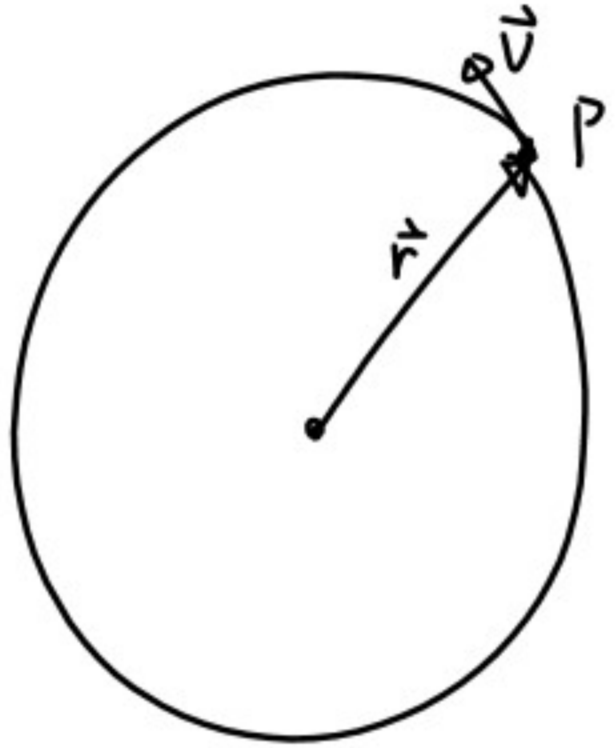
Work and Energy

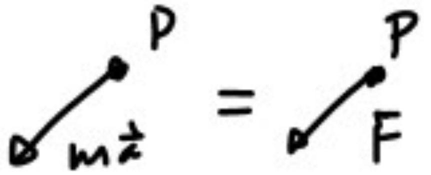
$d\vec{U} = \vec{F} \cdot d\vec{r}$

$\int \vec{F} \cdot d\vec{r} = W$

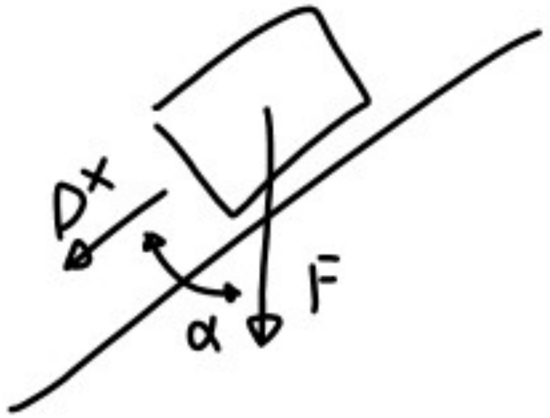
$\Delta U = W$

$W = F \Delta r \cos\alpha$

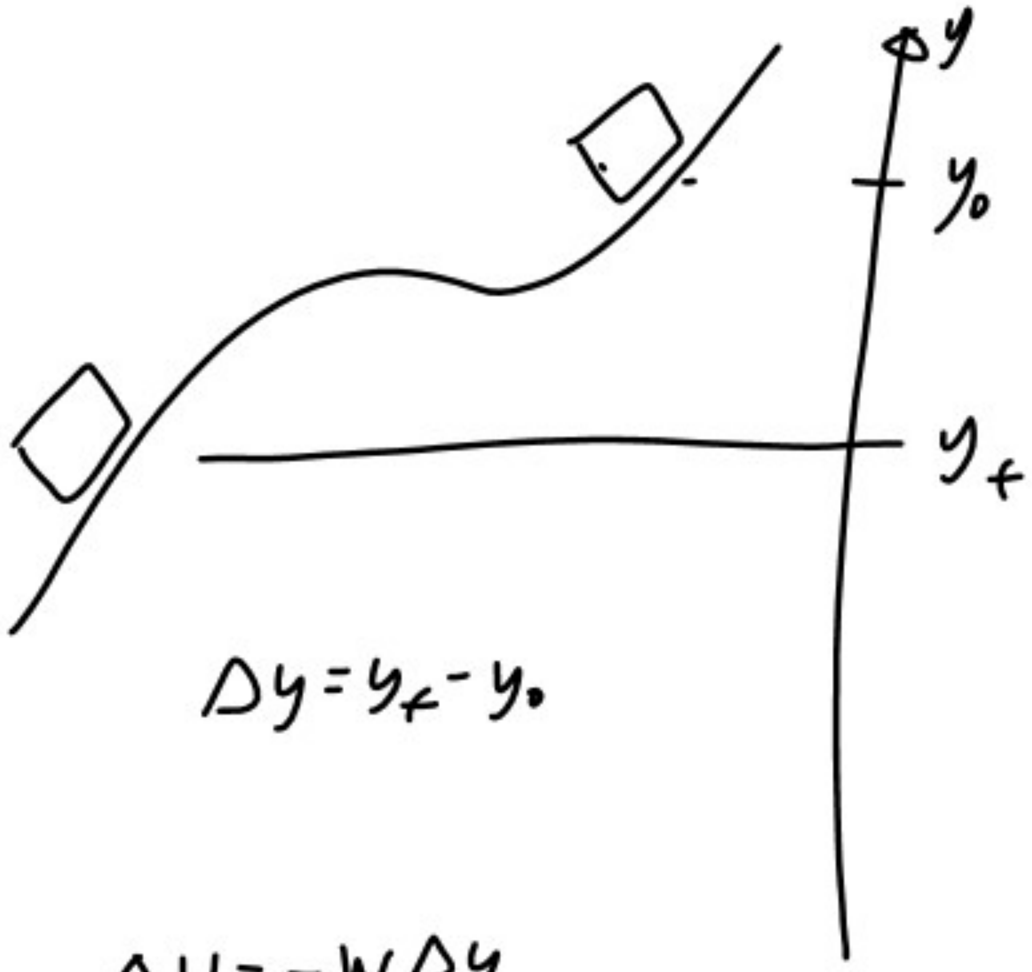




$W = F \Delta r \cos 90°$
$= 0$

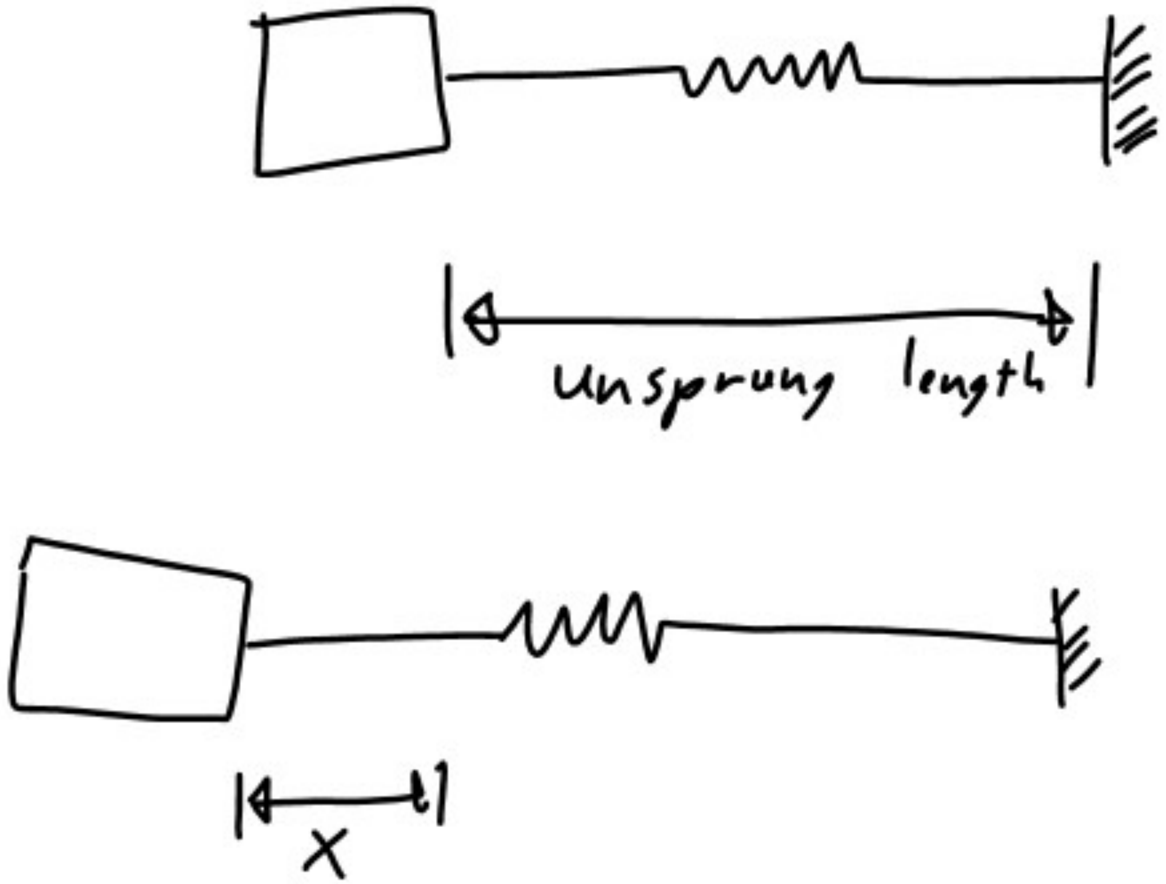


$$W = \Delta U = F \Delta x \cos\alpha$$



$$\Delta y = y_f - y_0$$

$$\Delta U = -W \Delta y$$



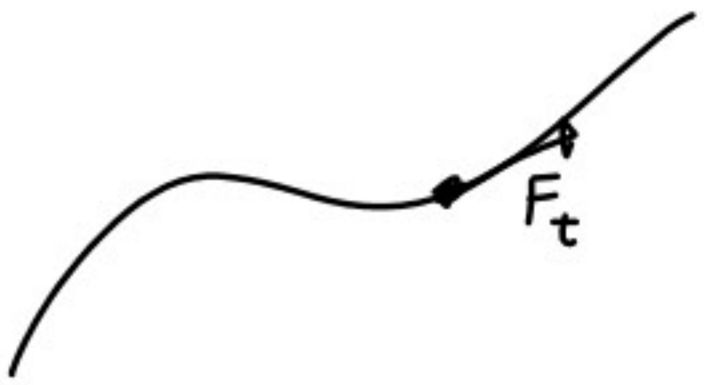
$$F = -kx$$

$$\Delta U = W = \int_{x_1}^{x_2} -F \cdot dx = \int_{x_1}^{x_2} -kx \, dx = -k\frac{x^2}{2}\Big|_{x_1}^{x} = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2$$

$$dU = -F\,dr$$

$$\int_{U_1}^{U_2} dU = \int_{r_1}^{r_2} -F\,dr$$

$$= \int_{r_1}^{r_2} \frac{-GMm}{r^2}\,dr = \frac{GMm}{r}\Bigg|_{r_1}^{r_2} = \frac{GMm}{r_2} - \frac{GMm}{r_1} = W = \Delta U$$



$$\vec{F}_t = m\vec{a} = m\frac{d\vec{v}}{dt}$$

$$F_t = m\frac{dv}{dt} = m\frac{dv}{ds}\frac{ds}{dt}$$

$$\int F_t\,ds = \int_{v_1}^{v_2} m v\,dv$$

$$W = m\frac{v^2}{2}\Big|_{v_1}^{v_2} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = \Delta U$$

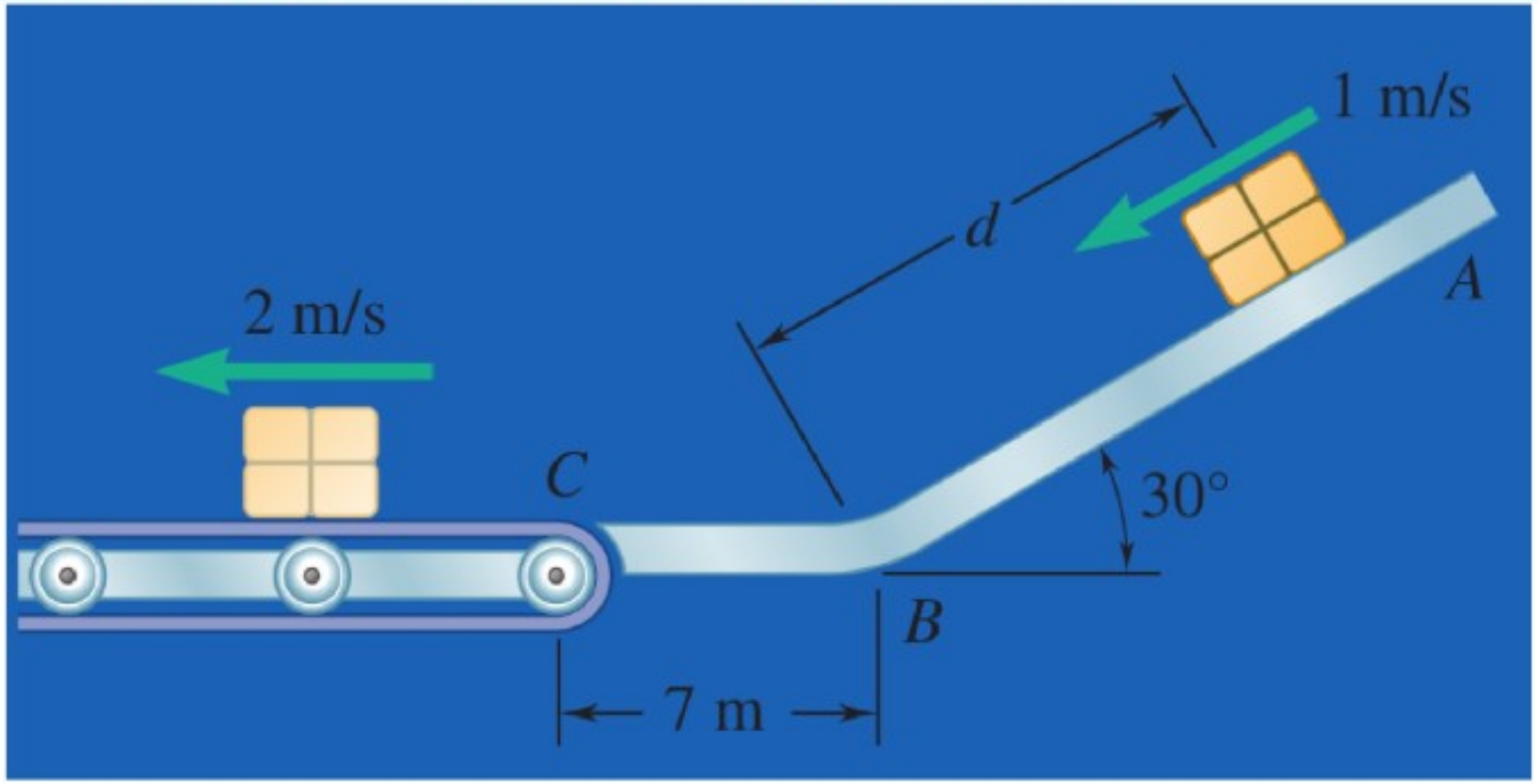
$$T = \frac{1}{2}mv^2 \qquad \Delta U = T_2 - T_1$$

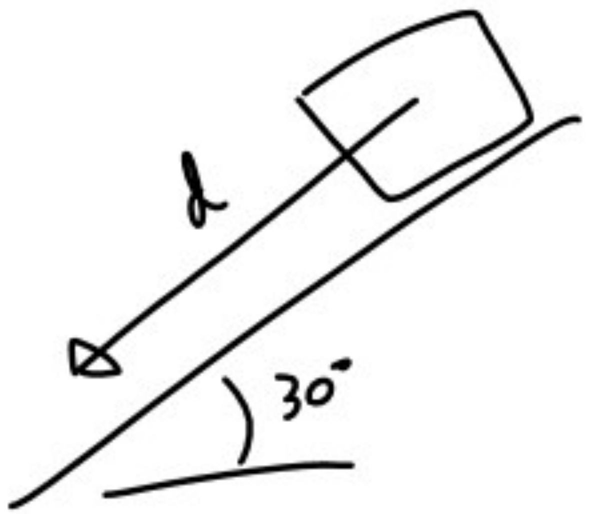
Power

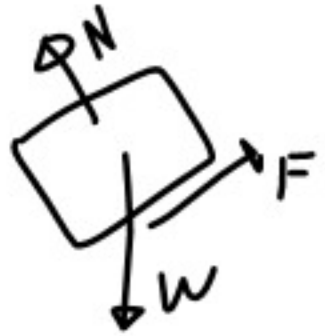
$$\text{Power} = \frac{dU}{dt} = \frac{F \cdot dr}{dt} = \frac{d}{dt} F \cdot dr = \cancel{\frac{d}{dt}}\, F \cdot dr + F \cdot \frac{dr}{dt} = F \cdot \frac{dr}{dt} = F \cdot v$$

$\searrow 0$

Packages are thrown down an incline at A with a velocity of 1 m/s. The packages slide along the surface ABC to a conveyor belt which moves with a velocity of 2 m/s. Knowing that $\mu_k = 0.25$ between the packages and the surface ABC, determine the distance d if the packages are to arrive at C with a velocity of 2 m/s.







$$F = 0.25\,N = 0.25\,W\cos 30$$

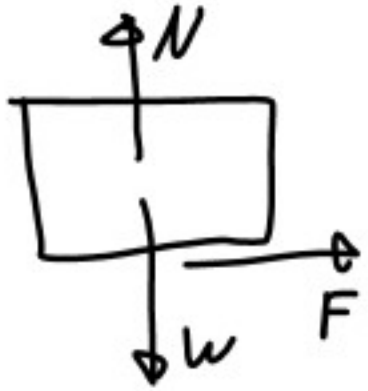
$$N = W\cos 30^\circ$$

$$W_d = W\cos 60^\circ$$

$$F_d = W_d - F = W\cos 60 - 0.25\,W\cos 30$$

$$W_{AB} = F\,d = (W\cos 60 - 0.25\,W\cos 30)\,d$$

$$\Delta U = T_2 - T_1 = \frac{1}{2} m v_C^2 - \frac{1}{2} m v_A^2 = \frac{1}{2} m\, 2^2 - \frac{1}{2} m\, 1^2 = 2m - \frac{m}{2} = \frac{3}{2} m$$



$N = W$

$F = 0.25N = 0.25W$

$W = mg$

$\frac{m}{W} = \frac{1}{g}$

$g = 9.8\ m/s^2$

$W_{BC} = Fd = -0.25W\,7 = -1.75W$

$\Delta U = \frac{3}{2}m$

$\Delta U = W_{AB} + W_{BC} = d(W\cos 60 - 0.25W\cos 30) - 1.75W = \frac{3}{2}m$

$d(\cos 60 - 0.25\cos 30) - 1.75 = \frac{3}{2}\frac{m}{W} = \frac{3}{2g}$

$d(\cos 60 - 0.25\cos 30) = \frac{3}{2g} + 1.75$

$d = \frac{3/2g + 1.75}{\cos 60 - 0.25\cos 30} = \boxed{6.71\ m}$