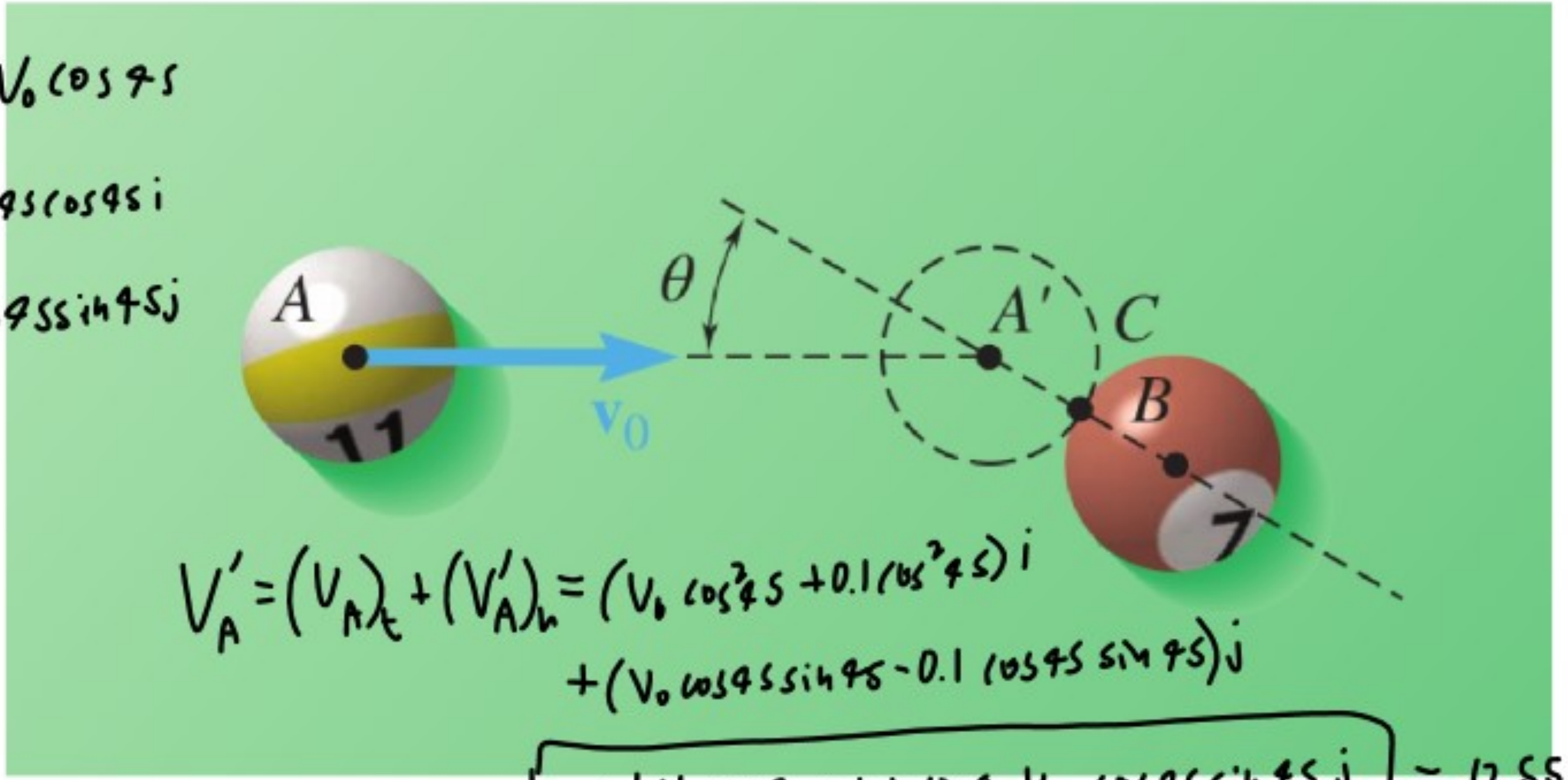
Two identical billiard balls can move freely on a horizontal table. Ball A has a velocity $\mathbf{v}_0$ as shown and hits ball B, which is at rest, at a point C defined by $\theta = 45°$. Knowing that the coefficient of restitution between the two balls is $e = 0.8$ and assuming no friction, determine the velocity of each ball after impact.

$V_A' = 0.1 V_0 \cos 45$

$(V_A)_t$ 45

$(V_A)_t = V_0 \cos 45$

$(\vec{V}_A)_t = V_0 \cos 45 \cos 45 i + V_0 \cos 45 \sin 45 j$

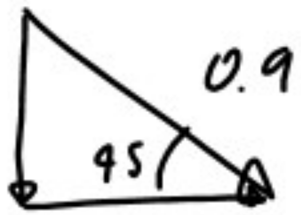


45 45 $(V_A')_n = V_A'$

$(\vec{V}_A')_n = 0.1 V_0 \cos 45 \cos 45 i - 0.1 V_0 \cos 45 \sin 45 j$

$V_A' = (V_A)_t + (V_A')_n = (V_0 \cos^2 45 + 0.1 \cos^2 45) i + (V_0 \cos 45 \sin 45 - 0.1 \cos 45 \sin 45) j$

$\boxed{= 1.1 V_0 \cos^2 45 i + 0.9 V_0 \cos 45 \sin 45 j} \simeq 0.55 V_0 i + 0.45 V_0 j$



$0.9\,V_0\cos45$

$$\boxed{\vec{V}_B' = 0.9V_0\cos45\cos45\,i - 0.9V_0\cos45\sin45\,j}$$

$$\cos^2 45 = 0.5$$

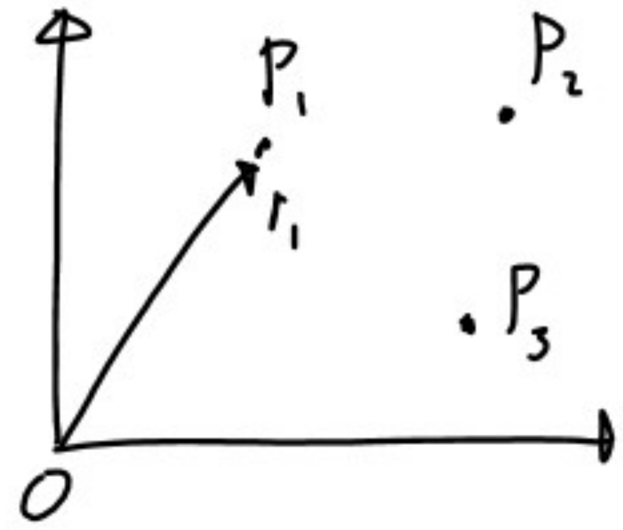
$$\cos45\sin45 = 0.5$$

$$= 0.45V_0\,i - 0.45V_0\,j$$

Systems of Particles

$\vec{F}_i$ external force on P_i

f_{12} force on P_1 from P_2



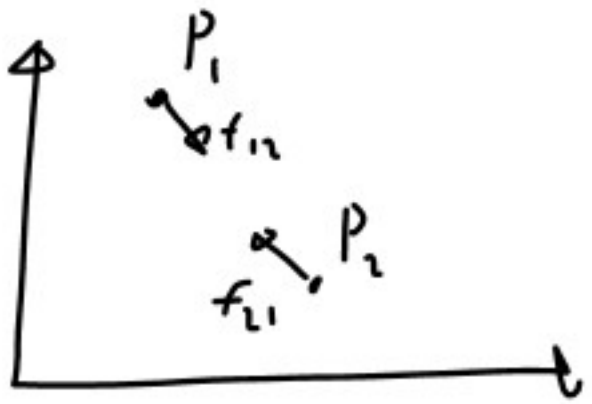
$$\vec{F}_i + \sum_{j=1}^{n} \vec{f}_{ij} = m_i \vec{a}_i$$

$$\vec{F}_1 + \vec{f}_{12} + \vec{f}_{13} = m_1 \vec{a}_1$$

moments about O

$$\vec{r}_j \times \vec{F}_i + \sum_{j=1}^{n} \vec{r}_i \times \vec{f}_{ij} = \vec{r}_i \times m_i \vec{a}_i$$

$$\sum_{i=1}^{n} \sum_{j=1}^{n} \vec{f}_{ij} = 0 \qquad \sum_{i=1}^{n} \vec{F}_i = \sum_{i=1}^{n} m_i \vec{a}_i$$



$$\sum_{i=1}^{n} \sum_{j=1}^{n} \vec{r}_i \times \vec{f}_{ij} = 0 \qquad \sum_{i=1}^{n} \vec{r}_i \times \vec{F}_i = \sum_{i=1}^{n} \vec{r}_i \times m_i \vec{a}_i$$

Momentum of a System of Particles

$$\vec{L} = \sum_{i=1}^{n} m_i \vec{v}_i = \sum_{i=1}^{n} \vec{L}_i$$

$$\vec{H}_0 = \sum_{i=1}^{n} \vec{r}_i \times m_i \vec{v}_i$$

$\vec{L}$ constant

$$\sum \vec{F}_i = \dot{\vec{L}}$$

$$\sum_{i=1}^{n} (\vec{M}_0)_i = \dot{\vec{H}}_0$$

$\vec{H}_0$ constant

Motion of the center of Mass

$$m\vec{r} = \sum_{i=1}^{n} m_i \vec{r}_i \qquad m\bar{x} = \sum_{i=1}^{n} m_i x_i$$

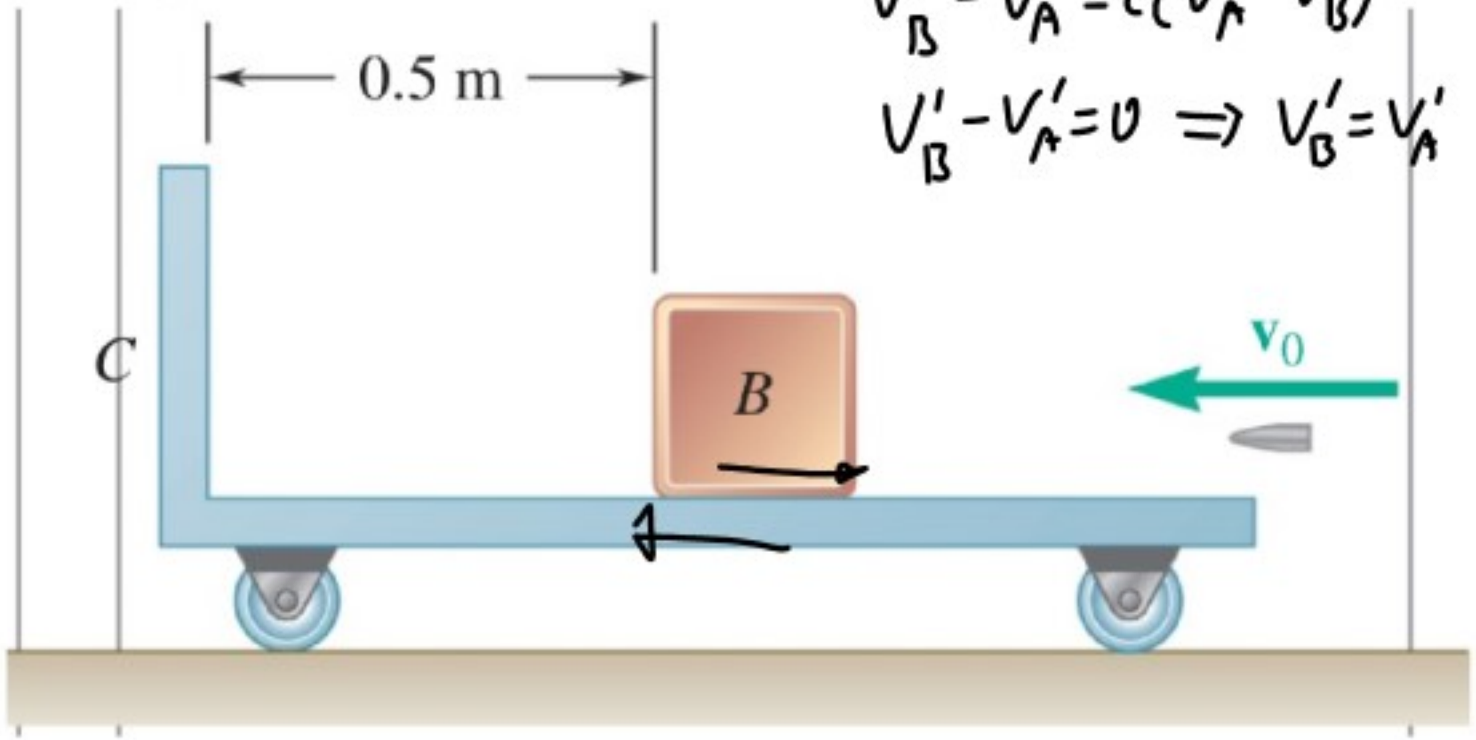
$$\sum_{i=1}^{n} \vec{F}_i = m\bar{a} = \dot{\vec{L}} \qquad \sum_{i=1}^{n} (M_G)_i = \dot{\vec{H}}_G \qquad \vec{H}_G = \sum_{i=1}^{n} \vec{r}_i{}' \times m_i \vec{V}_i = \sum_{i=1}^{n} \vec{r}_i{}' \times m_i \vec{V}_i{}'$$

$\vec{r}_i{}'$ Vector from center of mass to P_i

$$\vec{V}_i = \bar{V} + \vec{V}_i{}'$$

$\bar{V}$ Velocity of center of mass

A 30-g bullet is fired with a horizontal velocity of 450 m/s and becomes embedded in block *B*, which has a mass of 3 kg. After the impact, block *B* slides on 30-kg carrier *C* until it impacts the end of the carrier. Knowing the impact between *B* and *C* is perfectly plastic and the coefficient of kinetic friction between *B* and *C* is 0.2, determine (*a*) the velocity of the bullet and *B* after the first impact, (*b*) the final velocity of the carrier.



$$V_B' - V_A' = e(V_A - V_B)$$

$$V_B' - V_A' = 0 \Rightarrow V_B' = V_A'$$

$$m_A V_A + m_B \cancel{V_B}_0 = m_A V_A' + m_B V_B'$$

$$0.03 \cdot 450 = m_A V_A' + m_B V_A'$$

$$= (0.03 + 3)\, V_A'$$

$$\frac{0.03 \cdot 450}{0.03 + 3} = \boxed{V_A' = 4.5 \ \text{m/s}}$$

$$m_A V_A + \cancel{m_B V_B}_0 + \cancel{m_C V_C}_0 = L$$

$$0.03 \cdot 450 = L = 13.5$$

$$13.5 = (m_A + m_B + m_C)\, V$$

$$13.5 = (0.03 + 3 + 30)\, V$$

$$\frac{13.5}{0.03 + 3 + 30} = \boxed{0.4 \ \text{m/s}}$$

An airline employee tosses two suitcases in rapid succession, with a horizontal velocity of 7.2 ft/s, onto a 50-lb baggage carrier which is initially at rest. (*a*) Knowing that the final velocity of the baggage carrier is 3.6 ft/s and that the first suitcase the employee tosses onto the carrier has a weight of 30 lb, determine the weight of the other suitcase. (*b*) What would be the final velocity of the carrier if the employee reverses the order in which he tosses the suitcases?