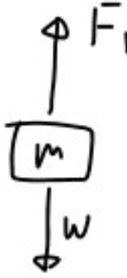
Free Vibration

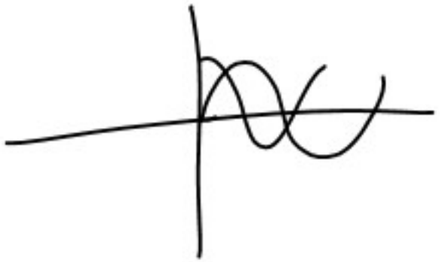


$$-W = F_K = Kx$$

$$-ma = Kx$$

$$-m\ddot{x} = Kx$$

$$m\ddot{x} + Kx = 0$$



$$x = \sin\left(\sqrt{\frac{K}{m}}\,t\right) \qquad \dot{x} = \sqrt{\frac{K}{m}}\cos\left(\sqrt{\frac{K}{m}}\,t\right) \qquad \ddot{x} = \frac{-K}{m}\sin\left(\sqrt{\frac{K}{m}}\,t\right)$$

$$m\left(\frac{-K}{m}\sin\left(\sqrt{\frac{K}{m}}\,t\right)\right) + K\sin\left(\sqrt{\frac{K}{m}}\,t\right) = 0 \quad \checkmark$$

$$x = \sin(\omega_n t)$$

ω_n natural frequency

$$\omega_n = \sqrt{\frac{K}{m}}$$

k	$\frac{N}{m}$
m	kg

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{N/m}{kg}} = \sqrt{\frac{\frac{\cancel{kg}\,\cancel{m}}{s^2}/\cancel{m}}{\cancel{kg}}} = \sqrt{1/s^2} = 1/s = rad/s$$

9	bounces	4.17	seconds	0.46	s/bounce
12	bounces	5.8	seconds	0.48	s/bounce
10	bounces	4.79	seconds	0.48	s/bounce

0.475 s/bounce

$$\frac{1}{0.475\ s/\text{bounce}} = 2.1\ \frac{\text{bounce}}{s}\ \frac{2\pi\ rad}{1\ \text{bounce}}$$

$$= 13.2\ rad/s = \omega_n$$

$$\omega_n = \sqrt{\frac{k}{m}}$$

$$\omega_n^2 = \frac{k}{m} \qquad k = \omega_n^2 m = \left(13.2\ rad/s\right)^2 0.2\ kg = 35\ \frac{kg}{s^2}\ \frac{1\ N}{1\ \frac{kg\ m}{s^2}} = 35\ N/m$$

11 bounces 7.92 s

$0.675\ \frac{s}{bounce}$

$1.98\ \frac{bounce}{s}\ \frac{2\pi\ rad}{1\ bounce} = 9.31\ \frac{rad}{s}$

11 bounces 7.92 s

$\omega_n = \sqrt{\frac{k}{m}}$ $\quad m = \frac{k}{\omega_n^2} = \frac{35\ N/m}{(9.31\ rad/s)^2} = 0.4\ kg$

$0.4\ kg - 0.2\ kg = \boxed{0.2\ kg}$

10	bounces	3.12	s
10	bounces	3.15	s

$0.314 \ s/bounce$

$3.19 \ \frac{bounces}{s} \ \frac{2\pi \ rad}{1 \ bounce} = 20 \ rad/s$

$$m = \frac{k}{\omega_n^2} = \frac{35}{20^2} = \boxed{0.087 \ kg}$$