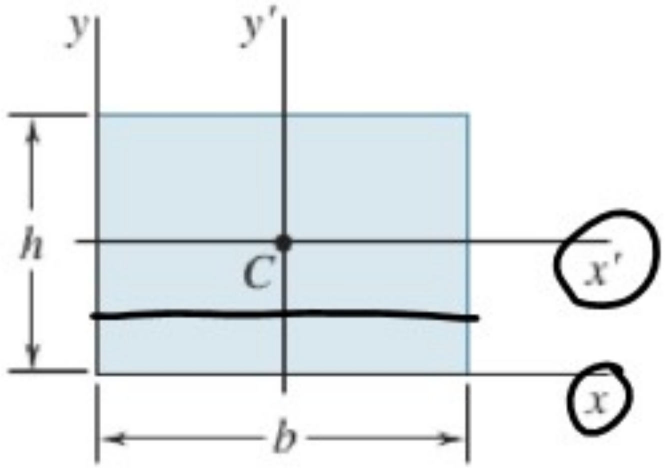
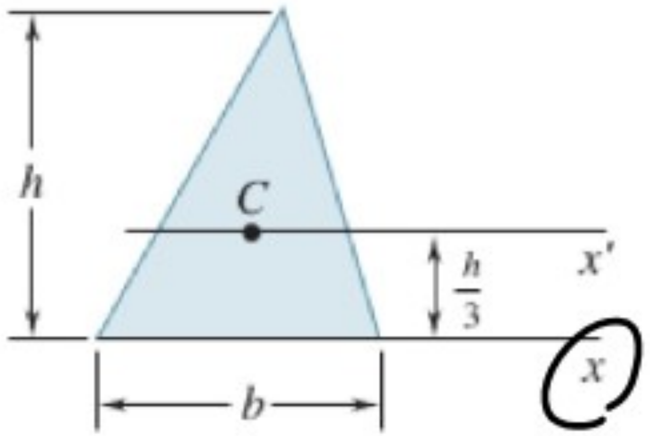
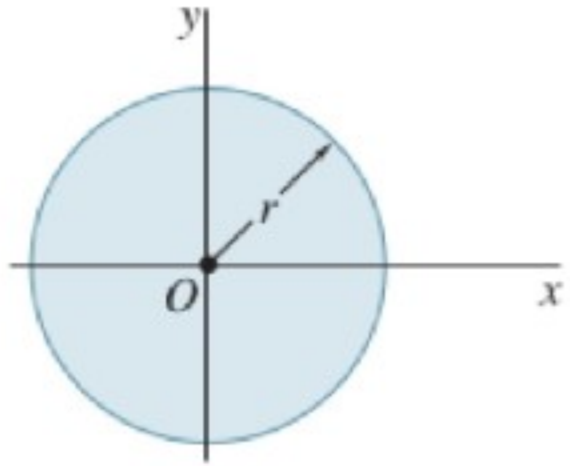
$$I' - Ad^2 = \bar{I}$$

Composite Areas



$$I_x = I_{1x} + I_{2x}$$

$$I_y = I_{1y} + I_{2y}$$

Rectangle		$\bar{I}_{x'} = \frac{1}{12} bh^3$ $\bar{I}_{y'} = \frac{1}{12} b^3h$ $I_x = \frac{1}{3} bh^3$ $I_y = \frac{1}{3} b^3h$ $J_C = \frac{1}{12} bh(b^2 + h^2)$
Triangle		$\bar{I}_{x'} = \frac{1}{36} bh^3$ $I_x = \frac{1}{12} bh^3$
Circle		$\bar{I}_x = \bar{I}_y = \frac{1}{4}\pi r^4$ $J_O = \frac{1}{2}\pi r^4$

$$\bar{I}_{x'} = \frac{1}{12} bh^3$$

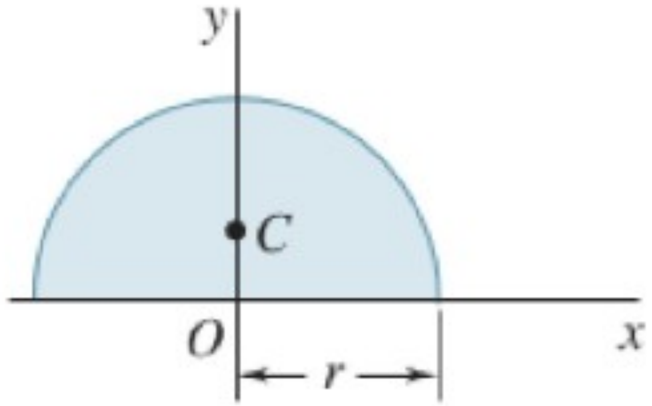
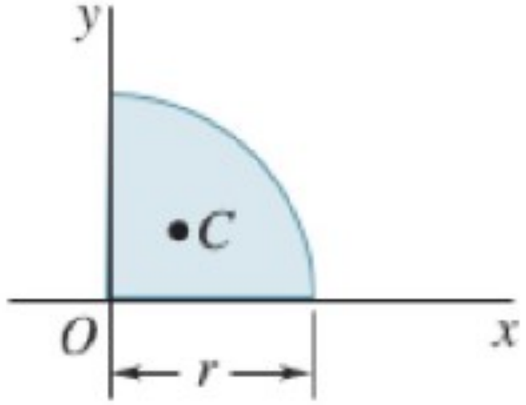
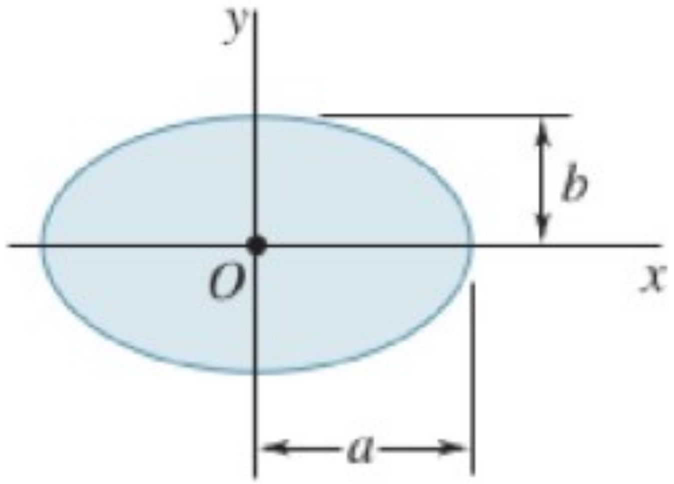
$$I_x = \bar{I}_{x'} + Ad^2$$

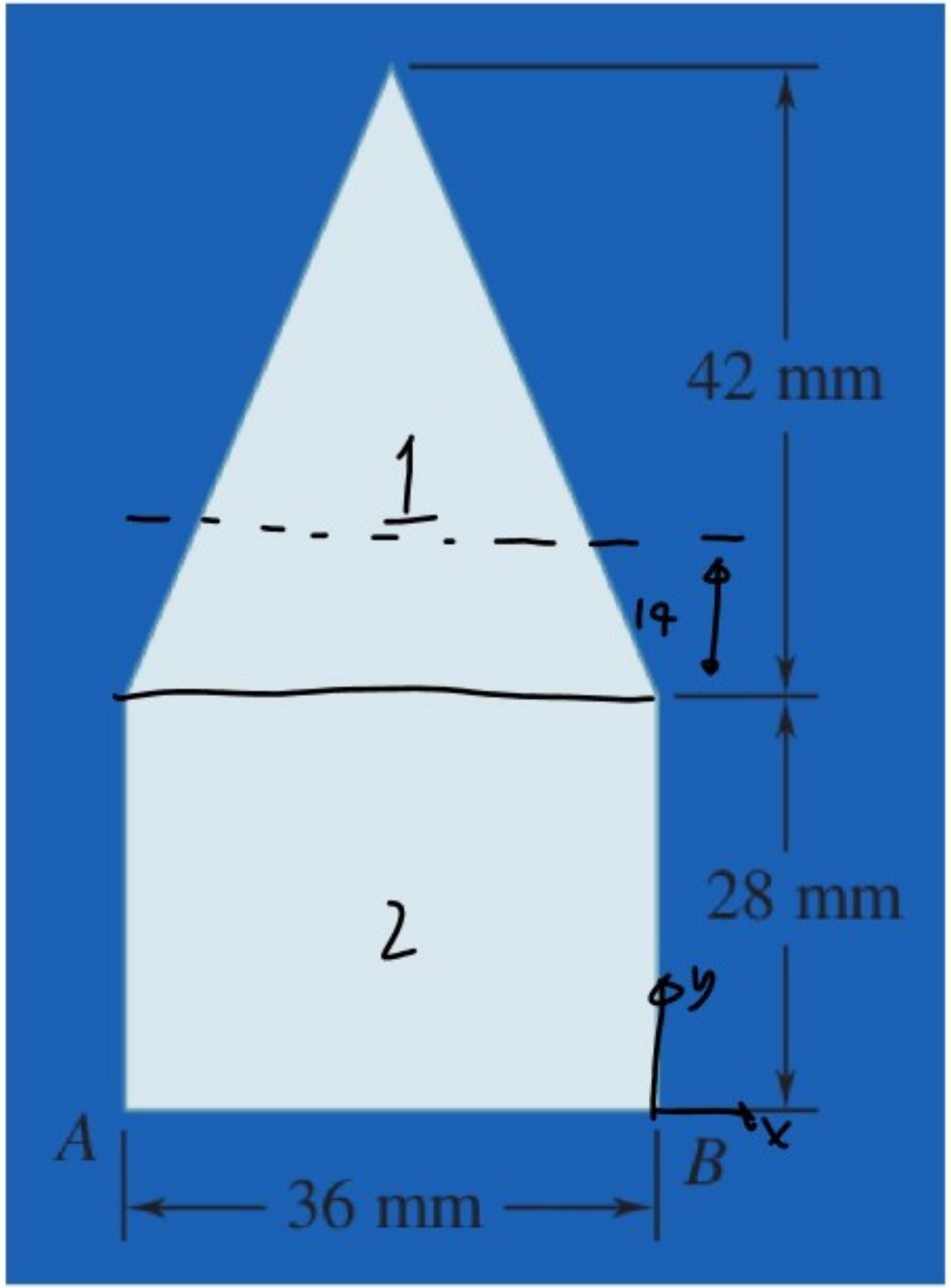
$$= \frac{1}{12} bh^3 + bh\left(\frac{h}{2}\right)^2$$

$$= \frac{1}{12} bh^3 + \frac{bh^3}{4}$$

$$= \left(\frac{1}{12} + \frac{1}{4}\right) bh^3$$

$$= \frac{bh^3}{3}$$

Semicircle		$I_x = I_y = \frac{1}{8}\pi r^4$ $J_O = \frac{1}{4}\pi r^4$
Quarter circle		$I_x = I_y = \frac{1}{16}\pi r^4$ $J_O = \frac{1}{8}\pi r^4$
Ellipse		$\bar{I}_x = \frac{1}{4}\pi a b^3$ $\bar{I}_y = \frac{1}{4}\pi a^3 b$ $J_O = \frac{1}{4}\pi a b(a^2 + b^2)$



Find I_x

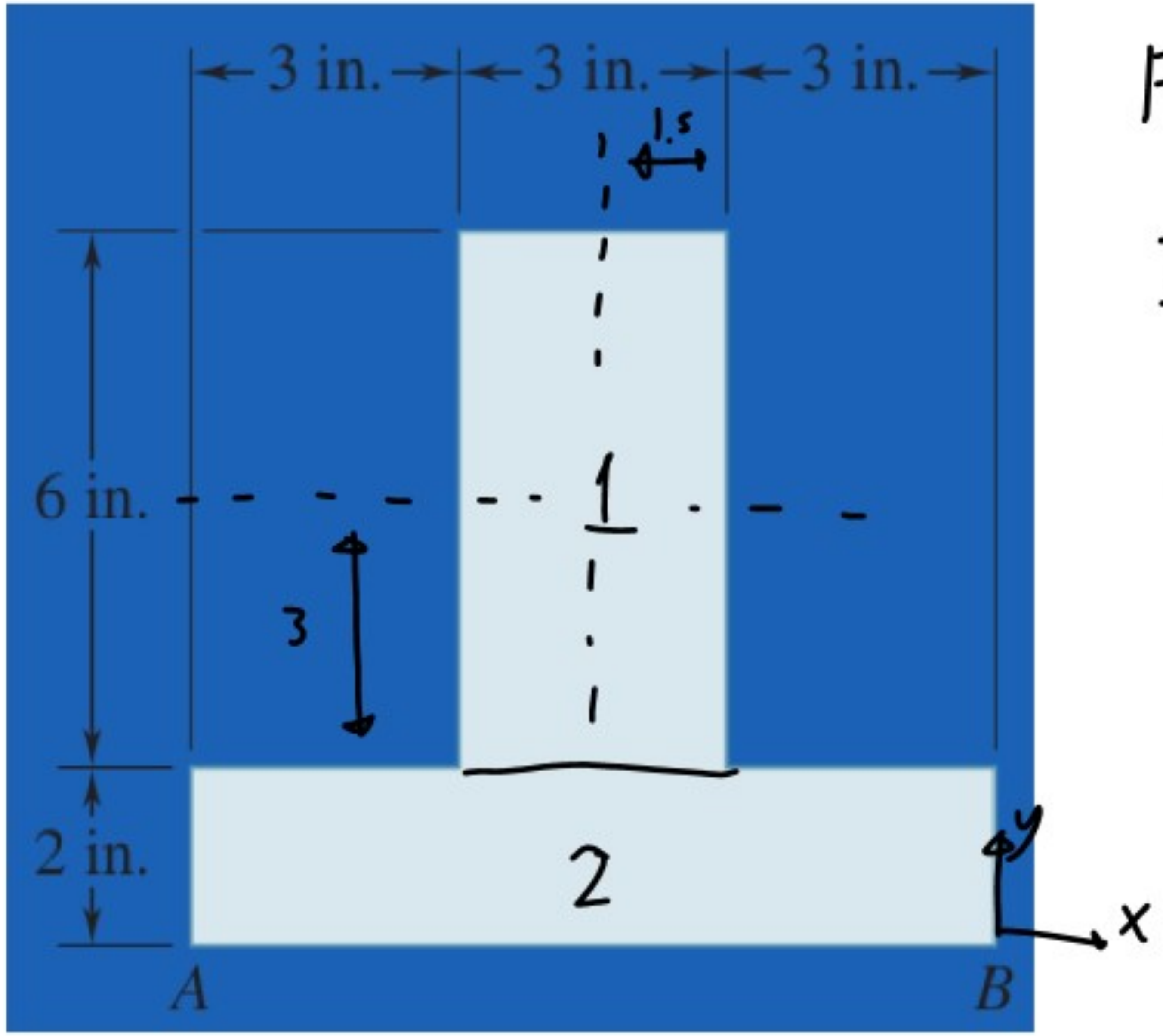
$$I_x = I_{x_1} + I_{x_2} = 1.4\times10^6 + 2.6\times10^5 = 1.67\times10^6\ mm^4$$

$$I_{x_2} = \frac{1}{3}bh^3 = \frac{1}{3}36(28)^3 = 2.6\times10^5\ mm^4$$

$$I_{x_1} = \frac{1}{36}bh^3 + Ad^2$$

$$= \frac{1}{36}36(42)^3 + \left(\frac{36\cdot42}{2}\right)(28+14)^2$$

$$= 1.4\times10^6\ mm^4$$



Find I_x and I_y

$$I_{x_2} = \frac{1}{3} bh^3 = \frac{1}{3} 9 (2)^3 = 24 \text{ in}^4$$

$$I_{x_1} = \frac{1}{12} bh^3 + A d^2$$

$$= \frac{1}{12} 3 (6)^3 + 6 \cdot 3 (3+2)^2 = 504 \text{ in}^4$$

$$I_x = I_{x_1} + I_{x_2} = 24 + 504 = 528 \text{ in}^4$$

$$I_{y_1} = \frac{1}{12} b^3 h + A d^2 = \frac{1}{12} (3)^3 6 + 6 \cdot 3 \cdot 4.5^2$$

$$I_{y_2} = \frac{1}{3} b^3 h = \frac{1}{3} 9^3 2$$

$$I_y = I_{y_1} + I_{y_2}$$