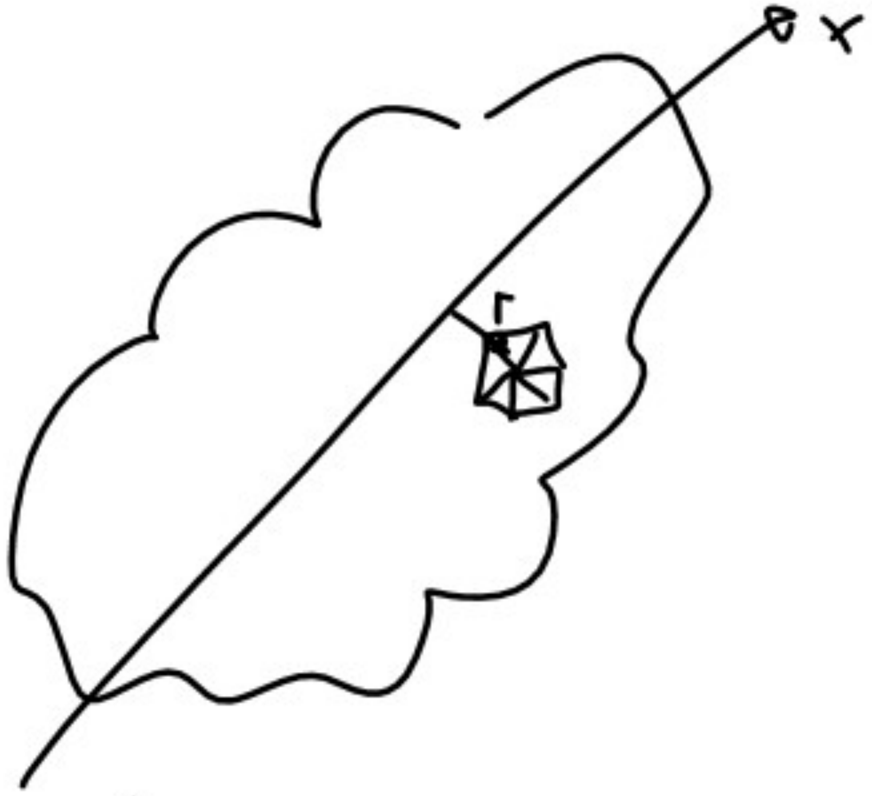
Mass Moment of Inertia

$F = ma$ translatoinal

$\tau = I\alpha$ rotational

$= I\ddot{\theta}$



$$I = \int r^2 \, dm$$

$$I_x = \int (y^2 + z^2) \, dm$$

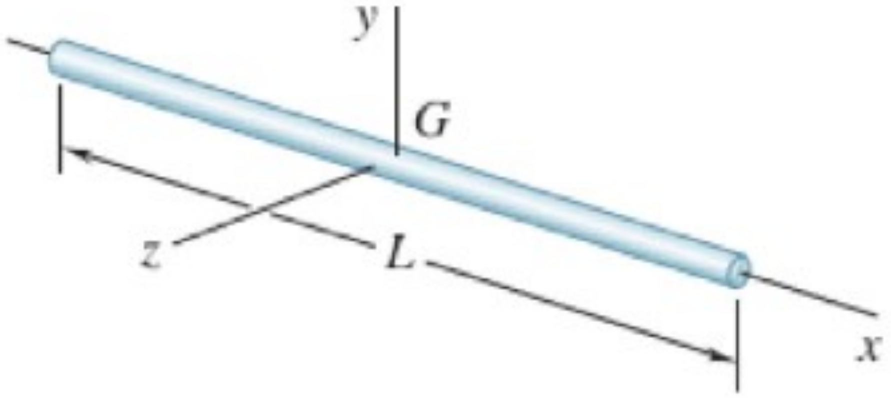
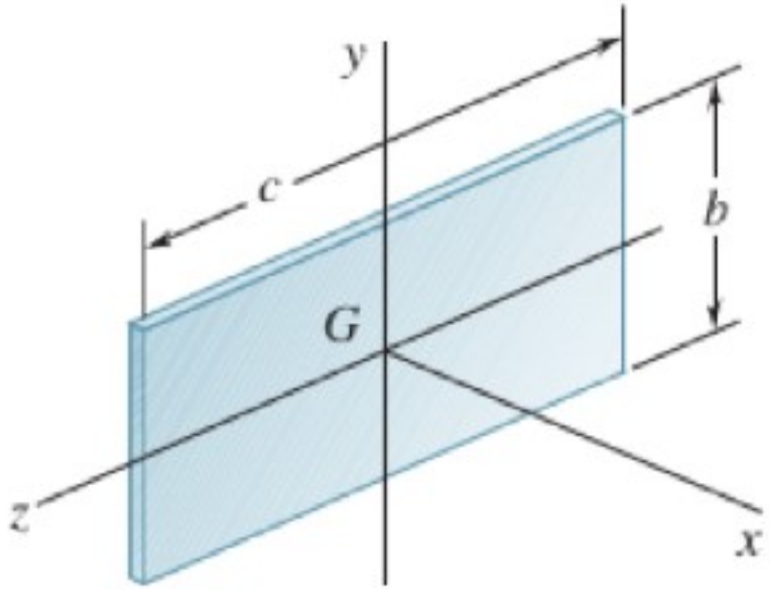
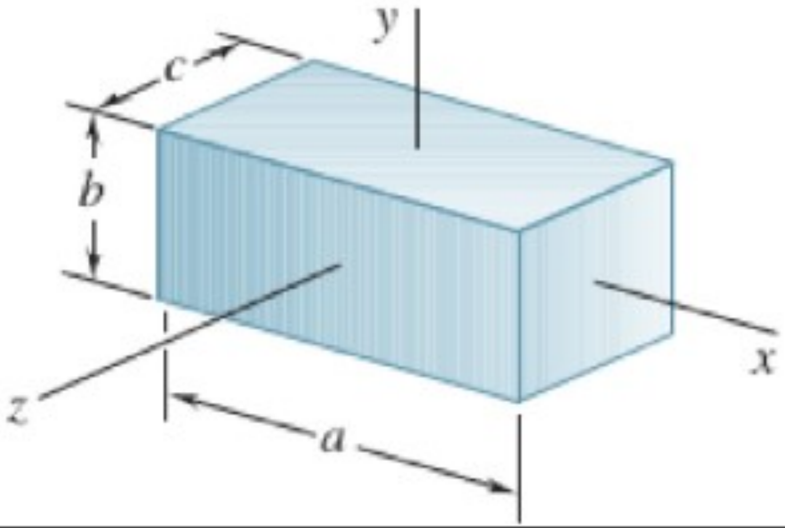
$$I_y = \int (z^2 + x^2) \, dm$$

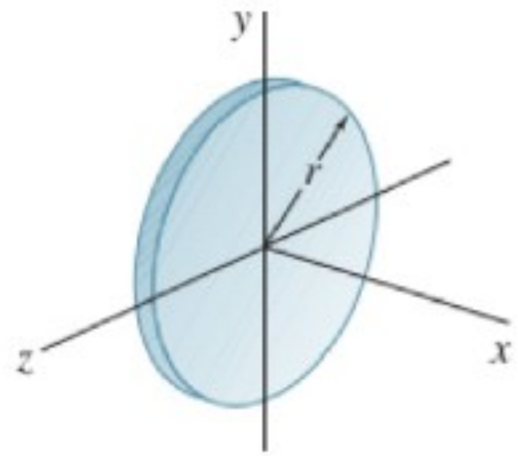
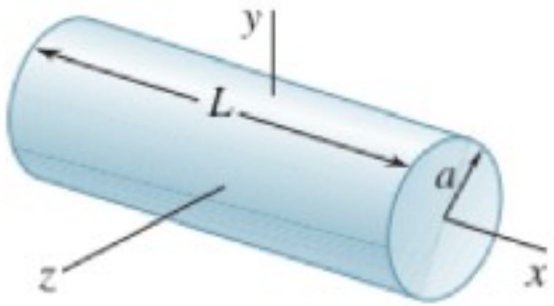
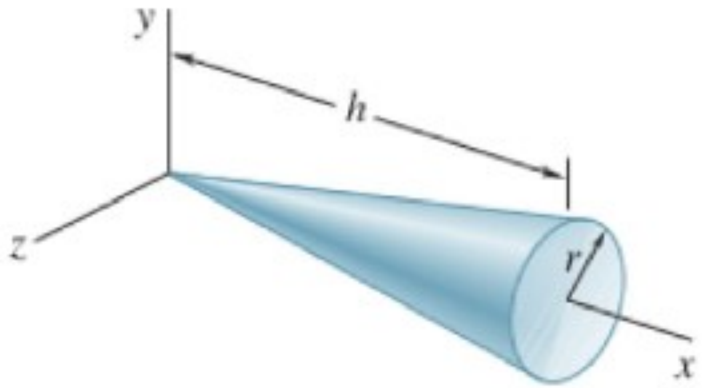
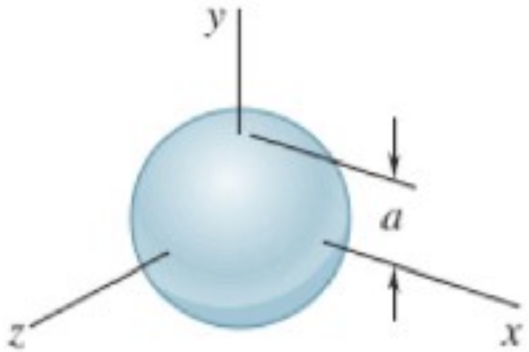
$$I_z = \int (x^2 + y^2) \, dm$$

$$I = \iiint \rho(x, y, z) \, r^2 \, dV$$

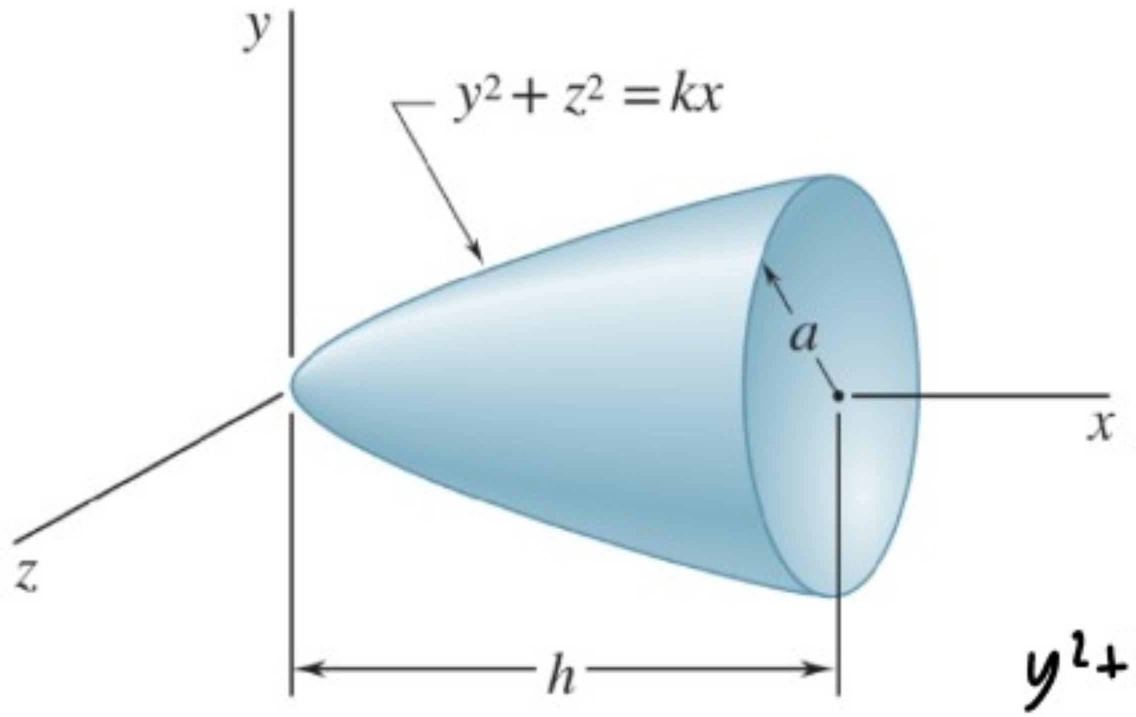
Parallel Axis Theorem

$$I = \bar{I} + md^2$$

Slender rod		$I_y = I_z = \frac{1}{12} mL^2$
Thin rectangular plate		$I_x = \frac{1}{12} m(b^2 + c^2)$ $I_y = \frac{1}{12} mc^2$ $I_z = \frac{1}{12} mb^2$
Rectangular prism		$I_x = \frac{1}{12} m(b^2 + c^2)$ $I_y = \frac{1}{12} m(c^2 + a^2)$ $I_z = \frac{1}{12} m(a^2 + b^2)$

Thin disk		$I_x = \frac{1}{2} mr^2$ $I_y = I_z = \frac{1}{4} mr^2$
Circular cylinder		$I_x = \frac{1}{2} ma^2$ $I_y = I_z = \frac{1}{12} m(3a^2 + L^2)$
Circular cone		$I_x = \frac{3}{10} ma^2$ $I_y = I_z = \frac{3}{5} m\left(\frac{1}{4}a^2 + h^2\right)$
Sphere		$I_x = I_y = I_z = \frac{2}{5} ma^2$

Find I_x

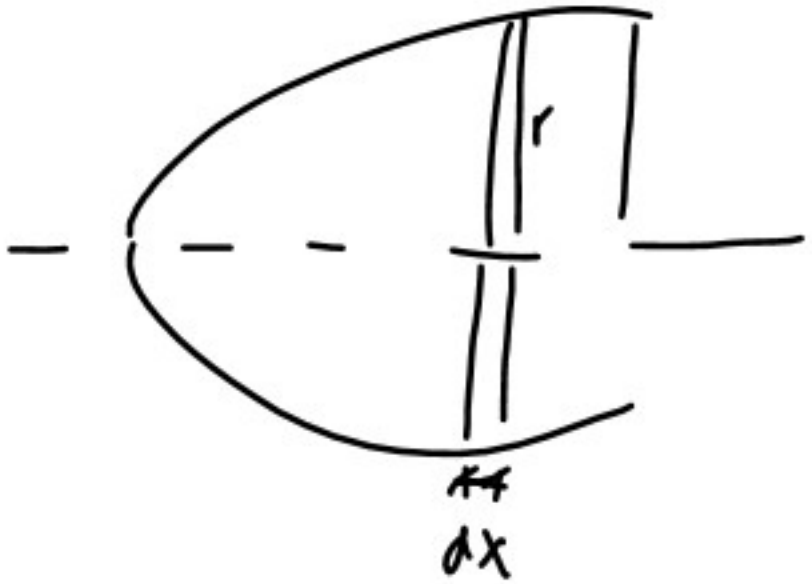


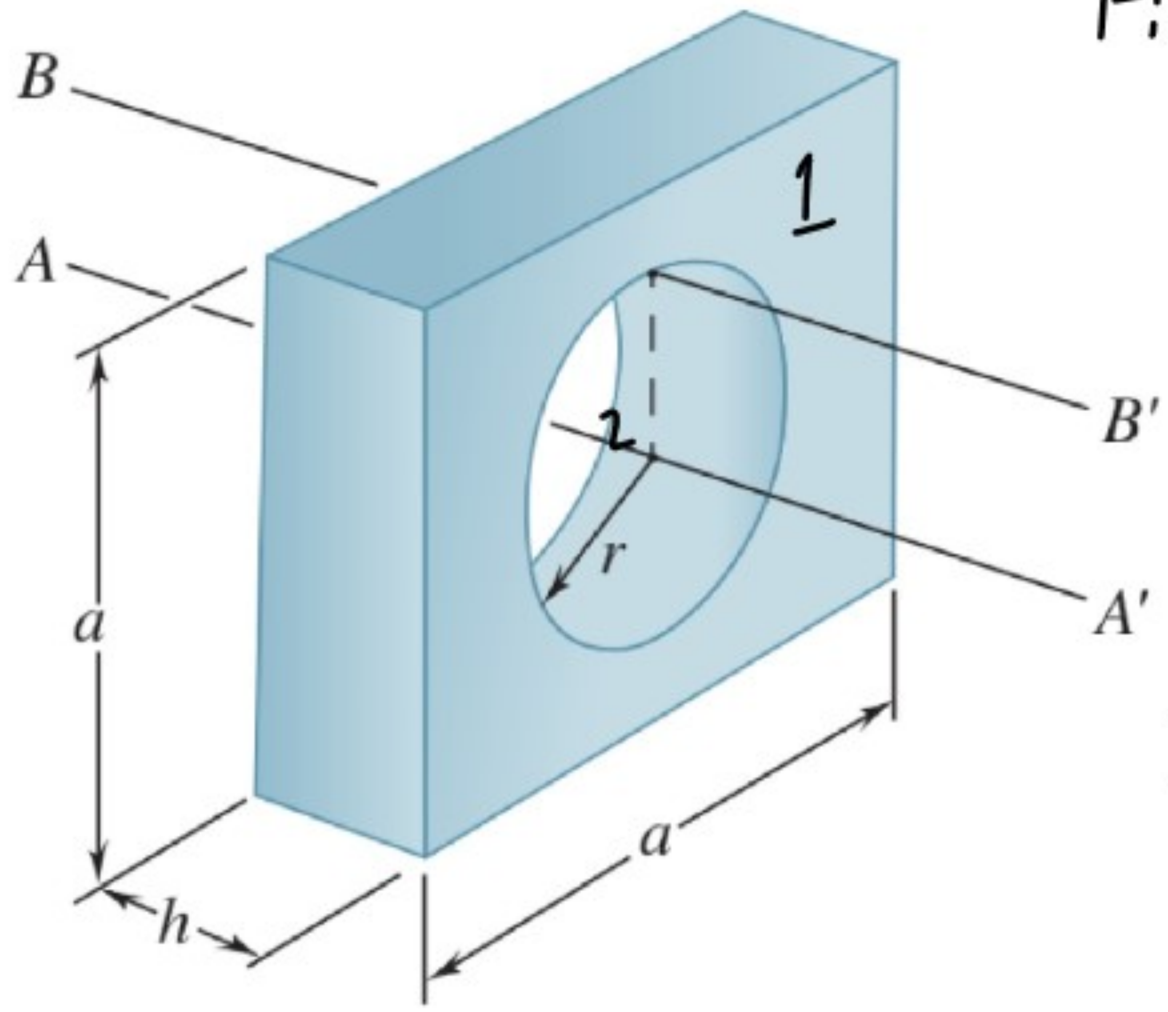
$$I_x = \int dI_x = \int \frac{1}{2} r^2\, dm = \int \frac{1}{2} kx\, dm = \int_0^h \frac{1}{2} kx\rho\pi kx\, dx$$

$$y^2 + z^2 = r^2 = kx$$

$$dm = \rho\, dV = \rho\pi r^2\, dx = \rho\pi kx\, dx$$

$$= \frac{\rho k^2 \pi}{2}\int_0^h x^2\, dx = \frac{\rho k^2 \pi x^3}{6}\Big|_0^h = \frac{\rho k^2 \pi h^3}{6}$$





Find I on axis B-B'

$$I_1 = \frac{1}{12} m_1 (a^2 + a^2)$$

$$I_2 = \frac{1}{2} m_2 r^2$$

$$\bar{I} = I_1 - I_2$$

$$I = \bar{I} + m r^2$$

$$I = \frac{m}{a^2 - \pi r^2}\left(\frac{a^4}{6} - \frac{3\pi r^4}{2} + a^2 r^2\right)$$