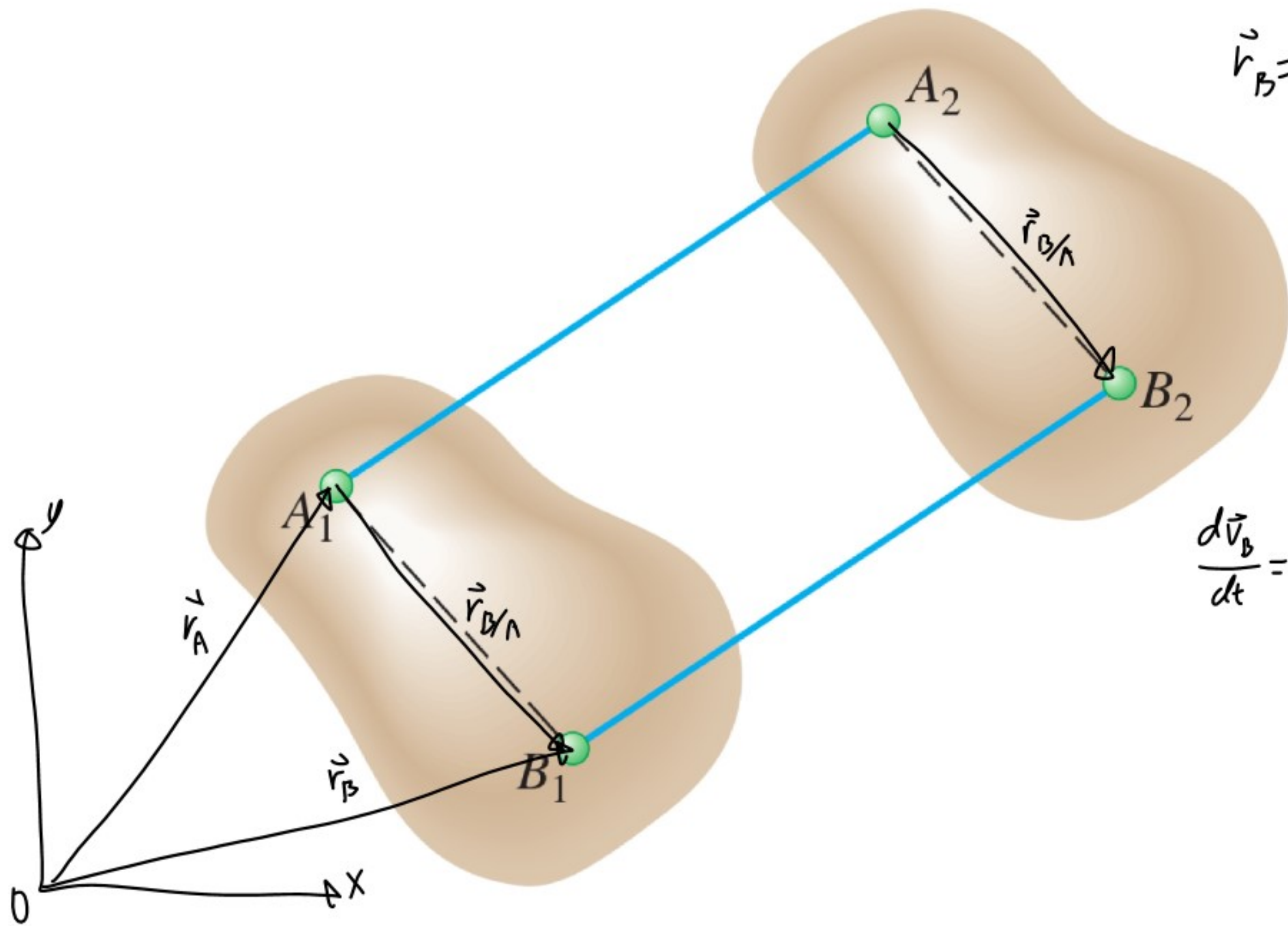


Translation



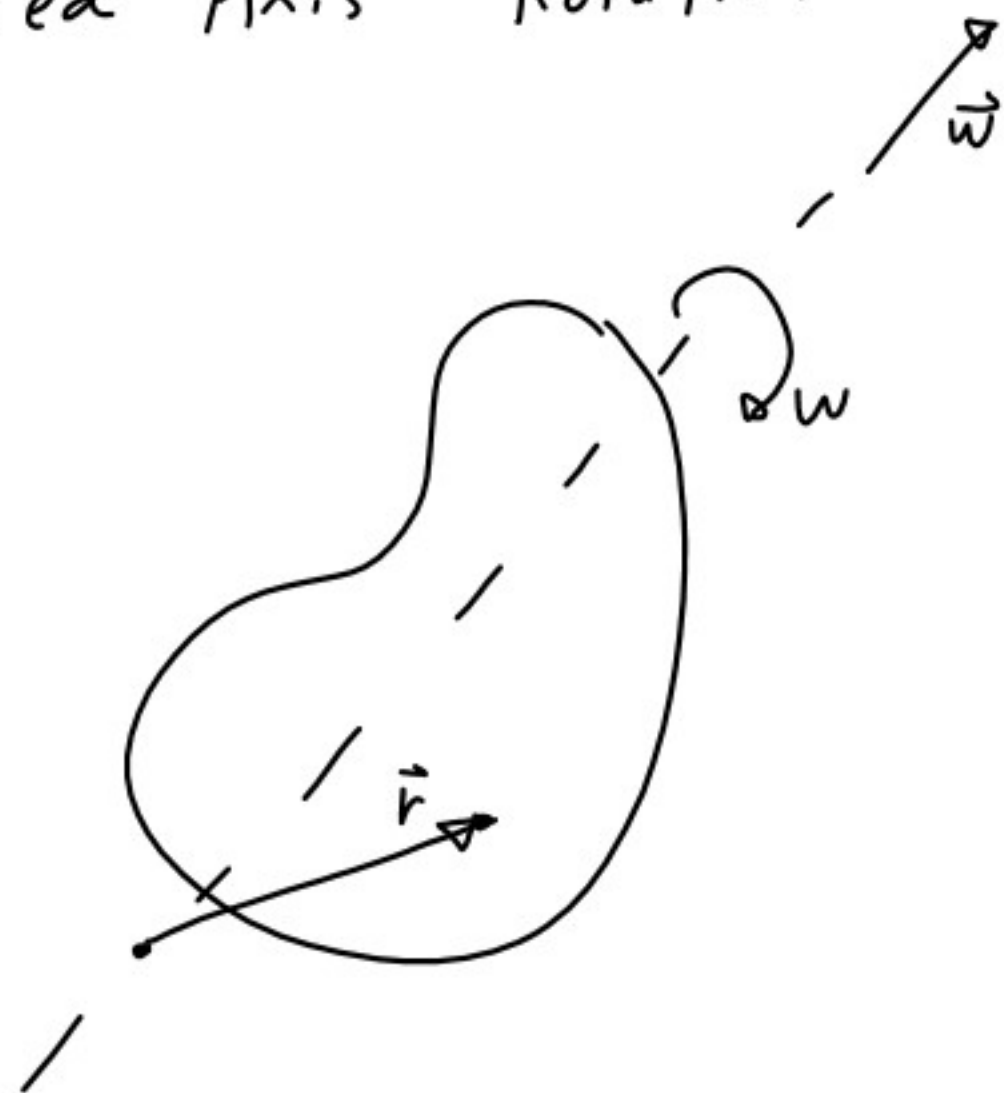
$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$

$$\vec{v}_{B/A} = \text{constant}$$

$$\frac{d\vec{r}_B}{dt} = \vec{v}_B = \vec{v}_A$$

$$\frac{d\vec{v}_B}{dt} = \vec{a}_B = \vec{a}_A$$

# Fixed Axis Rotation



$$\vec{v} = \vec{\omega} \times \vec{r}$$

$$\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})$$

$$\vec{\alpha} = \frac{d\vec{\omega}}{dt}$$

$$\vec{\omega} = \frac{d\vec{\theta}}{dt}$$

$$\vec{\alpha} = \vec{\omega} \frac{d\vec{\omega}}{d\vec{\theta}}$$

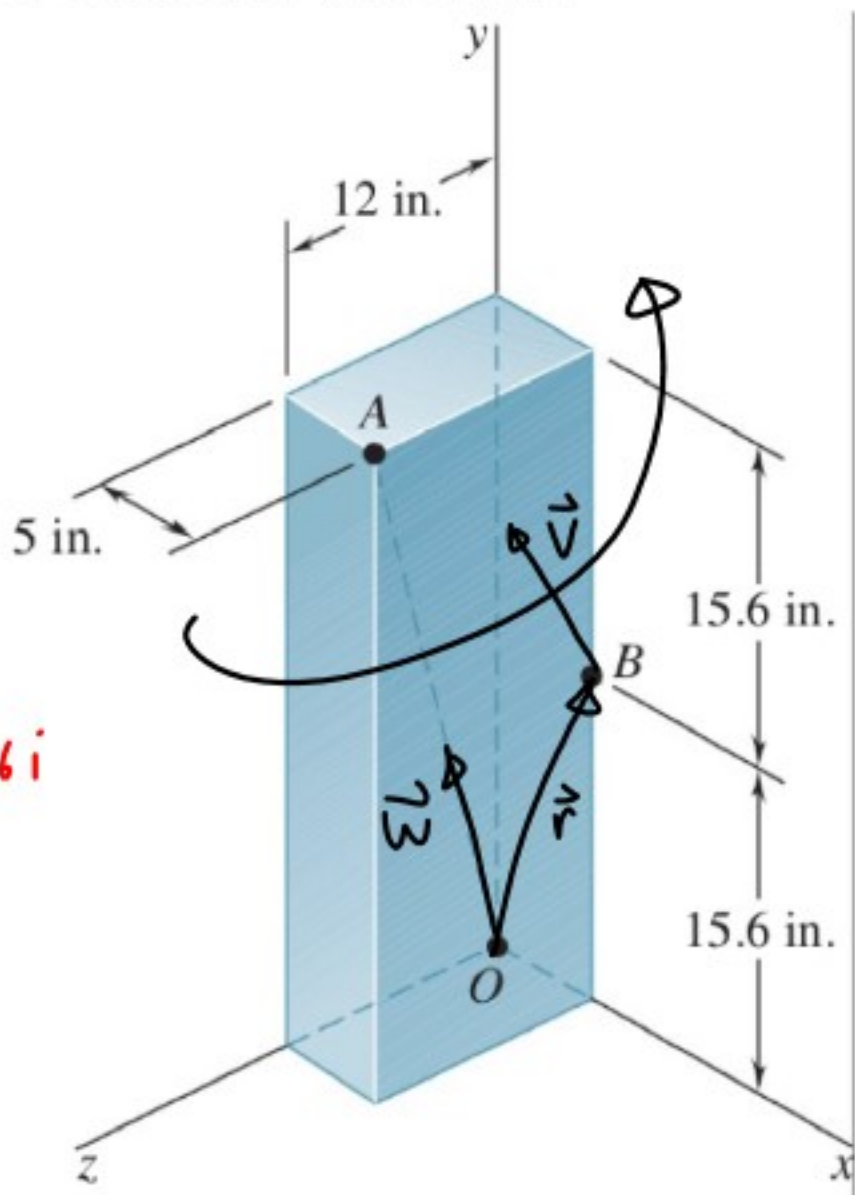
The rectangular block shown rotates about the diagonal  $OA$  with a constant angular velocity of  $6.76$  rad/s. Knowing that the rotation is counterclockwise as viewed from  $A$ , determine the velocity and acceleration of point  $B$  at the instant shown.

$$\vec{V} = \vec{\omega} \times \vec{r}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} & \mathbf{i} & \mathbf{j} \\ 1 & 6.24 & 2.4 & 5 & 15.6 \\ 5 & 15.6 & 0 & 5 & 15.6 \end{vmatrix}$$

$$= 2.4 - 5\mathbf{j} + 15.6\mathbf{k} - 5 \cdot 6.24\mathbf{k} - 2.4 \cdot 15.6\mathbf{i}$$

$$\vec{V} = -37.44\mathbf{i} + 12\mathbf{j} - 15.6\mathbf{k} \text{ m/s}$$



$$\vec{r} = 5\mathbf{i} + 15.6\mathbf{j} + 0\mathbf{k} \text{ in}$$

$$\vec{OA} = 5\mathbf{i} + 31.2\mathbf{j} + 12\mathbf{k} \text{ in}$$

$$OA = \sqrt{5^2 + 31.2^2 + 12^2} = 33.8 \text{ in}$$

$$\vec{\lambda} = \frac{5}{33.8}\mathbf{i} + \frac{31.2}{33.8}\mathbf{j} + \frac{12}{33.8}\mathbf{k}$$

$$\vec{\omega} = 6.76 \vec{\lambda}$$

$$= 6.76 \left( \frac{5}{33.8}\mathbf{i} + \frac{31.2}{33.8}\mathbf{j} + \frac{12}{33.8}\mathbf{k} \right)$$

$$= 1\mathbf{i} + 6.24\mathbf{j} + 2.4\mathbf{k} \text{ rad/s}$$

$$\vec{a} = \vec{\omega} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})$$

$$\vec{a} = \vec{\omega} \times (\vec{\omega} \times \vec{r})$$

$$= \begin{vmatrix} \cancel{i} & \cancel{j} & \cancel{k} & \cancel{i} & \cancel{j} \\ 1 & 6.24 & 2.9 & 1 & 6.24 \\ -37.49 & 12 & -15.6 & -37.49 & 12 \end{vmatrix}$$

$$= -0.24(15.6)i + 2.9(-37.49)j + 12k + 6.24(37.49)k - 2.9(12)i + 15.6j$$

$$\vec{a} = -126i - 79.3j + 246k \text{ m/s}^2$$

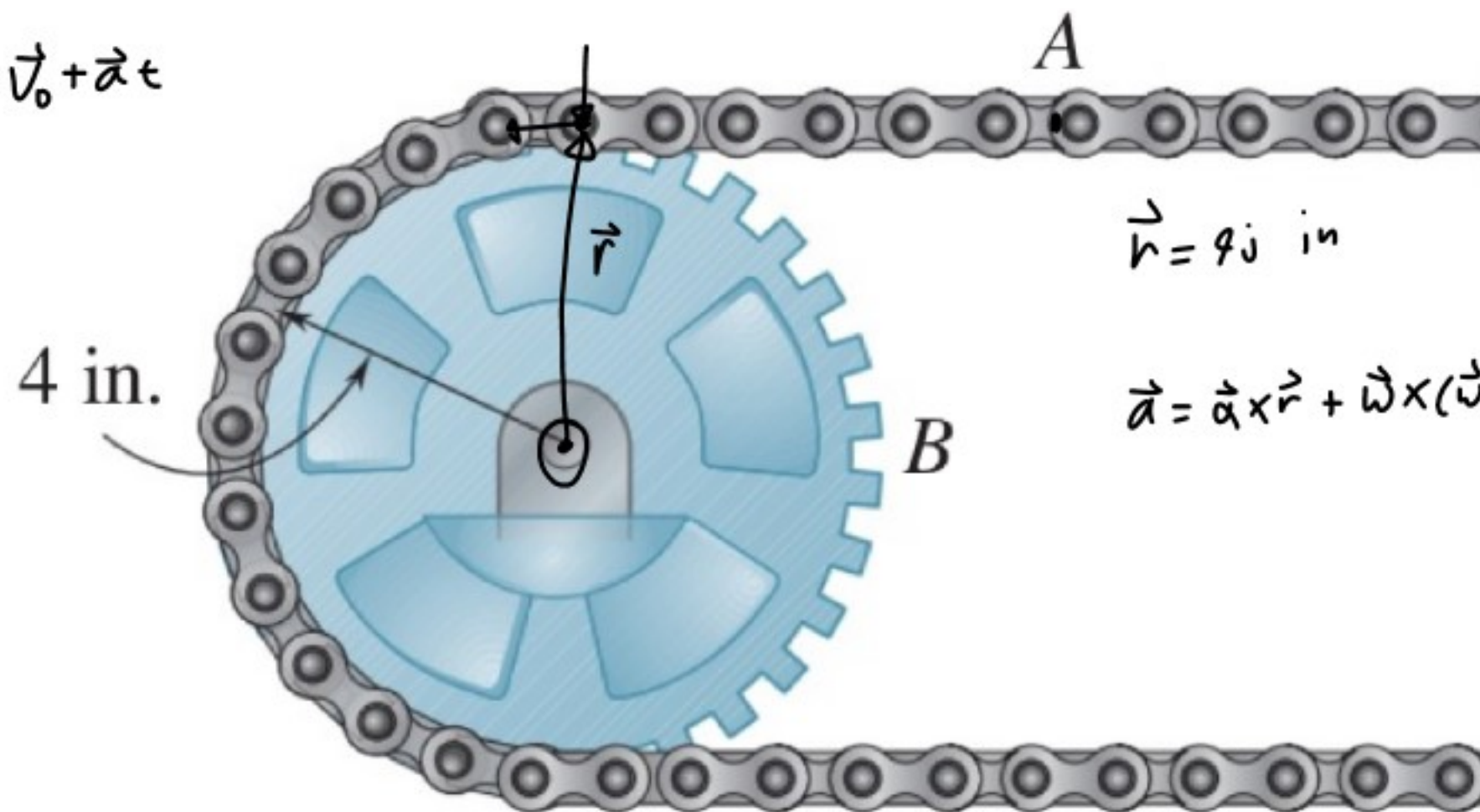
The sprocket wheel and chain shown are initially at rest. If the wheel has a uniform angular acceleration of 90 rad/s<sup>2</sup> counterclockwise, determine (a) the acceleration of point *A* of the chain, (b) the magnitude of the acceleration of point *B* of the wheel after 3 s.

$$\vec{\omega} = \vec{\omega}_0 + \vec{\alpha} t$$

$$\vec{v} = \vec{v}_0 + \vec{\alpha} t$$

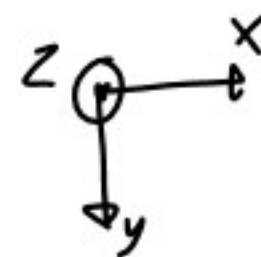
$$\vec{\alpha} = 90 \mathbf{k} \text{ rad/s}^2$$

$$\begin{aligned} \vec{\omega} &= \vec{\alpha} t = 90 \mathbf{k} (3) \\ &= 270 \mathbf{k} \text{ rad/s} \end{aligned}$$



$$\vec{r} = 4 \mathbf{j} \text{ in}$$

$$\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})$$



$$\vec{\omega} \times \vec{r} = \begin{vmatrix} i & j & k & | & i & j \\ 0 & 0 & 270 & | & 0 & 0 \\ 0 & 9 & 0 & | & 0 & 9 \end{vmatrix} = -4(270)i = 1080i \text{ m/s}$$

$$\vec{\omega} \times (\vec{\omega} \times \vec{r}) = \begin{vmatrix} i & j & k & | & i & j \\ 0 & 0 & 270 & | & 0 & 0 \\ 1080 & 0 & 0 & | & 1080 & 0 \end{vmatrix} = 270(1080)j = 291600j \text{ m/s}^2$$

$$\vec{\alpha} \times \vec{r} = \begin{vmatrix} i & j & k & | & i & j \\ 0 & 0 & 90 & | & 0 & 0 \\ 0 & 9 & 0 & | & 0 & 9 \end{vmatrix} = -9(90)i = -360i \text{ m/s}^2$$

$$\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})$$

$$= -360i + 291600j \text{ m/s}^2$$

$$\boxed{\vec{a}_A = -360i \text{ m/s}^2}$$