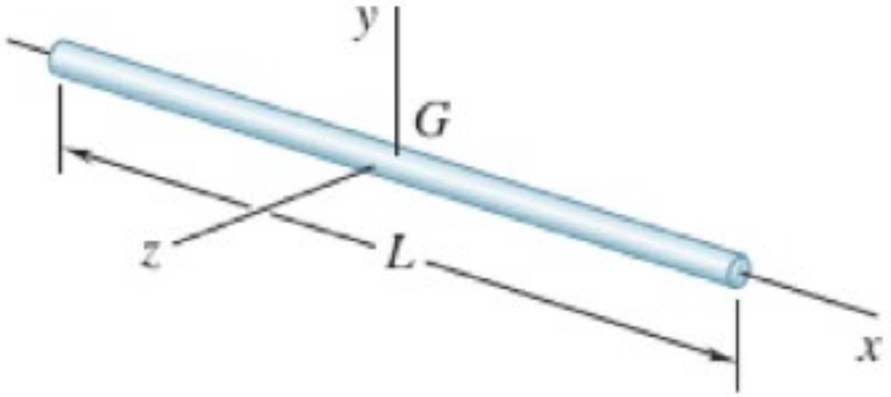
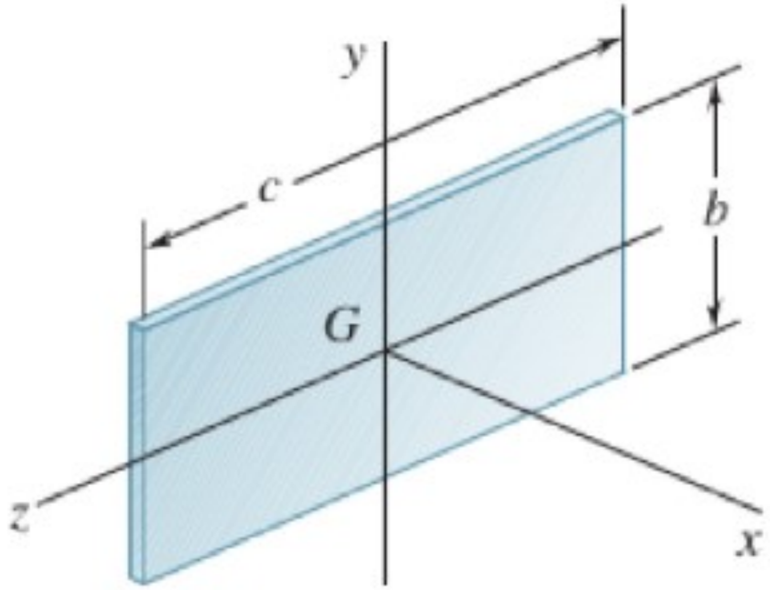
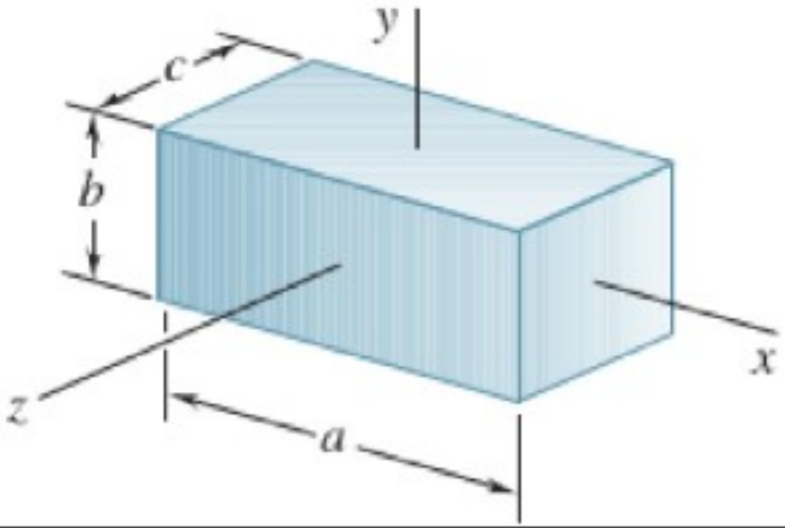
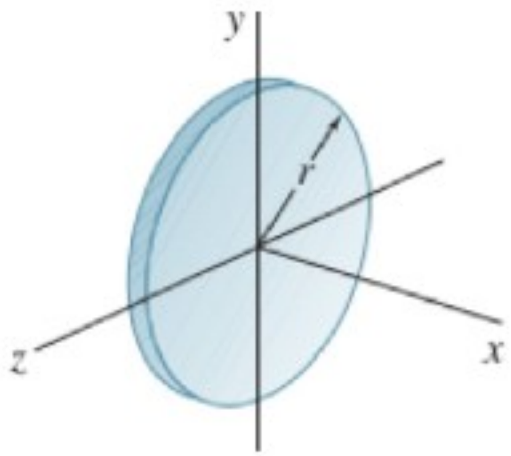
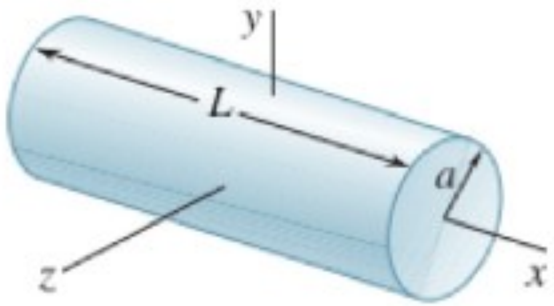
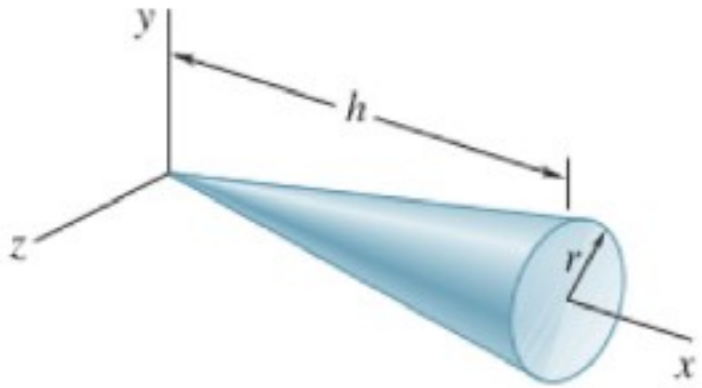
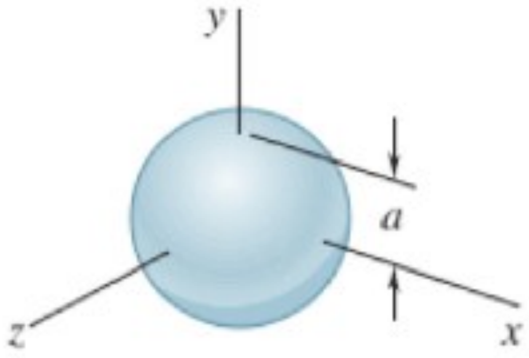
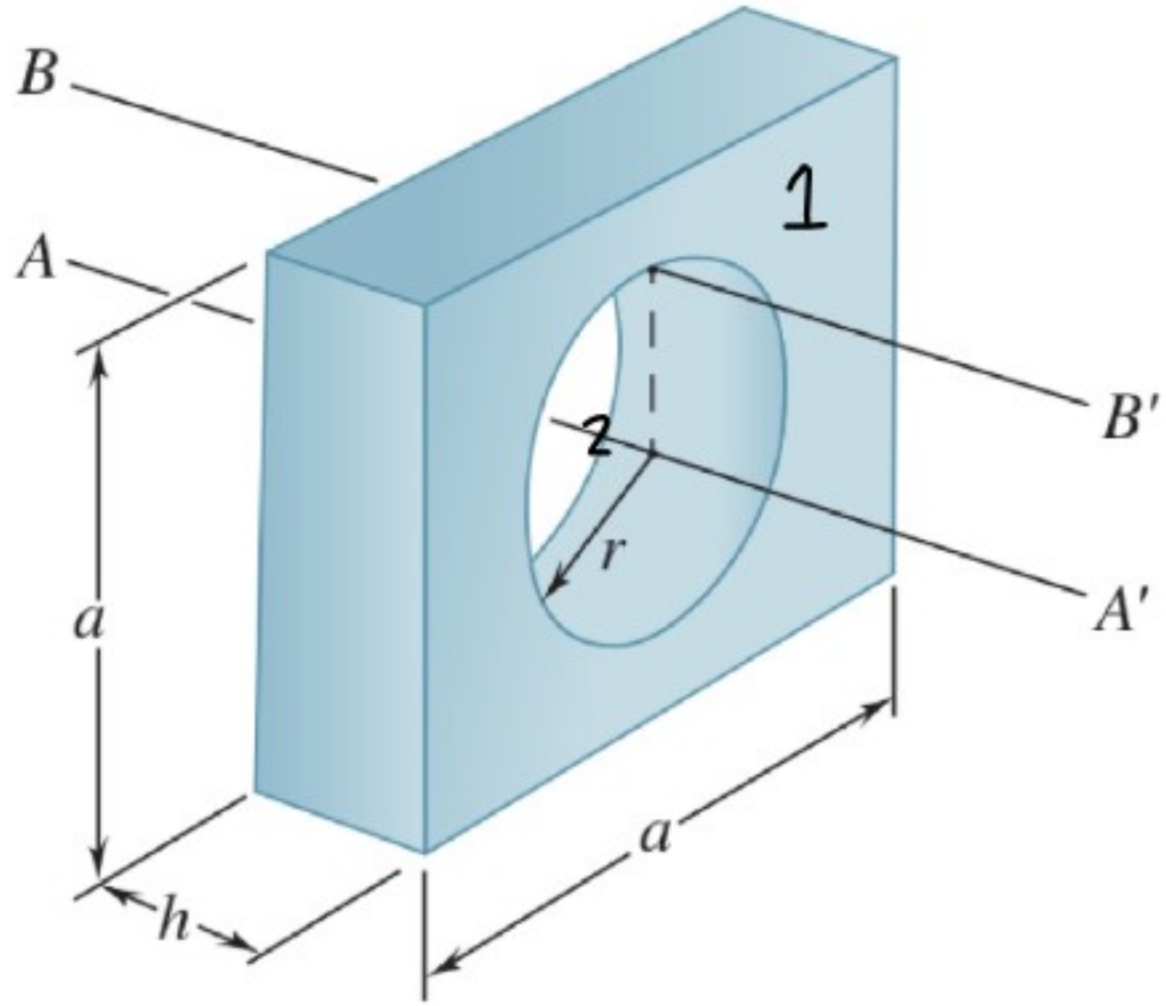
| | | |
|---|---|---|
| Slender rod |  | $I_y = I_z = \frac{1}{12} mL^2$ |
| Thin rectangular plate |  | $I_x = \frac{1}{12} m(b^2 + c^2)$<br>$I_y = \frac{1}{12} mc^2$<br>$I_z = \frac{1}{12} mb^2$ |
| Rectangular prism |  | $I_x = \frac{1}{12} m(b^2 + c^2)$<br>$I_y = \frac{1}{12} m(c^2 + a^2)$<br>$I_z = \frac{1}{12} m(a^2 + b^2)$ |

| | | |
|---|---|---|
| Thin disk |  | $I_x = \frac{1}{2} mr^2$<br>$I_y = I_z = \frac{1}{4} mr^2$ |
| Circular cylinder |  | $I_x = \frac{1}{2} ma^2$<br>$I_y = I_z = \frac{1}{12} m(3a^2 + L^2)$ |
| Circular cone |  | $I_x = \frac{3}{10} ma^2$<br>$I_y = I_z = \frac{3}{5} m\left(\frac{1}{4} a^2 + h^2\right)$ |
| Sphere |  | $I_x = I_y = I_z = \frac{2}{5} ma^2$ |




$$I_1 = \frac{1}{6} m_1 a^2 \qquad \bar{I}_1 = \frac{1}{12} m (a^2 + a^2) = \frac{1}{12} m\, 2a^2$$

$$I_2 = \frac{1}{2} m_2 r^2$$

$$\bar{I} = I_1 - I_2 = \frac{1}{6} m_1 a^2 - \frac{1}{2} m_2 r^2$$

$$I = \bar{I} + m d^2 = \frac{1}{6} m_1 a^2 - \frac{1}{2} m_2 r^2 + (m_1 - m_2) r^2$$

$$m_1 = \rho a^2 h \qquad m_2 = \rho \pi r^2 h$$

$$= \frac{1}{6} \rho a^2 h a^2 - \frac{1}{2} \rho \pi r^2 h r^2 + (\rho a^2 h - \rho \pi r^2 h) r^2$$

$$= \rho h \left( \frac{1}{6} a^4 - \frac{1}{2} \pi r^4 + a^2 r^2 - \pi r^4 \right)$$

$$= \rho h \left( \frac{1}{6} a^4 - \frac{3}{2} \pi r^4 + a^2 r^2 \right)$$

$$V \rho = m$$

$$\rho = \frac{m}{V} = \frac{m}{h(a^2 - \pi r^2)}$$

$$= \frac{m}{\cancel{h}(a^2 - \pi r^2)} \cancel{h} \left( \frac{1}{6} a^4 - \frac{3}{2} \pi r^4 + a^2 r^2 \right) = \boxed{\frac{m}{a^2 - \pi r^2} \left( \frac{1}{6} a^4 - \frac{3}{2} \pi r^4 + a^2 r^2 \right)}$$

# Virtual Work

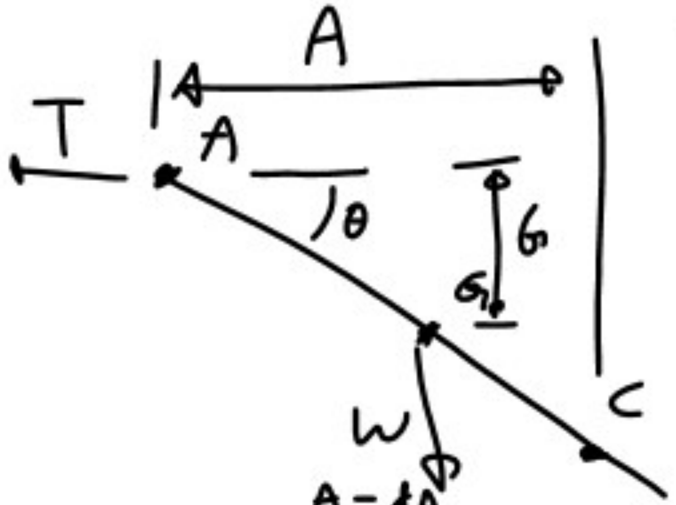
$$W = \int F \cdot dr \quad \Rightarrow \quad dW = F \cdot dr$$

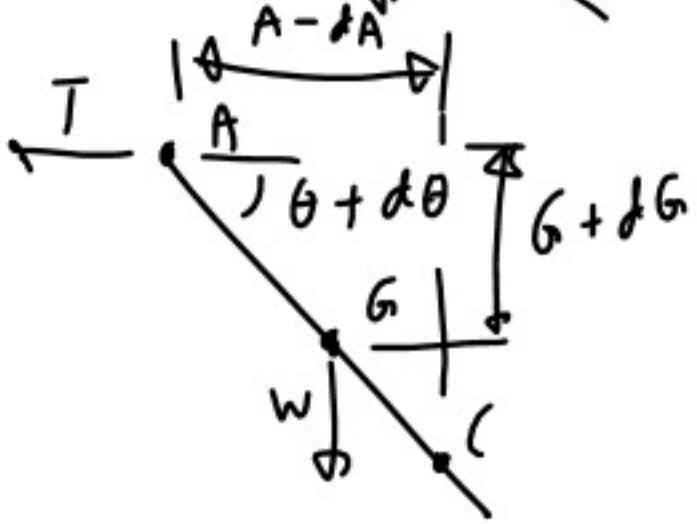
$$dW = F_1 \cdot dr_1 + F_2 \cdot dr_2 + \cdots$$

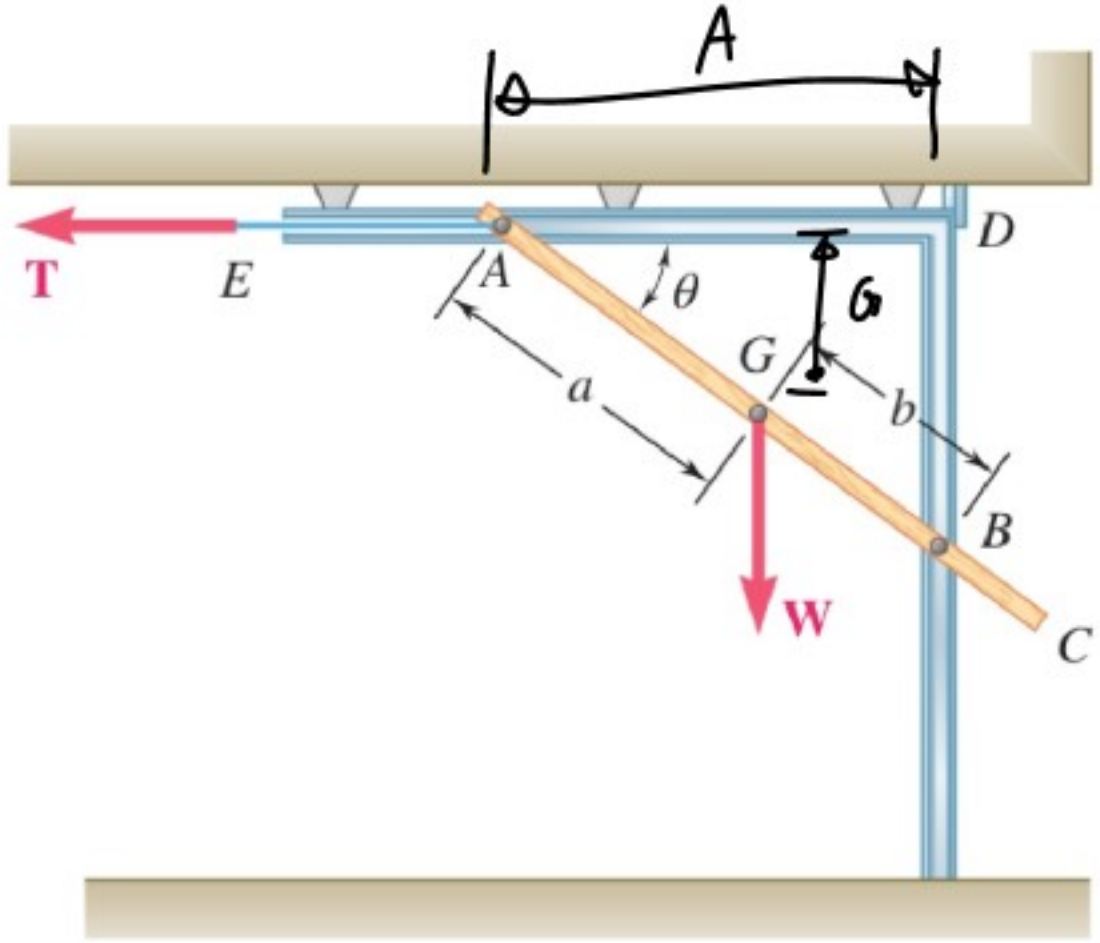
Static equilibrium $\quad dr = 0$

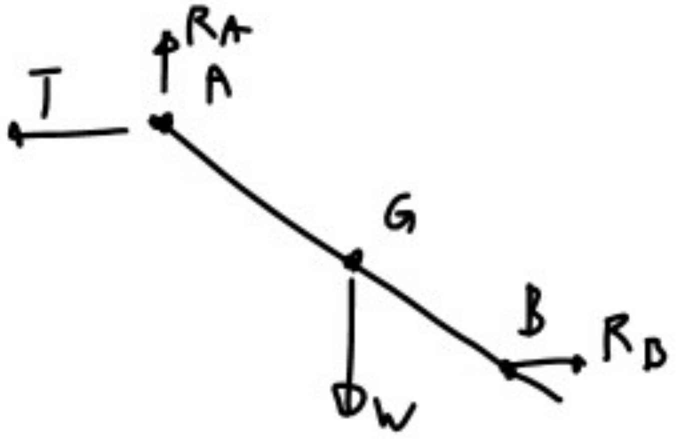
$dW_{in} = dW_{out}$ $\quad$ conservation of energy

**10.9** An overhead garage door of weight $W$ consists of a uniform rectangular panel $AC$ supported by a cable $AE$ attached at the middle of the upper edge of the door and by two sets of frictionless rollers $A$ and $B$ that can slide in horizontal and vertical channels. Express the tension $T$ in cable $AE$ in terms of $W$, $a$, $b$, and $\theta$.




$$\cos(\theta) = \frac{A}{a+b}$$

$$\cos(\theta + d\theta) = \frac{A - dA}{a+b}$$

$$\sin(\theta) = \frac{G}{a}$$

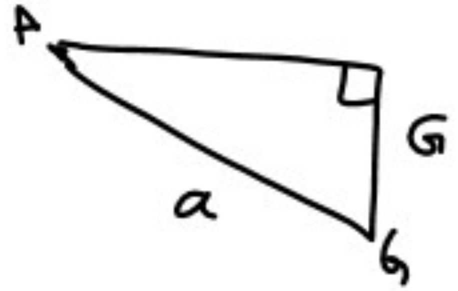
$$\sin(\theta + d\theta) = \frac{G + dG}{a}$$

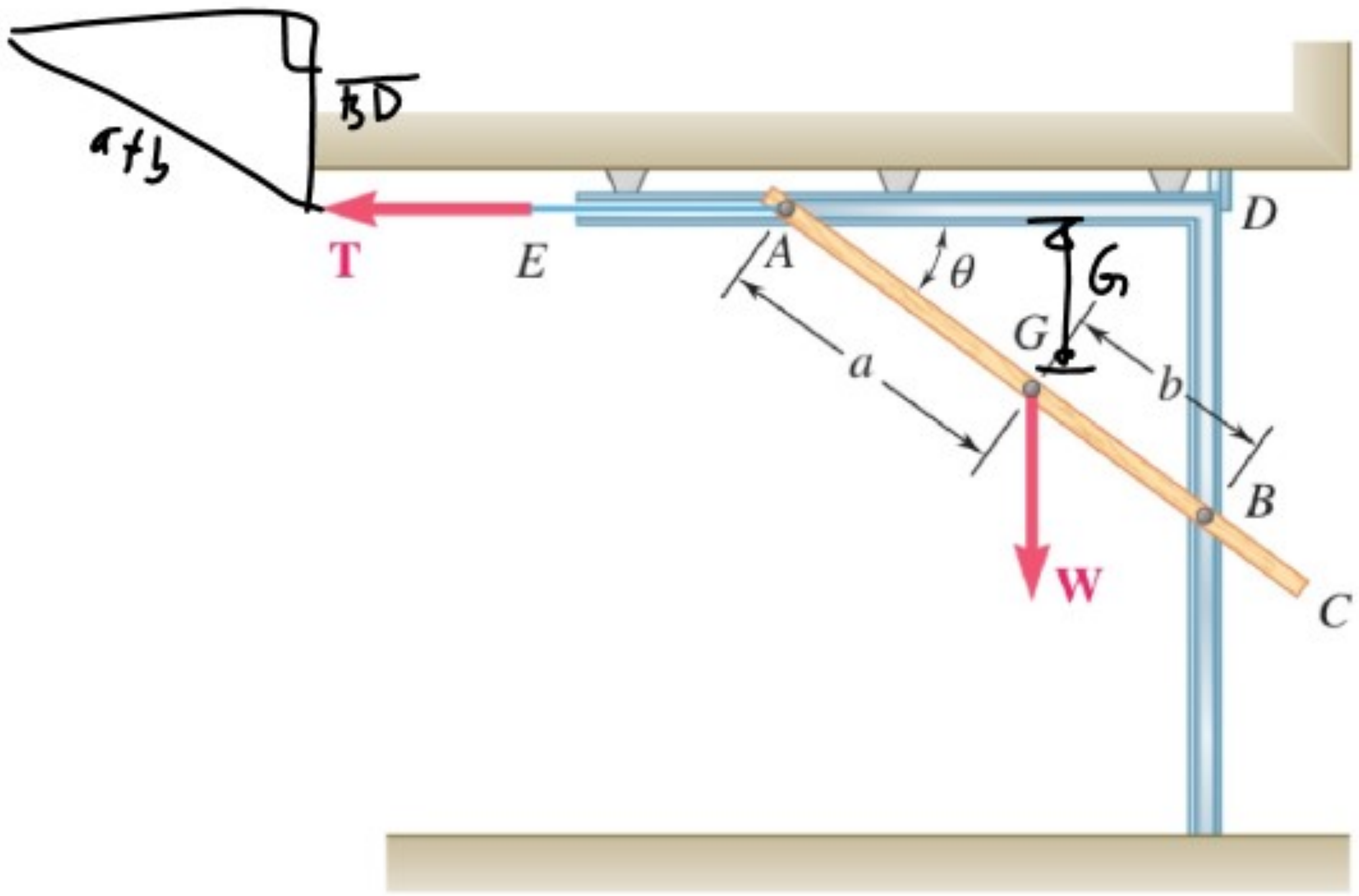
$$\cos^2(\theta + d\theta) + \sin^2(\theta + d\theta) = 1$$

$$\left(\frac{A - dA}{a+b}\right)^2 + \left(\frac{G + dG}{a}\right)^2 = 1$$

$$A^2 - 2AdA + dA^2$$

**10.9** An overhead garage door of weight $W$ consists of a uniform rectangular panel $AC$ supported by a cable $AE$ attached at the middle of the upper edge of the door and by two sets of frictionless rollers $A$ and $B$ that can slide in horizontal and vertical channels. Express the tension $T$ in cable $AE$ in terms of $W$, $a$, $b$, and $\theta$.




$$\frac{dG}{a} = \frac{dB}{a+b}$$

$$dB = \frac{(a+b)\,dG}{a}$$

$$\overline{AD}^2 + \overline{BD}^2 = (a+b)^2$$

$$(\overline{AD}+dA)^2 + (\overline{BD}-dB)^2 = (a+b)^2$$

$$(\overline{AD}+dA)^2 + \left(\overline{BD}-\left(\frac{(a+b)\,dG}{a}\right)\right)^2 = (a+b)^2$$

$$dW_{in} = dW_{out}$$

$$T\,dA = W\,dG$$

$$T = W\frac{dG}{dA}$$

$$\left(\overline{AD} + dA\right)^2 + \left(\overline{BD} - \frac{(a+b)\,d\sigma}{a}\right)^2 = (a+b)^2$$

**10.1** Determine the vertical force **P** that must be applied at $C$ to maintain the equilibrium of the linkage.