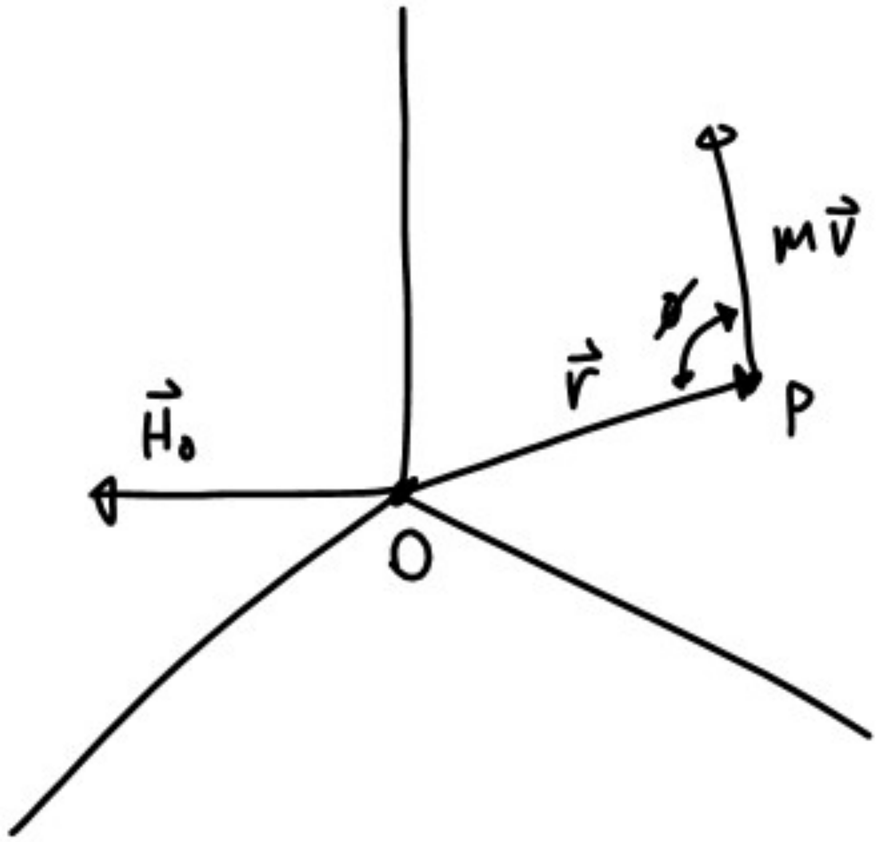
Angular Momentum

$\vec{H}_o = \vec{r} \times m\vec{v}$

$H_o = r m v \sin\phi$

$H_o = m r^2 \dot{\theta}$

H_o constant (conservation of energy)



A space vehicle is in a circular orbit with a 1400-mi radius around the moon. To transfer to a smaller orbit with a 1300-mi radius, the vehicle is first placed in an elliptic path *AB* by reducing its speed by 86 ft/s as it passes through *A*. Knowing that the mass of the moon is 5.03×10^{21} lb·s^2/ft, determine (*a*) the speed of the vehicle as it approaches *B* on the elliptic path, (*b*) the amount by which its speed should be reduced as it approaches *B* to insert it into the smaller circular orbit.

$$1400 \text{ mi}\left(\frac{5280 \text{ ft}}{1 \text{ mi}}\right) = 7.39\times10^6 \text{ ft}$$

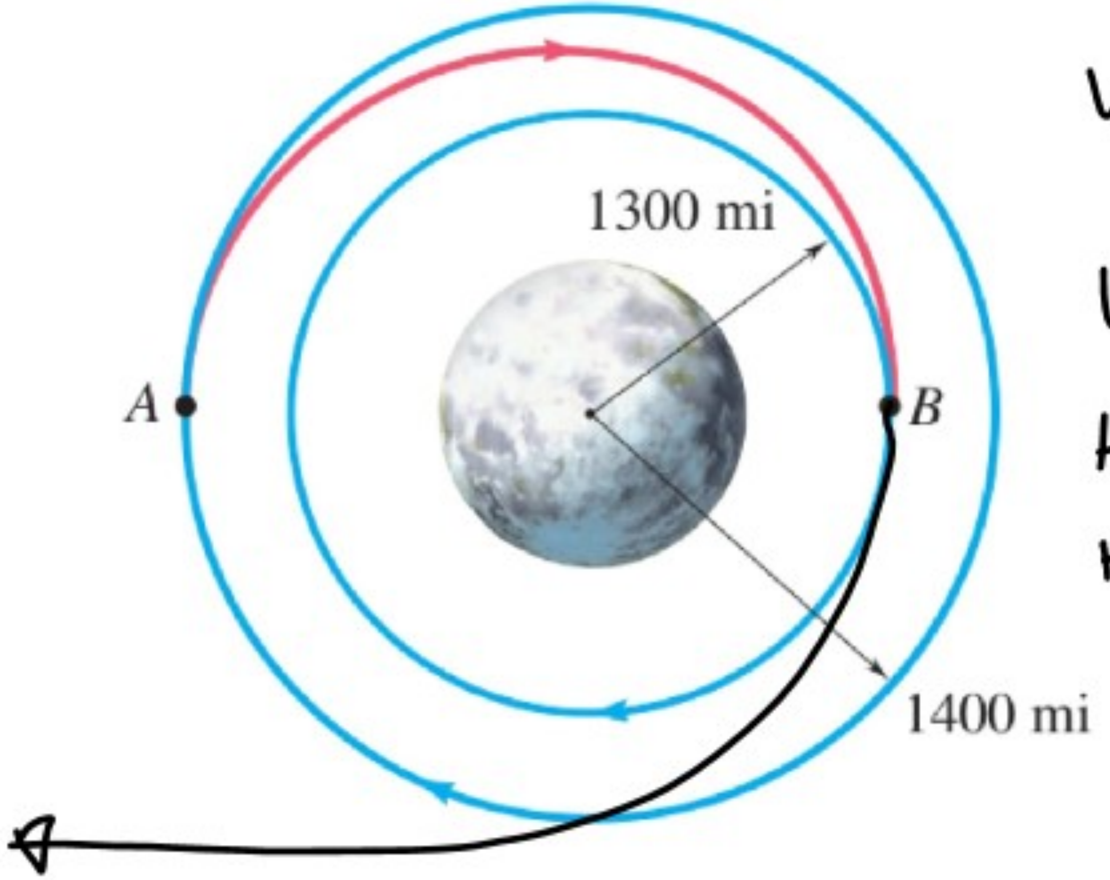
$$1300 \text{ mi}\left(\frac{5280 \text{ ft}}{1 \text{ mi}}\right) = 6.86\times10^6 \text{ ft}$$

$$\boxed{m} \xrightarrow{F} = \boxed{m} \xrightarrow{ma}$$

$$a = \frac{v^2}{r} \qquad F = m\frac{v^2}{r}$$

$$F = \frac{GM\cancel{m}}{r^{\cancel{2}}} = \cancel{m}\frac{v^2}{\cancel{r}}$$

$$\frac{GM}{r} = v^2 \qquad \boxed{v = \sqrt{\frac{GM}{r}}}$$



$$V_A = \sqrt{\frac{34.4\times10^{-9}\; 5.03\times10^{21}}{7.39\times10^6}} = 4838 \text{ ft/s}$$

$$V_{ABO} = 4752 \text{ ft/s}$$

$$H_{ABO} = 7.39\times10^6\, m\, 4752$$

$$H_{ABf} = 6.86\times10^6\, m\, V_{ABf}$$

$$7.39\times10^6\, \cancel{m}\, 4752 = 6.86\times10^6\, \cancel{m}\, V_{ABf}$$

$$\frac{7.39\times10^6\; 4752}{6.86\times10^6} = \boxed{5116 \text{ ft/s}}$$

$$V_B = \sqrt{\frac{34.4\times10^{-9}\ 5.03\times10^{21}}{6.86\times10^{6}}} = 5022\ ft/s$$

$$5022 - 5116 = \boxed{-94\ ft/s}$$

To place a communications satellite into a geosynchronous orbit (see Prob. 12.80) at an altitude of 22,240 mi above the surface of the earth, the satellite first is released from a space shuttle, which is in a circular orbit at an altitude of 185 mi, and then is propelled by an upper-stage booster to its final altitude. As the satellite passes through A, the booster's motor is fired to insert the satellite into an elliptical transfer orbit. The booster is again fired at B to insert the satellite into a geosynchronous orbit. Knowing that the second firing increases the speed of the satellite by 4810 ft/s, determine (*a*) the speed of the satellite as it approaches B on the elliptic transfer orbit, (*b*) the increase in speed resulting from the first firing at A.

$GM = 14.08 \times 10^{15}\ \frac{ft^3}{s^2}$

$r_A = R + 185$

$= 4145\ mi \left(\frac{5280\ ft}{1\ mi}\right)$

$= 21.9 \times 10^6\ ft$

$r_B = R + 22240$

$= 26200\ mi \left(\frac{5280\ ft}{1\ mi}\right)$

$= 138 \times 10^6\ ft$

$V_A = \sqrt{\frac{GM}{r_A}} = \sqrt{\frac{14.08 \times 10^{15}}{21.9 \times 10^6}} = 25.4 \times 10^3\ ft/s$

$V_B = \sqrt{\frac{GM}{r_B}} = \sqrt{\frac{14.08 \times 10^{15}}{138 \times 10^6}} = 10.1 \times 10^3\ ft/s$

$V_{ABf} = 10.1 \times 10^3 - 4810 = \boxed{5291\ ft/s}$

$r_A m V_{ABo} = r_B m V_{ABf}$

$\frac{r_B V_{ABf}}{r_A} = \frac{138 \times 10^6\ 5291}{21.9 \times 10^6} = 33.3 \times 10^6\ ft/s$

$33.3 \times 10^6 - 21.4 \times 10^6 = \boxed{11.4 \times 10^6\ ft/s}$