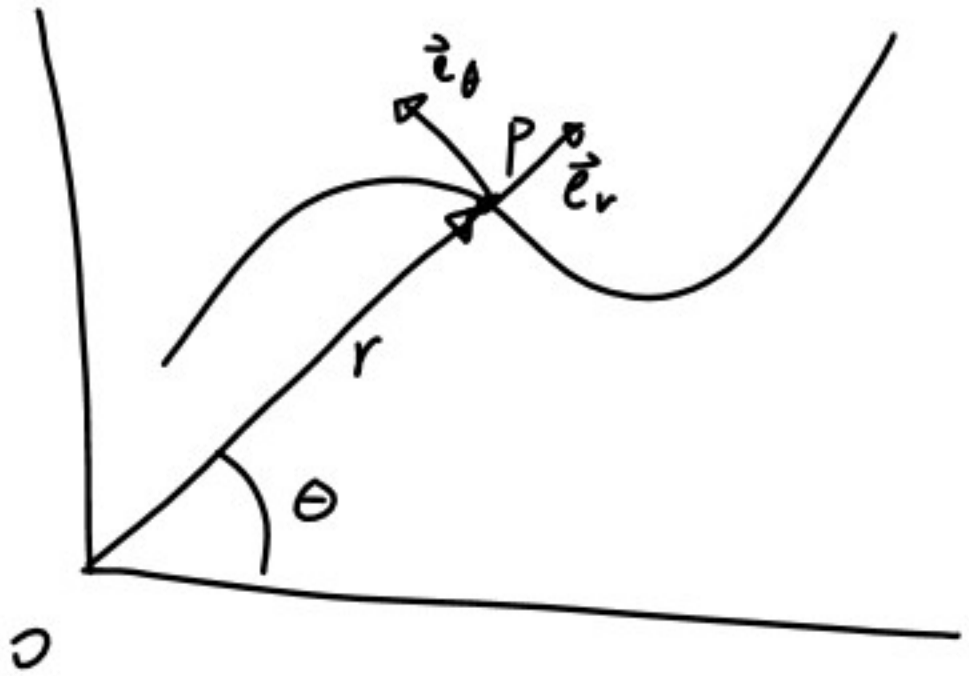
Radial and Transverse Components



$\vec{e}_r$ and $\vec{e}_\theta$ unit vectors

$$\frac{d\vec{e}_r}{d\theta} = \vec{e}_\theta \qquad \frac{d\vec{e}_\theta}{d\theta} = -\vec{e}_r$$

$$\frac{d\vec{e}_r}{dt} = \frac{d\vec{e}_r}{d\theta}\frac{d\theta}{dt} \qquad \frac{d\vec{e}_\theta}{dt} = -\vec{e}_r\frac{d\theta}{dt}$$

$$= \vec{e}_\theta \frac{d\theta}{dt} \qquad \dot{\vec{e}}_\theta = -\dot{\theta}\,\vec{e}_r$$

$$\dot{\vec{e}}_r = \dot{\theta}\,\vec{e}_\theta$$

$$\vec{V} = \frac{d}{dt} r\vec{e}_r = \dot{r}\vec{e}_r + r\dot{\vec{e}}_r = \dot{r}\vec{e}_r + r\dot{\theta}\vec{e}_\theta$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt} \dot{r}\vec{e}_r + r\dot{\theta}\vec{e}_\theta$$

$$= \ddot{r}\vec{e}_r + \dot{r}\dot{\vec{e}}_r + \dot{r}\dot{\theta}\vec{e}_\theta + r\ddot{\theta}\vec{e}_\theta + r\dot{\theta}\dot{\vec{e}}_\theta$$

$$= (\ddot{r} - r\dot{\theta}^2)\vec{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\vec{e}_\theta$$

$$r\dot{\theta}\dot{\vec{e}}_\theta = r\dot{\theta}(-\dot{\theta}\vec{e}_r)$$

$$= -r\dot{\theta}^2\vec{e}_r$$

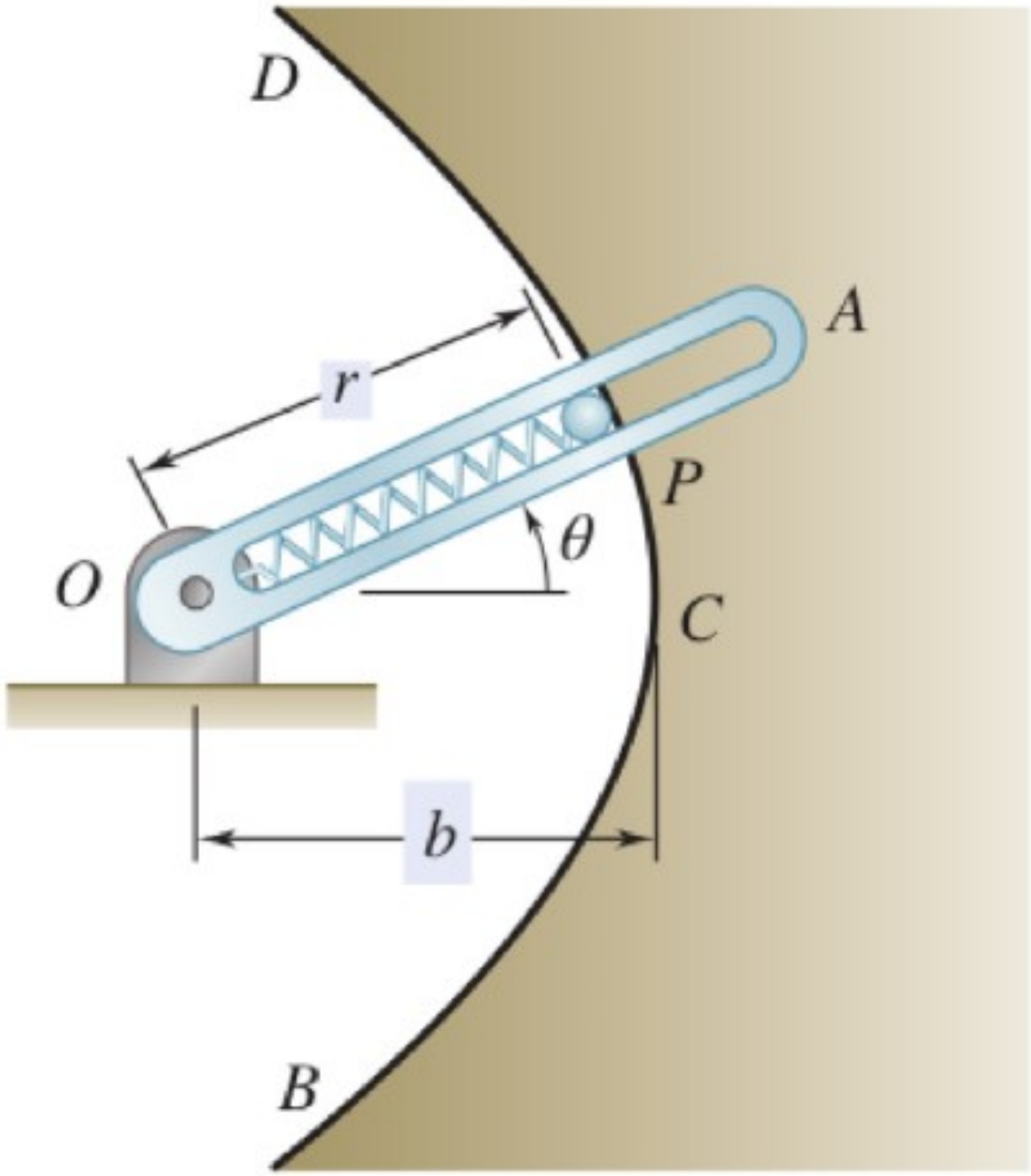
As rod OA rotates, pin P moves along the parabola BCD. Knowing that the equation of this parabola is $r = 2b/(1 + \cos\theta)$ and that $\theta = kt$, determine the velocity and acceleration of P when (*a*) $\theta = 0$, (*b*) $\theta = 90°$.

$$\vec{V} = \dot{r}\vec{e}_r + r\dot{\theta}\vec{e}_\theta$$

$$= \left(\frac{2bK\sin Kt}{(1+\cos Kt)^2}\right)\vec{e}_r + \frac{2b}{1+\cos Kt}K\,\vec{e}_\theta$$

$$\vec{V}(0) = 0\vec{e}_r + bK\vec{e}_\theta$$

$$\vec{V}\left(\frac{90°}{K}\right) = 2bK\vec{e}_r + 2bK\vec{e}_\theta$$



$$r = \frac{2b}{1+\cos\theta} \qquad \theta = Kt$$

$$r = \frac{2b}{1+\cos Kt} \qquad \dot{\theta} = K$$

$$\dot{r} = \frac{2bK\sin Kt}{(1+\cos Kt)^2} \qquad 90° = Kt$$

$$t = \frac{90°}{K}$$

$$\ddot{\theta} = 0$$

$$\ddot{r} = 2b\left(\frac{K^2\cos Kt}{(1+\cos Kt)^2} + \frac{2K^2\sin^2 Kt}{(1+\cos Kt)^3}\right)$$

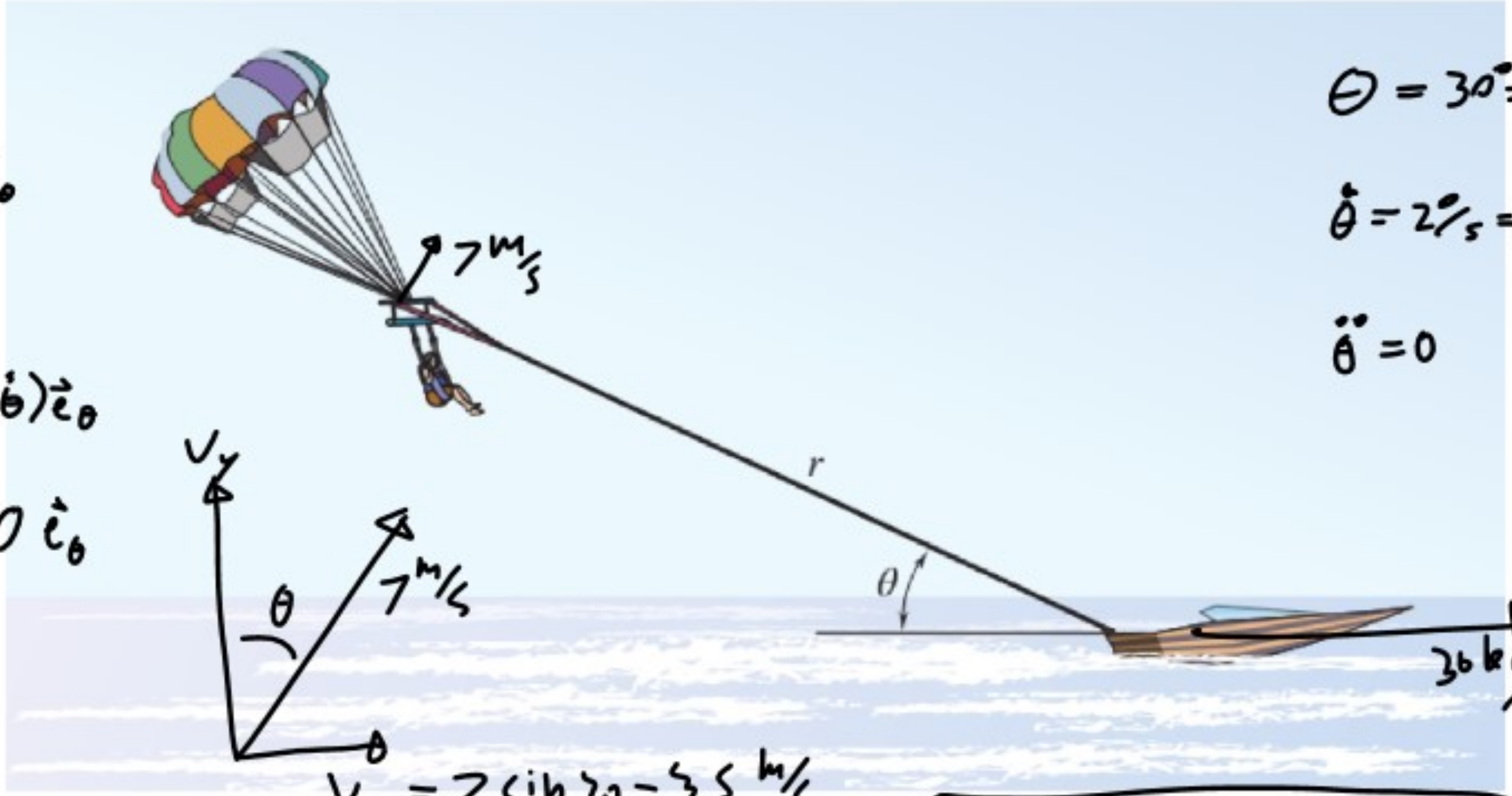
$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\vec{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\vec{e}_\theta$$

$$= \left(2b\left(\frac{k^2\cos kt}{(1+\cos kt)^2} + \frac{2k^2\sin^2 kt}{(1+\cos kt)^3}\right) - \frac{2bk\sin kt}{(1+\cos kt)^2}k^2\right)\vec{e}_r + \left(\frac{2b}{1+\cos kt}\,0 + 2\,\frac{2bk\sin kt}{(1+\cos kt)^2}\,k\right)\vec{e}_\theta$$

$$\vec{a}(0) = \left(2b\left(\frac{k^2}{4}\right)\right)\vec{e}_r + 0\,\vec{e}_\theta$$

$$a\left(90^\circ/k\right) = \left(2b\,(2k^2) - 2bk^3\right)\vec{e}_r + (4bk^2)\vec{e}_\theta$$

During a parasailing ride, the boat is traveling at a constant 30 km/hr with a 200-m long tow line. At the instant shown, the angle between the line and the water is 30° and is increasing at a constant rate of 2°/s. Determine the velocity and acceleration of the parasailer at this instant.



$\vec{V} = \dot{r}\vec{e}_r + r\dot{\theta}\vec{e}_\theta$

$= 0\vec{e}_r + 200\,m \; 0.035\,\frac{rad}{s}\,\vec{e}_\theta$

$= 7\vec{e}_\theta \ \frac{m}{s}$

$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\vec{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\vec{e}_\theta$

$= 200\,m\,(0.035\,\frac{rad}{s})^2\vec{e}_r + 0\,\vec{e}_\theta$

$\theta = 30° = 0.52$ rad $\quad r = 200$ m

$\dot{\theta} = 2°/s = 0.035\ \frac{rad}{s} \quad \dot{r} = 0$

$\ddot{r} = 0$

$\ddot{\theta} = 0$

$30\,\frac{km}{h}\left(\frac{1000\,m}{1\,km}\right)\left(\frac{1\,h}{60\,min}\right)\left(\frac{1\,min}{60\,s}\right)$

$= 8.33\ \frac{m}{s}$

$V_x = 7\sin 30 = 3.5\ \frac{m}{s}$

$V_y = 7\cos 30 = 6.02$

$\boxed{V_{tot} = (3.5 + 8.33)\,i + 6.02\,j}$