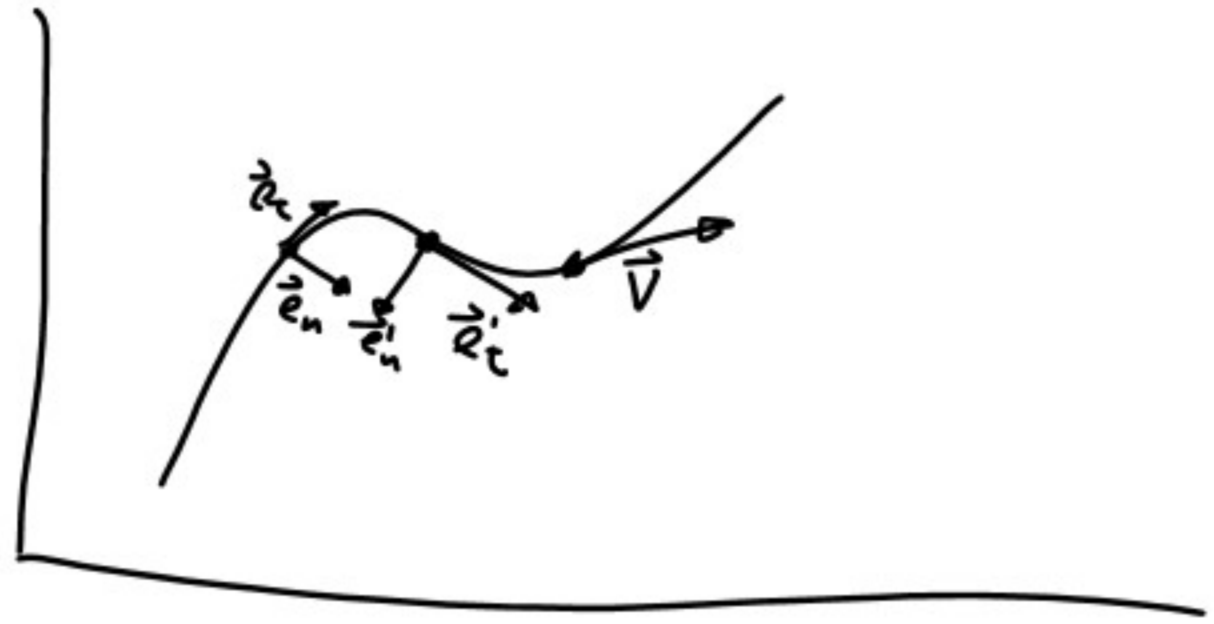
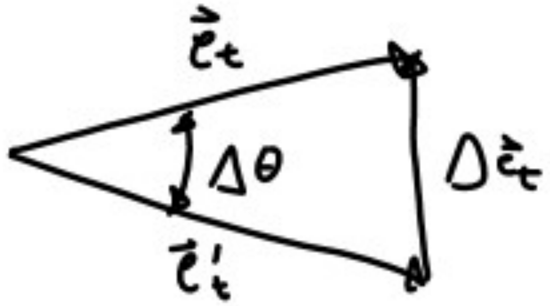
Tangential and Normal Components



$\vec{e}_t$ and $\vec{e}_n$ unit vectors

$$\Delta \vec{e}_t = \vec{e}_t' - \vec{e}_t$$



$$|\Delta \vec{e}_t| = 2\sin\left(\Delta\theta/2\right)$$

$$\lim_{\Delta\theta \to 0} \frac{|\Delta \vec{e}_t|}{\Delta\theta} = \lim_{\Delta\theta \to 0} \frac{2\sin(\Delta\theta/2)}{\Delta\theta} = 1$$

$$\vec{e}_n = \frac{d\vec{e}_t}{d\theta}$$

$\vec{V} = V\vec{e}_t$

$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{d}{dt} V\vec{e}_t = \frac{dV}{dt}\vec{e}_t + V\frac{d\vec{e}_t}{dt}$$

$$= \frac{dV}{dt}\vec{e}_t + \frac{V^2}{\rho}\vec{e}_n$$

$$\frac{d\vec{e}_t}{dt} = \frac{d\vec{e}_t}{d\theta}\frac{d\theta}{ds}\frac{ds}{dt}$$

$$= \vec{e}_n \frac{1}{\rho} V$$

ρ curvature

$\rho = \frac{1}{r}$

straight line

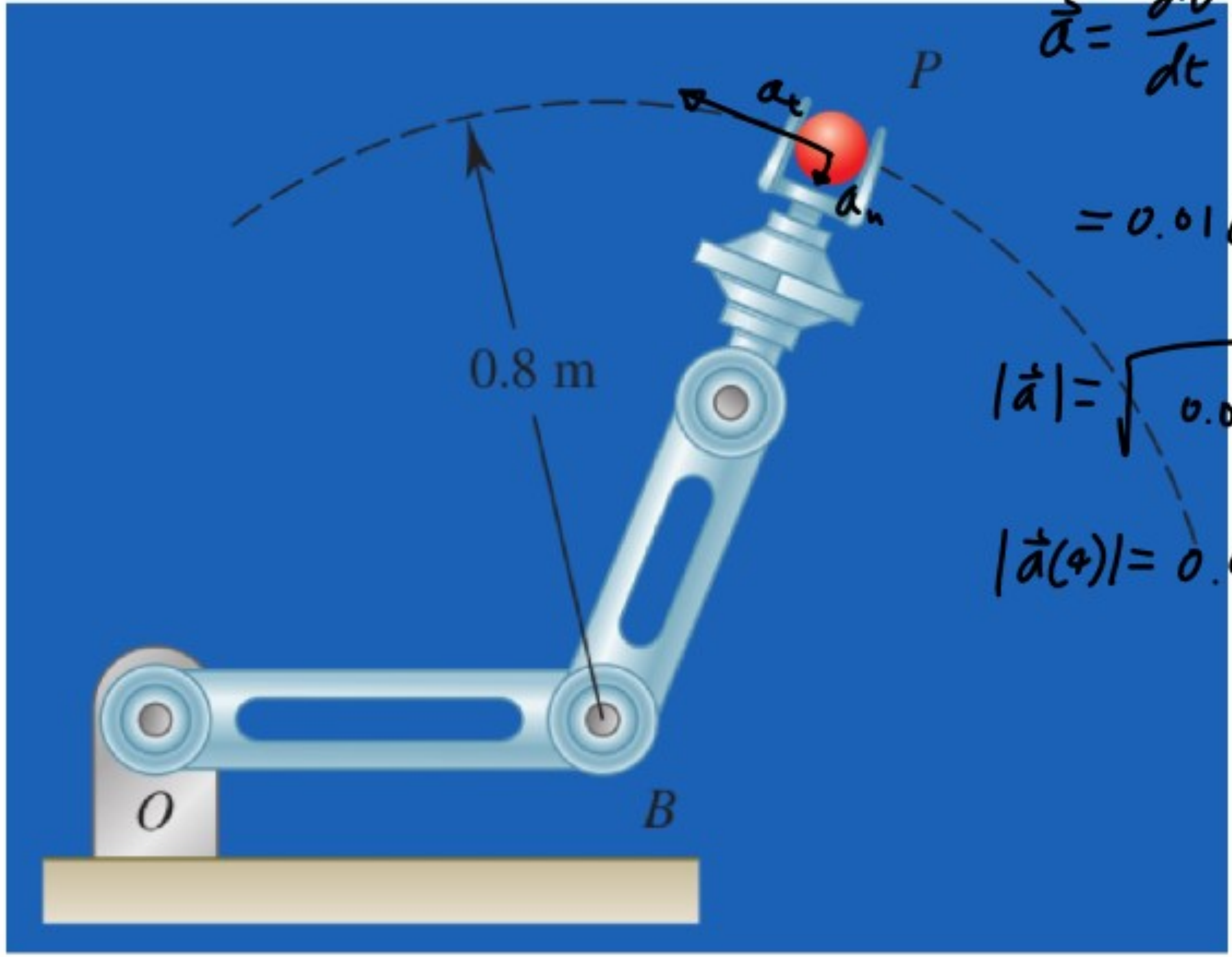
$r = \infty \quad \rho = \frac{1}{\infty} = 0$

A robot arm moves so that P travels in a circle about point B, which is not moving. Knowing that P starts from rest, and its speed increases at a constant rate of 10 mm/s^2, determine (*a*) the magnitude of the acceleration when $t = 4$ s, (*b*) the time for the magnitude of the acceleration to be 80 mm/s^2.

$V = V_0 + at$

$= 0.01t$

$\rho = \frac{1}{r} = \frac{1}{0.8} = 1.25 \ {}^{1}/_{m}$



$\vec{a} = \frac{dv}{dt}\vec{e}_t + \frac{v^2}{\rho}\vec{e}_n$

$= 0.01\vec{e}_t + \frac{(0.01t)^2}{1.25}\vec{e}_n$

$|\vec{a}| = \sqrt{0.01^2 + \left(\frac{(0.01t)^2}{1.25}\right)^2}$

$|\vec{a}(4)| = 0.01008 \ {}^{m}/_{s^2}$

$$0.08 = \sqrt{0.01^2 + \left(\frac{(0.01t)^2}{1.25}\right)^2}$$

$$0.0064 = 0.01^2 + \left(\frac{(0.01t)^2}{1.25}\right)^2$$

$$0.0064 - 0.0001 = \left(\frac{(0.01t)^2}{1.25}\right)^2$$

$$0.0063 = \left(\frac{(0.01t)^2}{1.25}\right)^2$$

$$0.079 = \frac{(0.01t)^2}{1.25}$$

$$0.099 = 0.01^2 t^2$$

$$\frac{0.099}{0.01^2} = t^2$$

$$t^2 = 992$$

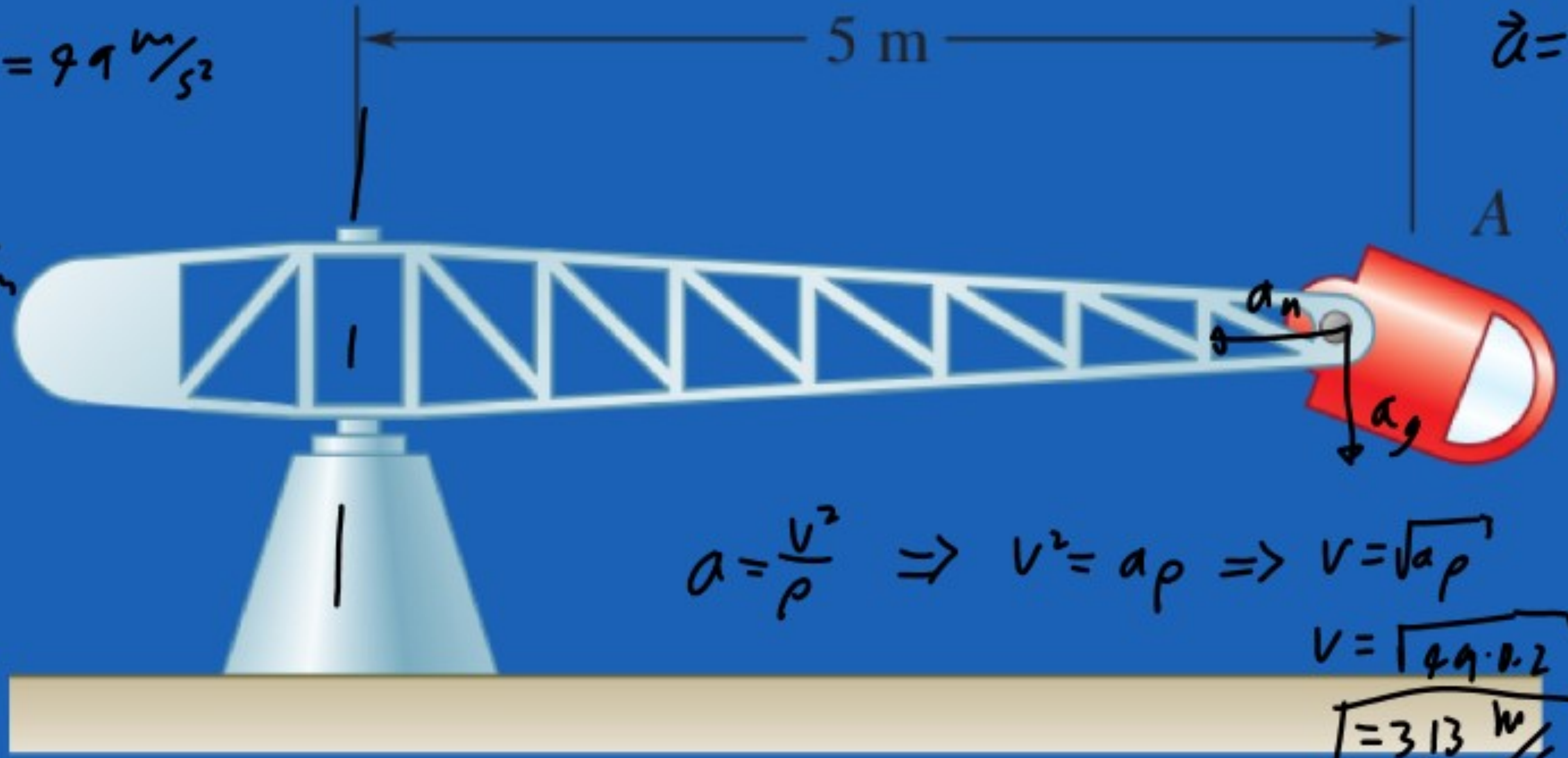
$$\boxed{t = 31.5}$$

Human centrifuges are often used to simulate different acceleration levels for pilots and astronauts. Pilots typically face inward toward the center of the gondola in order to experience a simulated forward acceleration. Knowing that the pilots sits 5 m from the axis of rotation and experiences 5 g's inward, determine her velocity.

$5g = 5 \cdot 9.8\ \text{m}/\text{s}^2 = 49\ \text{m}/\text{s}^2$

$\rho = 1/r = 1/5 = 0.2\ 1/\text{m}$

$\vec{a} = \frac{dv}{dt}\vec{e}_t + \frac{v^2}{\rho}\vec{e}_n$

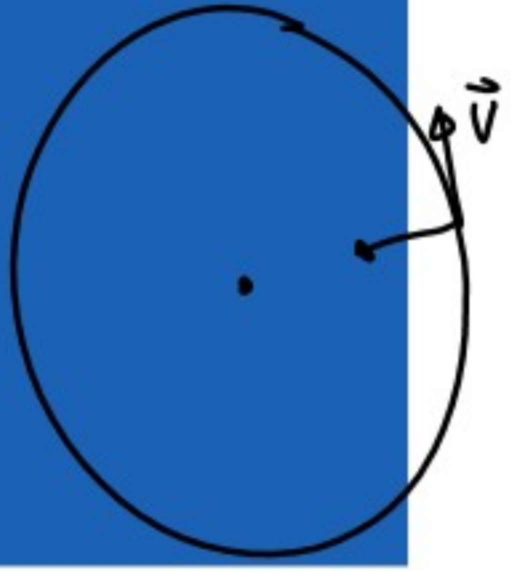


$\vec{a} = \frac{v^2}{\rho}\vec{e}_n$

$a = \frac{v^2}{\rho} \Rightarrow v^2 = a\rho \Rightarrow v = \sqrt{a\rho}$

$v = \sqrt{49 \cdot 0.2}$

$\boxed{= 3.13\ \text{m}/\text{s}}$



$\sim \boxed{7\ \text{mph}}$