

The motion of a particle is defined by the equations $x = 10t - 5 \sin t$ and $y = 10 - 5 \cos t$, where x and y are expressed in feet and t is expressed in seconds. Sketch the path of the particle for the time interval $0 \leq t \leq 2\pi$, and determine (a) the magnitudes of the smallest and largest velocities reached by the particle, (b) the corresponding times, positions, and directions of the velocities.

t	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	$\vec{r} = (10t - 5 \sin t)\mathbf{i} + (10 - 5 \cos t)\mathbf{j}$
x	0	10.7	31.4	52.1	62.8	$\vec{v} = (10 - 5 \cos t)\mathbf{i} + (5 \sin t)\mathbf{j}$
y	5	10	15	10	5	

$$\vec{v} = (10 - 5 \cos t)\mathbf{i} + 5 \sin t \mathbf{j}$$

$$|\vec{v}| = \sqrt{(10 - 5 \cos t)^2 + (5 \sin t)^2}$$

$$(10 - 5 \cos t)(10 - 5 \cos t)$$

$$= 100 - 50 \cos t - 50 \cos t + 25 \cos^2 t$$

$$|\vec{v}| = \sqrt{100 - 100 \cos t + 25 \cos^2 t + 25 \sin^2 t}$$

$$= \sqrt{100 - 100 \cos t + 25}$$

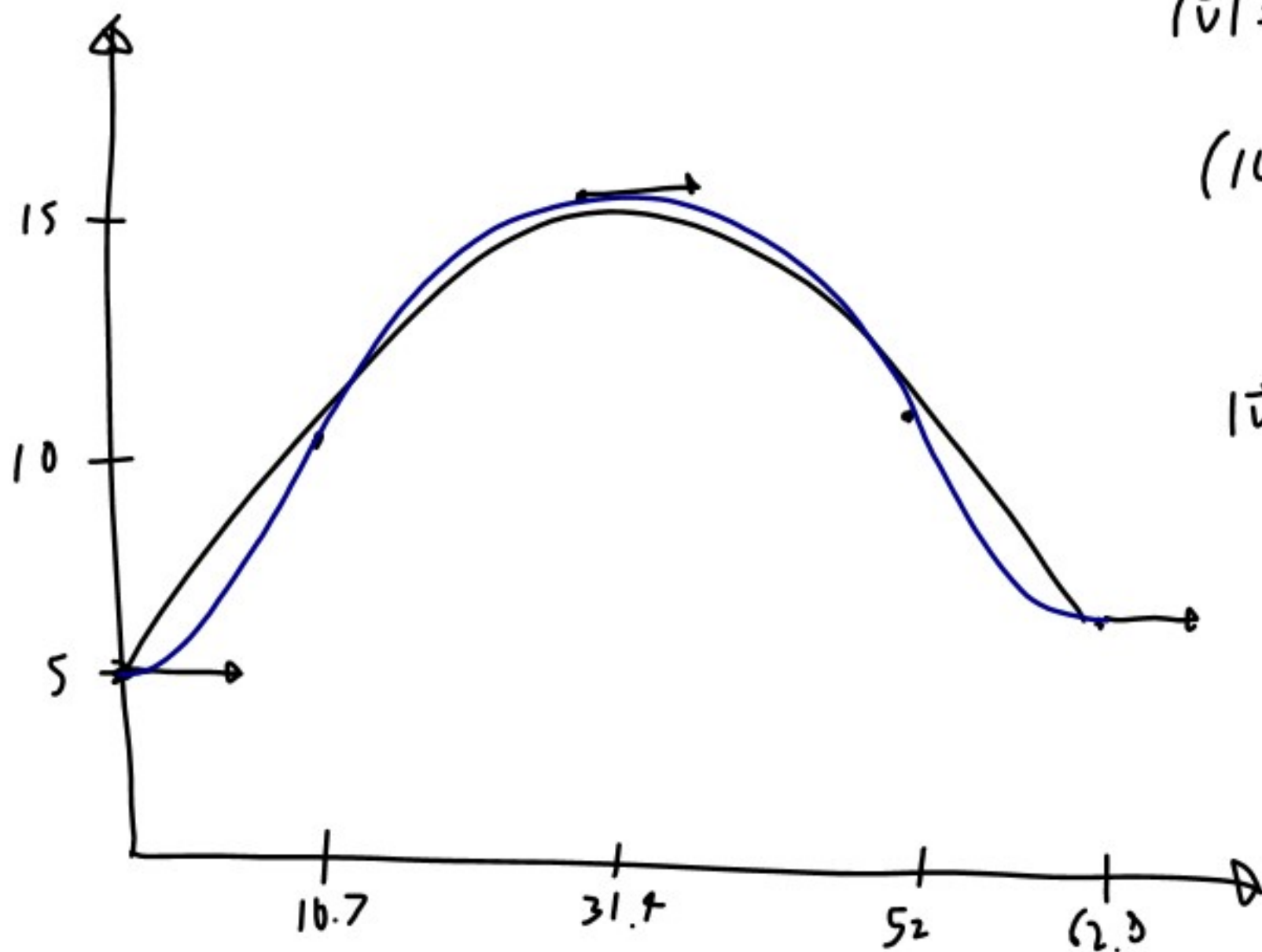
$$= \sqrt{125 - 100 \cos t}$$

$$V_{\max} \text{ when } \cos t = -1 \\ t = \pi \text{ s}$$

$$V_{\max} = \sqrt{225} \\ = 15 \text{ ft/s}$$

$$V_{\min} \text{ when } \cos t = 1 \\ t = 0, 2\pi \text{ s}$$

$$V_{\min} = \sqrt{25} = 5 \text{ ft/s}$$



v_{max} at $t = 3.14$ s

$$\vec{r} = (10t - 5 \sin t) i + (10 - 5 \cos t) j$$

$$\vec{r}(\pi) = (10\pi - 5 \sin \pi) i + (10 - 5 \cos \pi) j$$

$$= 31.4 i + 15 j \quad \frac{ft}{s}$$

$$\vec{v} = (10 - 5 \cos t) i + (5 \sin t) j$$

$$\vec{v}(\pi) = (10 - 5 \cos \pi) i + (5 \sin \pi) j$$

$$= 15 i + 0 j \quad \frac{ft}{s}$$

v_{min} at $t = 0$ s

$$\vec{r}(0) = 0 i + 5 j \quad \frac{ft}{s}$$

$$\vec{v}(0) = (10 - 5 \cos 0) i + (5 \sin 0) j$$

$$= 5 i + 0 j \quad \frac{ft}{s}$$

v_{min} at $t = 2\pi$ s

$$\vec{r}(2\pi) = 62.8 i + 5 j \quad \frac{ft}{s}$$

$$\vec{v}(2\pi) = (10 - 5 \cos 2\pi) i + (5 \sin 2\pi) j$$

$$= 5 i + 0 j \quad \frac{ft}{s}$$

A ball is thrown so that the motion is defined by the equations $x = 5t$ and $y = 2 + 6t - 4.9t^2$, where x and y are expressed in meters and t is expressed in seconds. Determine (a) the velocity at $t = 1$ s, (b) the horizontal distance the ball travels before hitting the ground.

