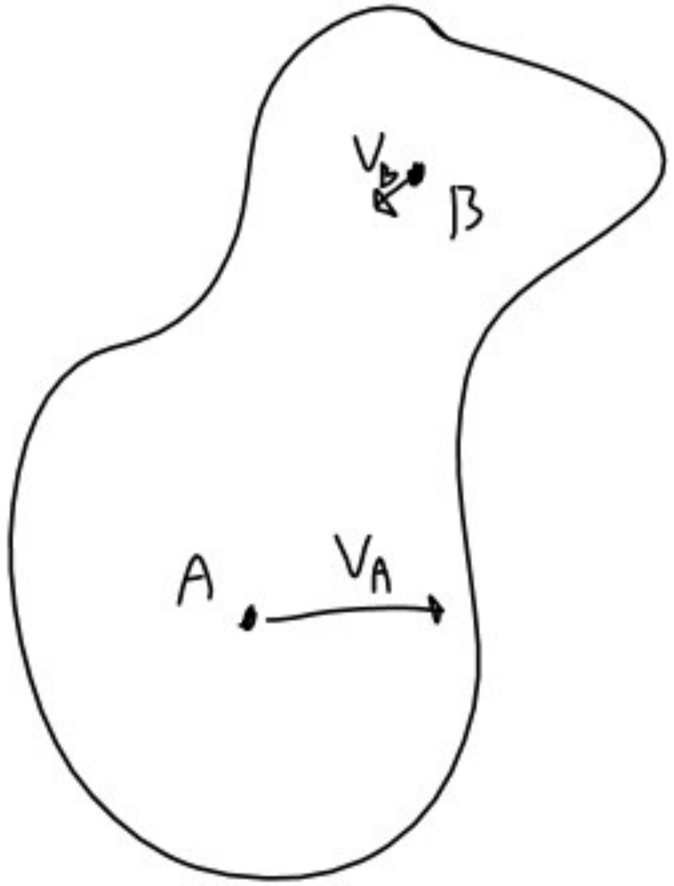
Energy Methods for Rigid Bodies

$$T_1 + V_1 + U_{1\rightarrow 2} = T_2 + V_2$$

$$T = \frac{1}{2}\sum_{i=1}^{n} \Delta m_i v_i^2 = \frac{1}{2}\int v^2\, dm = \frac{1}{2} m \bar{v}^2 + \frac{1}{2}\bar{I}\omega^2$$

$\bar{v}$ velocity of center of mass

$\bar{I}$ mass moment of inertia about the center of mass



$$U_{1\rightarrow 2} = \int_{r_1}^{r_2} F \cdot dr$$

$$U_{1\rightarrow 2} = \int_{\theta_1}^{\theta_2} M\, d\theta$$

$$= M(\theta_2 - \theta_1)$$ if M costant

A 200-kg flywheel is at rest when a constant 300 N·m couple is applied. After executing 560 revolutions, the flywheel reaches its rated speed of 2400 rpm. Knowing that the radius of gyration of the flywheel is 400 mm, determine the average magnitude of the couple due to kinetic friction in the bearing.

$$\bar{I} = mk^2 = 200(0.4)^2 = 32 \text{ kg m}^2$$

$$2400 \frac{\text{rev}}{\text{min}} \frac{2\pi \text{ rad}}{1 \text{ rev}} \frac{1 \text{ min}}{60 \text{ s}} = 251 \text{ rad/s}$$

$$\cancel{T_1}^{0} + U_{1\to 2} = T_2$$

$$T_2 = \frac{1}{2} m \cancel{\bar{v}^2}^{0} + \frac{1}{2}\bar{I}\omega^2$$

$$= \frac{1}{2} 32 (251)^2 = 1.01\times10^6 \frac{\text{kg m}^2}{\text{s}^2}$$

$$N = \frac{\text{kg m}}{\text{s}^2}$$

$$U_g + U_f = T_2$$

$$1.06\times10^6 \text{ Nm} + U_f = 1.01\times10^6 \frac{\text{kg m}^2}{\text{s}^2}$$

$$U_g = M(\theta_2 - \theta_1) = 300 \text{ N-m} \cdot 560 \text{ rev} \frac{2\pi \text{ rad}}{1 \text{ rev}} = 1.06\times10^6 \text{ N m}$$

$$U_f = 1.01\times10^6 - 1.06\times10^6 = -45\times10^3 \text{ Nm}$$

$$U_f = -45\times10^3 \text{ Nm} = M(\theta_2 - \theta_1) = M(560 \text{ rev})$$

$$M = \frac{-45\times10^3 \text{ Nm}}{560 \text{ rev}} \frac{1 \text{ rev}}{2\pi \text{ rad}} = \boxed{-12.8 \text{ Nm}}$$

The rotor of an electric motor has an angular velocity of 3600 rpm when the load and power are cut off. The 110-lb rotor, which has a centroidal radius of gyration of 9 in., then coasts to rest. Knowing that the kinetic friction of the rotor produces a couple with a magnitude of 2.5 lb·ft, determine the number of revolutions that the rotor executes before coming to rest.

$T_1 + U_{1\to 2} = \cancel{T_2}^{\,0}$

$T_1 = -U_{1\to 2} \Rightarrow U_{1\to 2} = -136\times 10^3 \text{ lb ft}$

$T_1 = \frac{1}{2} m \cancel{\bar{v}^2}^{\,0} + \frac{1}{2}\bar{I}\omega^2$

$= \frac{1}{2} 1.92 \text{ lb s}^2\text{ft} \left(3600 \frac{\text{rev}}{\text{min}}\right)^2$

$= 12.4\times 10^6 \frac{\text{lb s}^2\text{ft rev}^2}{\text{min}^2} \left(\frac{1 \text{ min}}{60 \text{ s}}\right)^2 \left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)^2$

$= 136\times 10^3 \text{ lb ft}$

$\bar{I} = mk^2$

$= 3.4 \frac{\text{lb s}^2}{\text{ft}} (9 \text{ in})^2$

$= 277 \frac{\text{lb s}^2 \text{ in}^2}{\text{ft}} \left(\frac{1 \text{ ft}}{12 \text{ in}}\right)^2$

$= 1.92 \text{ lb s}^2 \text{ ft}$

$W = 110 \text{ lb}$

$m = \frac{W}{g} = \frac{110 \text{ lb}}{32.2 \text{ ft/s}^2} = 3.4 \frac{\text{lb s}^2}{\text{ft}}$

$U_{1\to 2} = -136\times 10^3 \text{ lb ft} = M(\theta_2 - \theta_1)$

$\frac{-136\times 10^3 \text{ lb ft}}{-2.5 \text{ lb ft}} = 54.6\times 10^3 \text{ rad} \quad \frac{1 \text{ rev}}{2\pi \text{ rad}} \boxed{= 8686 \text{ rev}}$