$$\begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix} = \begin{bmatrix} \bar{I}_x & -\bar{I}_{xy} & -\bar{I}_{xz} \\ -\bar{I}_{yx} & \bar{I}_y & -\bar{I}_{yz} \\ -\bar{I}_{zx} & -\bar{I}_{zy} & \bar{I}_z \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

Mass moment of inertia

$I_x \quad I_y \quad I_z$

Mass product of inertia

$I_{yx} \quad I_{xz}$ etc

(Zero if part is symmetric)

$$\begin{bmatrix} H_{x'} \\ H_{y'} \\ H_{z'} \end{bmatrix} = \underbrace{\begin{bmatrix} \bar{I}_{x'} & 0 & 0 \\ 0 & \bar{I}_{y'} & 0 \\ 0 & 0 & \bar{I}_{z'} \end{bmatrix}}_{I} \begin{bmatrix} \omega_{x'} \\ \omega_{y'} \\ \omega_{z'} \end{bmatrix}$$

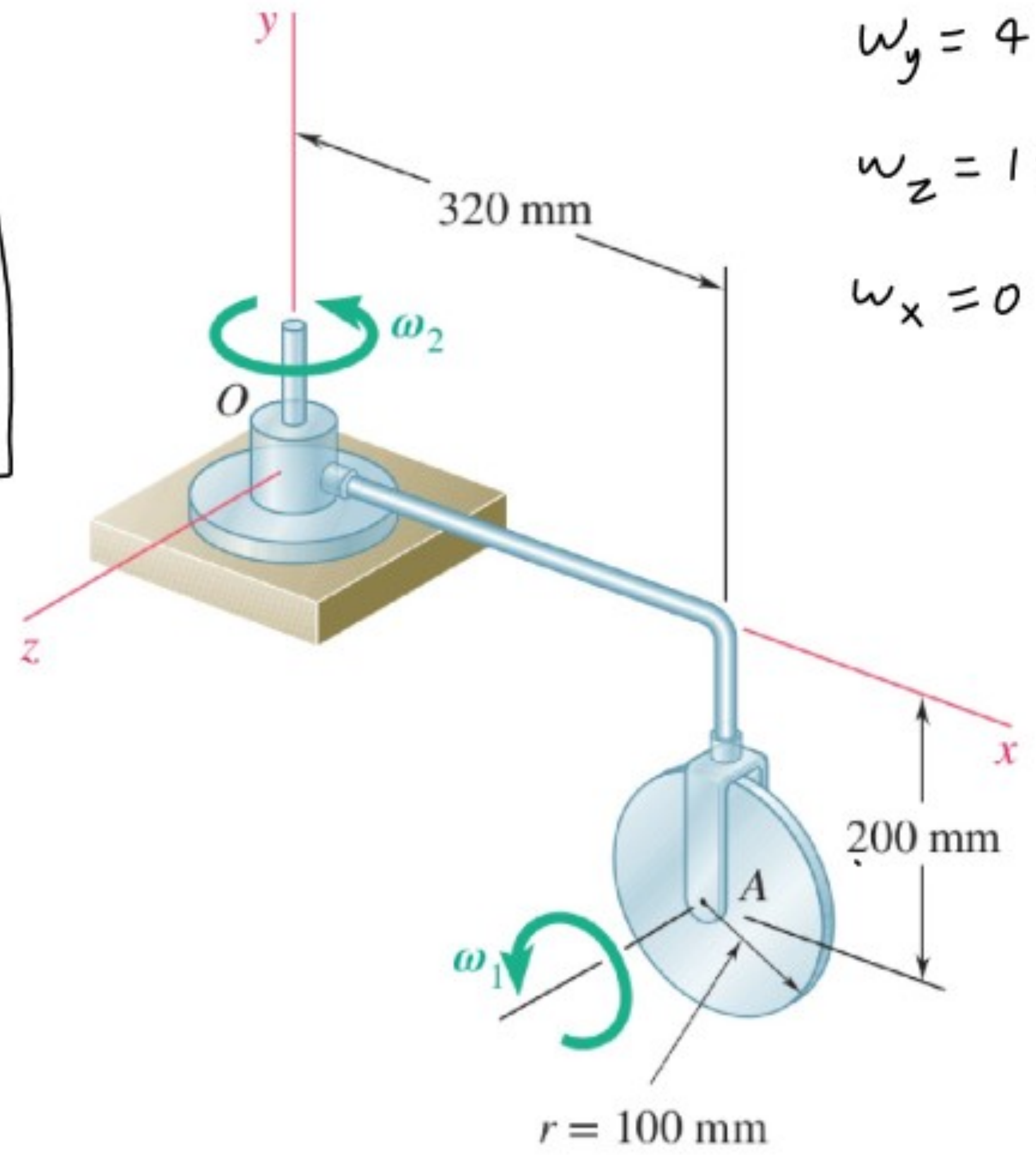
$$\Sigma M = I\alpha$$

$$H_{x'} = \bar{I}_{x'}\,\omega_x$$

$$H_{y'} = \bar{I}_{y'}\,\omega_y$$

$$H_{z'} = \bar{I}_{z'}\,\omega_z$$

A homogeneous disk of mass $m = 8$ kg rotates at the constant rate $\omega_1 = 12$ rad/s with respect to arm OA, which itself rotates at the constant rate $\omega_2 = 4$ rad/s about the y axis. Determine the angular momentum $\mathbf{H}_A$ of the disk about its center A.



$$w_y = 4 \text{ rad/s}$$

$$w_z = 12 \text{ rad/s}$$

$$w_x = 0$$

$$I_x = \frac{1}{4} m r^2 = \frac{1}{4} 8 (0.1)^2$$

$$I_y = \frac{1}{4} m r^2 = \frac{1}{4} 8 (0.1)^2$$

$$I_z = \frac{1}{2} m r^2 = \frac{1}{2} 8 (0.1)^2$$

$$H_A = \begin{bmatrix} I_x & -I_{xy} & -I_{xz} \\ -I_{yx} & I_y & -I_{yz} \\ -I_{zx} & -I_{zy} & I_z \end{bmatrix} \begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix}$$

$$H_A = \begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix} \begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix}$$

$$\begin{bmatrix} 0.02 & 0 & 0 \\ 0 & 0.02 & 0 \\ 0 & 0 & 0.04 \end{bmatrix} \begin{bmatrix} 0 \\ 4 \\ 12 \end{bmatrix} = \boxed{\begin{bmatrix} 0 \\ 0.08 \\ 0.48 \end{bmatrix} \frac{\text{kg m}^2}{\text{s}}}$$

A thin, homogeneous disk of mass m and radius r spins at the constant rate ω_1 about an axle held by a fork-ended vertical rod that rotates at the constant rate ω_2. Determine the angular momentum $\mathbf{H}_G$ of the disk about its mass center G.

$\omega_x = 0$

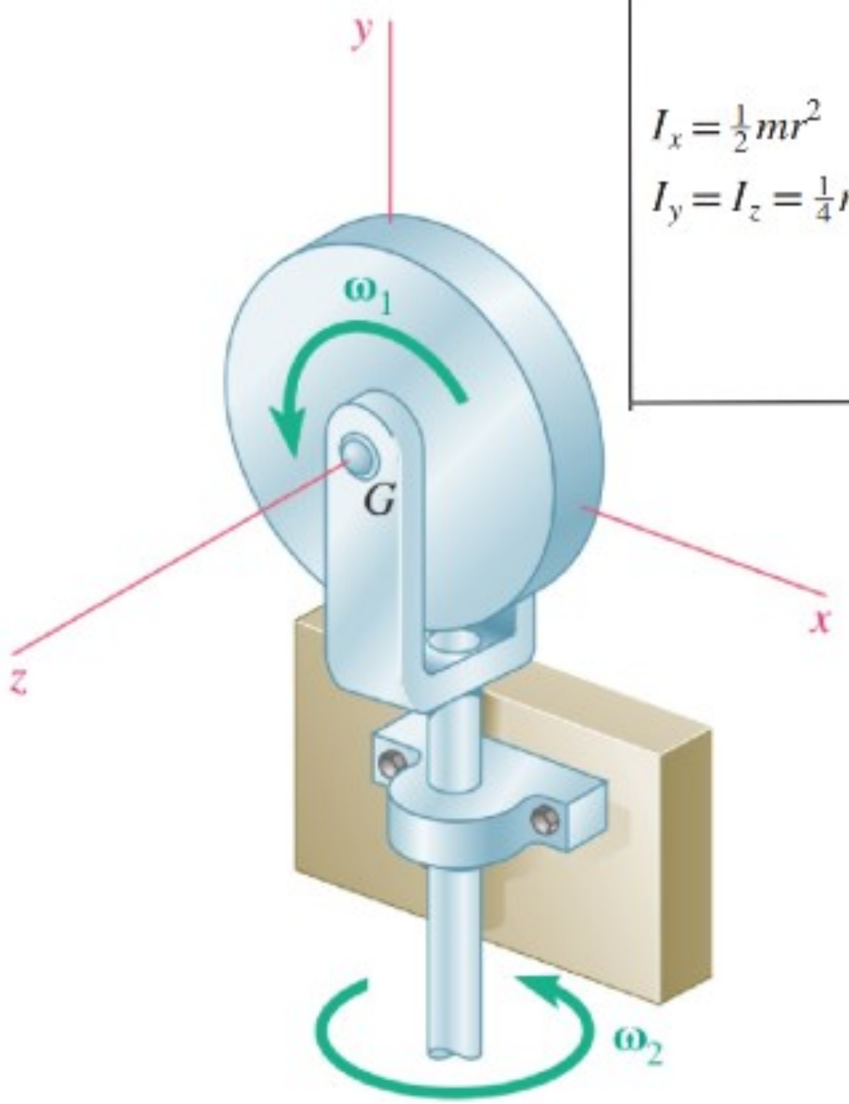
$\omega_y = \omega_2$

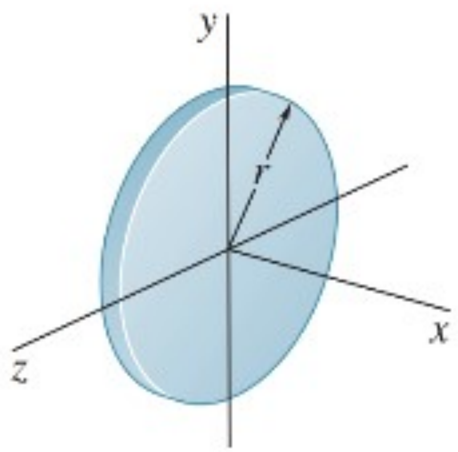
$\omega_z = \omega_1$

$\bar{I}_x = \frac{1}{4} m r^2$

$\bar{I}_y = \frac{1}{4} m r^2$

$\bar{I}_z = \frac{1}{2} m r^2$



Thin disk	
$I_x = \frac{1}{2} mr^2$ $I_y = I_z = \frac{1}{4} mr^2$	

$$\boxed{\begin{aligned} H_x &= \bar{I}_x \omega_x = 0 \\ H_y &= \bar{I}_y \omega_y = \frac{1}{4} m r^2 \omega_2 \\ H_z &= \bar{I}_z \omega_z = \frac{1}{2} m r^2 \omega_1 \end{aligned}}$$