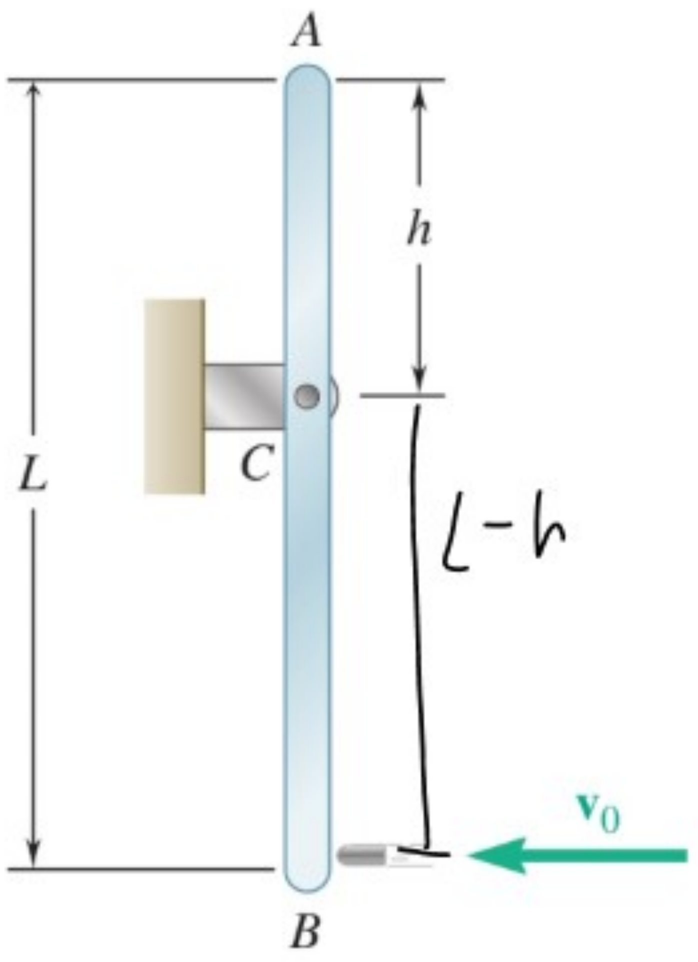
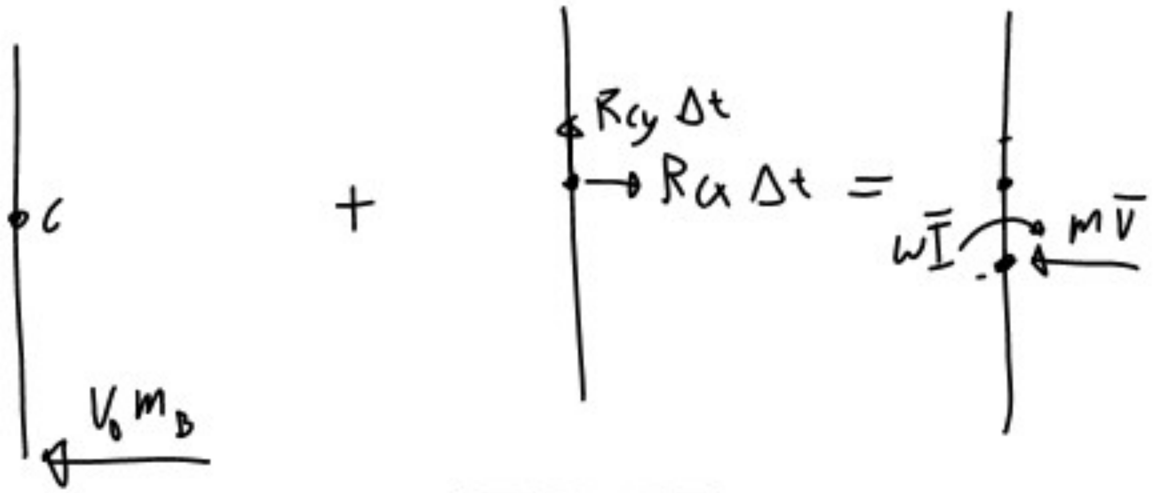
A bullet weighing 0.08 lb is fired with a horizontal velocity of 1800 ft/s into the lower end of a slender 15-lb bar of length $L = 30$ in. Knowing that $h = 12$ in. and that the bar is initially at rest, determine (*a*) the angular velocity of the bar immediately after the bullet becomes embedded, (*b*) the impulsive reaction at C, assuming that the bullet becomes embedded in 0.001 s.

$$\bar{I} = \frac{1}{12} m L^2$$





$$\boxed{R_{cy} = 0}$$

$$-V_0 m_B + R_{cx}\Delta t = m\bar{v}$$

$$H_{C1} + \cancel{M_C \Delta t} = H_{C2}$$

0

$$-V_0 m_B (L-h) = H_{C2}$$

$$= -m\bar{v}\left(\frac{L}{2}-h\right) - \bar{I}\omega$$

$$= -m\omega\left(\frac{L}{2}-h\right)^2 - \bar{I}\omega$$

$$= -m\omega\left(\frac{L}{2}-h\right)^2 - \frac{1}{12} m L^2 \omega$$

$$\frac{-V_0 m_B (L-h)}{-m\left(\frac{L}{2}-h\right)^2 - \frac{1}{12} m L^2} = \omega$$

$$\frac{1800\,\text{ft}/\text{s}\;0.08\,\text{lb}\,(30\,\text{in} - 12\,\text{in})}{15\,\text{lb}\left(\frac{30\,\text{in}}{2} - 12\,\text{in}\right)^2 + \frac{1}{12}\,15\,\text{lb}\,(30\,\text{in})^2} = 2.057\,\frac{\text{ft}\,\cancel{\text{lb}}\,\cancel{\text{in}}}{\text{s}\,\cancel{\text{lb}}\,\text{in}^{\cancel{2}}} = 2.057\,\frac{\cancel{\text{ft}}}{\text{s}\,\cancel{\text{in}}}\;\frac{12\,\cancel{\text{in}}}{1\,\cancel{\text{ft}}} = \boxed{24.7\,\text{rad}/\text{s}}$$

$$\frac{0.08\,\text{lb}}{32.2\,\text{ft}/\text{s}^2} = 2.5\times10^{-3}\,\frac{\text{lb}\,\text{s}^2}{\text{ft}} \qquad \frac{15\,\text{lb}}{32.2\,\text{ft}/\text{s}^2} = 0.466\,\frac{\text{lb}\,\text{s}^2}{\text{ft}}$$

$$-V_0 m_B + R_{cx}\Delta t = m\bar{V}$$

$$R_{cx}\Delta t = V_0 m_B - m\bar{V}$$

$$\bar{V} = \left(\frac{L}{2} - h\right)\omega = \left(\frac{30\,\text{in}}{2} - 12\,\text{in}\right)24.7\,\text{rad}/\text{s} = 74.1\,\text{in}/\text{s}\;\frac{1\,\text{ft}}{12\,\text{in}} = 6.18\,\text{ft}/\text{s}$$

$$R_{cx} = \frac{V_0 m_B - m\bar{V}}{\Delta t}$$

$$= \frac{1800\,\text{ft}/\text{s}\;2.5\times10^{-3}\,\frac{\text{lb}\,\text{s}^2}{\text{ft}} - 0.466\,\frac{\text{lb}\,\text{s}^2}{\text{ft}}\,6.18\,\text{ft}/\text{s}}{0.001\,\text{s}} = \boxed{1620\,\text{lb}}$$

3D Kinematics of Rigid Bodies

$$\Sigma \vec{F} = m\bar{a} \qquad \Sigma \vec{M}_G = \dot{\vec{H}}_G$$

$$H_G = \sum_{i=1}^{n} (r_i' \times v_i' \Delta m_i)$$

$$= \sum_{i=1}^{n} (r_i' \times (\omega \times r_i') \Delta m_i)$$

$$H_x = \sum_{i=1}^{n} (y_i (\omega \times r_i')_z - z_i (\omega \times r_i')_y) \Delta m_i$$

$$= \sum_{i=1}^{n} (y_i (\omega_x y_i - \omega_y x_i) - z_i (\omega_z x_i - \omega_x z_i)) \Delta m_i$$

$$= \omega_x \sum_i (y_i^2 - z_i^2) \Delta m_i - \omega_y \sum_i x_i y_i \Delta m_i - \omega_z \sum_i z_i x_i \Delta m_i$$

$$= \omega_x \int (y^2 - z^2) dm - \omega_y \int xy\, dm - \omega_z \int zx\, dm$$

$$= \bar{I}_x \omega_x - \bar{I}_{xy} \omega_y - \bar{I}_{xz} \omega_z$$

$$H_y = -\bar{I}_{yx} \omega_x + \bar{I}_y \omega_y - \bar{I}_{yz} \omega_z$$

$$H_z = -\bar{I}_{zx} \omega_x - \bar{I}_{zy} \omega_y + \bar{I}_z \omega_z$$

$$H = \begin{bmatrix} \quad & \quad & \quad \\ & & \\ & & \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$