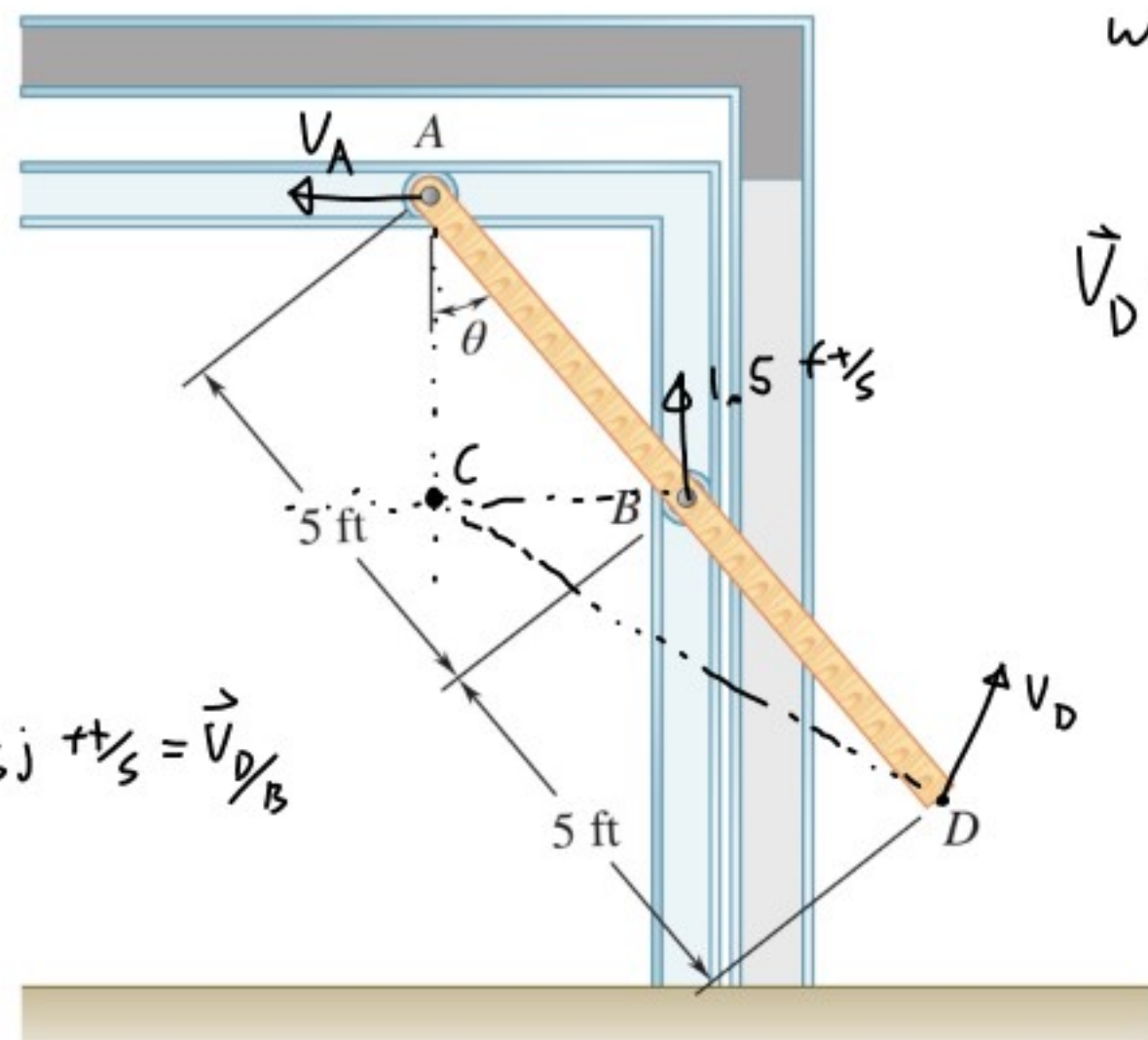


An overhead door is guided by wheels at  $A$  and  $B$  that roll in horizontal and vertical tracks. Knowing that when  $\theta = 40^\circ$  the velocity of wheel  $B$  is  $1.5 \text{ ft/s}$  upward, determine (a) the angular velocity of the door, (b) the velocity of end  $D$  of the door.



$$\vec{\omega} \times \vec{r}_{D/B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} & | & \mathbf{i} & \mathbf{j} \\ 0 & 0 & 0.47 & | & 0 & 0 \\ 3.2 & -3.8 & 0 & | & 3.2 & -3.8 \end{vmatrix}$$

$$= 0.47 \cdot 3.8 \mathbf{i} + 0.47 \cdot 3.2 \mathbf{j} = 1.8 \mathbf{i} + 1.5 \mathbf{j} \text{ ft/s} = \vec{v}_{D/B}$$



$$\omega = \frac{v_B}{BC} = \frac{1.5 \text{ ft/s}}{5 \sin 40^\circ \text{ ft}} = \boxed{0.47 \text{ rad/s}}$$

$$\vec{v}_D = \vec{v}_B + \vec{v}_{D/B} \quad \vec{\omega} = 0.47 \mathbf{k} \text{ rad/s}$$

$$= \vec{v}_B + \vec{\omega} \times \vec{r}_{D/B}$$

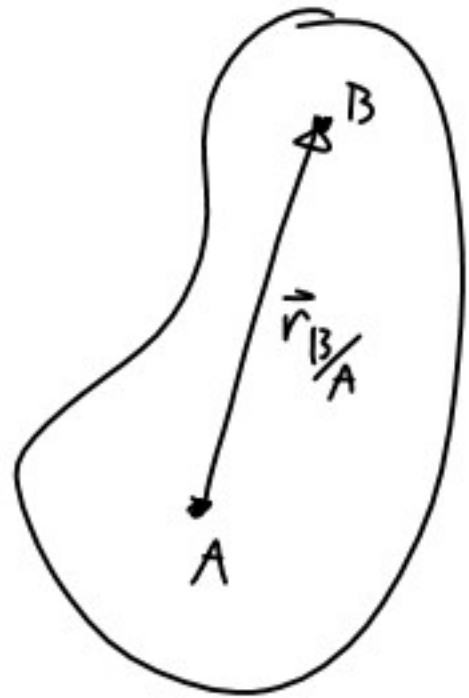
$$\vec{r}_{D/B} = 5 \sin 40^\circ \mathbf{i} - 5 \cos 40^\circ \mathbf{j} \text{ ft}$$

$$= 3.2 \mathbf{i} - 3.8 \mathbf{j} \text{ ft}$$

$$\vec{v}_D = 1.5 \mathbf{j} + 1.8 \mathbf{i} + 1.5 \mathbf{j}$$

$$= \boxed{1.8 \mathbf{i} + 3 \mathbf{j} \text{ ft/s}}$$

# General Plane Motion : acceleration



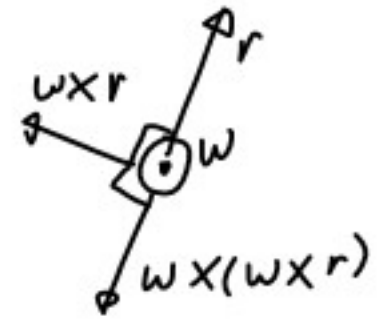
$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

$$\vec{a}_{B/A} = \vec{\alpha} \times \vec{r}_{B/A} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{B/A}) \quad 3D$$

$$= \alpha \mathbf{k} \times \vec{r}_{B/A} - \omega^2 \vec{r}_{B/A} \quad 2D$$

$$(a_{B/A})_n = -\vec{r}_{B/A} \omega^2$$

$$(a_{B/A})_t = \alpha \mathbf{k} \times \vec{r}_{B/A}$$



$$|\omega \times (\omega \times r)| = \omega^2 r$$

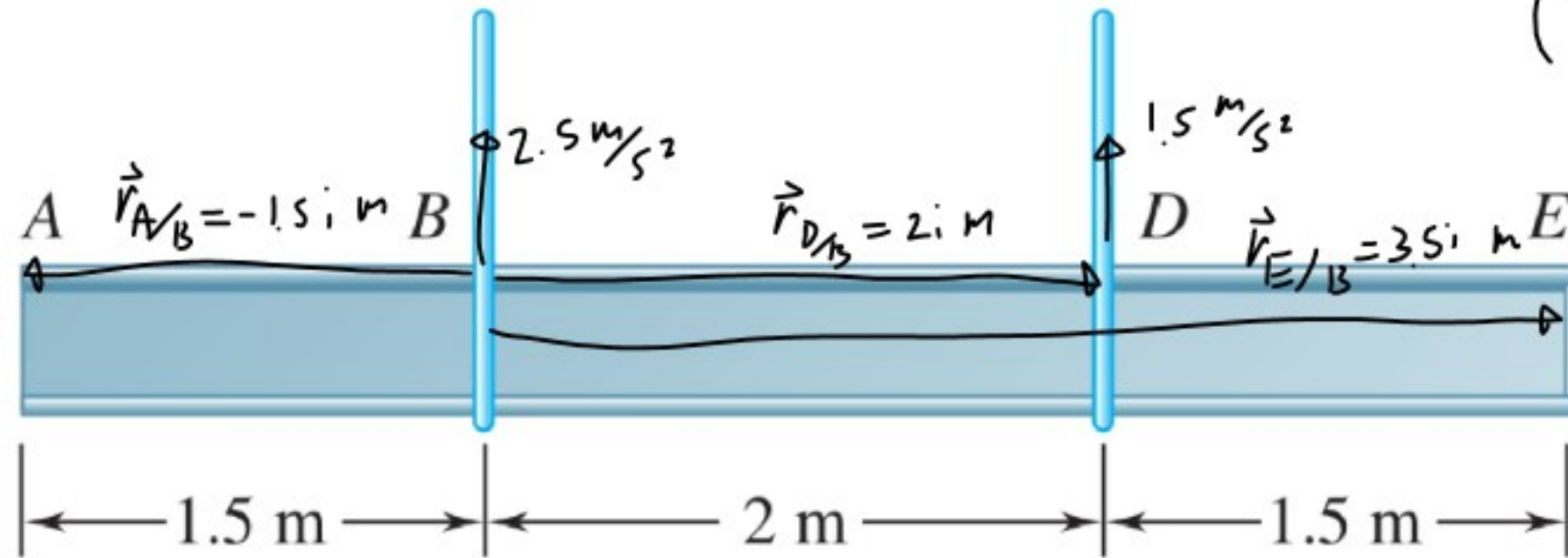
A 5-m steel beam is lowered by means of two cables unwinding at the same speed from overhead cranes. As the beam approaches the ground, the crane operators apply brakes to slow the unwinding motion. At the instant considered, the deceleration of the cable attached at  $B$  is  $2.5 \text{ m/s}^2$ , while that of the cable attached at  $D$  is  $1.5 \text{ m/s}^2$ . Determine (a) the angular acceleration of the beam, (b) the acceleration of points  $A$  and  $E$ .

$\omega = 0$

$$\vec{a}_D = \vec{a}_B + \vec{a}_{D/B}$$

$$\vec{a}_D - \vec{a}_B = \vec{a}_{D/B}$$

$$1.5 \text{ m/s}^2 - 2.5 \text{ m/s}^2 = -1 \text{ m/s}^2 = \vec{a}_{D/B}$$



$$\left( \vec{a}_{D/B} \right)_t = \alpha \mathbf{k} \times \vec{r}_{D/B}$$

$$-1 \mathbf{j} = \alpha \mathbf{k} \times 2 \mathbf{i}$$

$$-1 = \alpha \cdot 2$$

$$\alpha = -\frac{1}{2} = \boxed{-0.5 \text{ m/s}^2}$$

$$\vec{a}_A = \vec{a}_B + \vec{a}_{A/B}$$

$$= 2.5j + \alpha k \times \vec{v}_{A/B} - \omega^2 \vec{v}_{A/B}$$

$$= 2.5j - 0.5k \times -1.5i$$

$$= 2.5j + 0.75j = \boxed{3.25j \text{ m/s}^2}$$

$$\vec{a}_E = \vec{a}_B + \vec{a}_{E/B} = 2.5j + \alpha k \times \vec{v}_{E/B}$$

$$= 2.5j - 0.5k \times 3.5i$$

$$= 2.5j - 1.75j = \boxed{0.75j \text{ m/s}^2}$$

$$-0.5k \times -1.5i = \begin{vmatrix} i & j & k & | & i & j \\ 0 & 0 & -0.5 & | & 0 & 0 \\ -1.5 & 0 & 0 & | & -1.5 & 0 \end{vmatrix} = 0.75j$$

$$-0.5k \times 3.5i = \begin{vmatrix} i & j & k & | & i & j \\ 0 & 0 & -0.5 & | & 0 & 0 \\ 3.5 & 0 & 0 & | & 3.5 & 0 \end{vmatrix} = -1.75j$$

An automobile travels to the left at a constant speed of 90 km/h. Knowing that the diameter of the wheel is 650 mm, determine the acceleration of (a) point  $B$ , (b) point  $C$ , (c) point  $D$ .

