

$$\vec{R}^T = \begin{matrix} \nearrow \\ \circ \\ \searrow \end{matrix} = \begin{matrix} \nearrow m a_r \\ \circ \\ \searrow m a_t \end{matrix}$$

$$\sum F_t = m a_t$$

$$-W \sin \theta = m a_t$$

$$-W \sin \theta = m \ddot{\theta} l$$

$$-mg \sin \theta = m l \ddot{\theta}$$

$$-g \sin \theta = l \ddot{\theta}$$

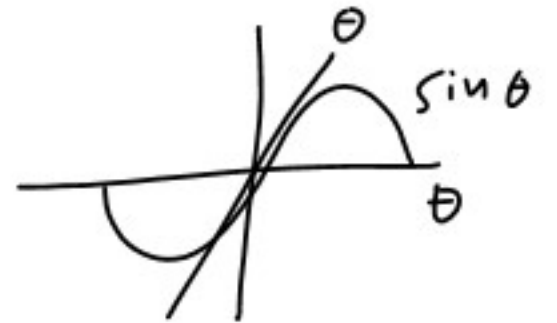
$$l \ddot{\theta} + g \sin \theta = 0$$

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$$\theta = \theta_m \sin(\omega_n t)$$

$$\tau_n = \frac{2\pi}{\omega_n} = 2\pi \sqrt{\frac{l}{g}}$$

$$\omega_n = \sqrt{\frac{g}{l}}$$



$\sin \theta \approx \theta$ if θ small
Small angle theorem

Exact sol'n

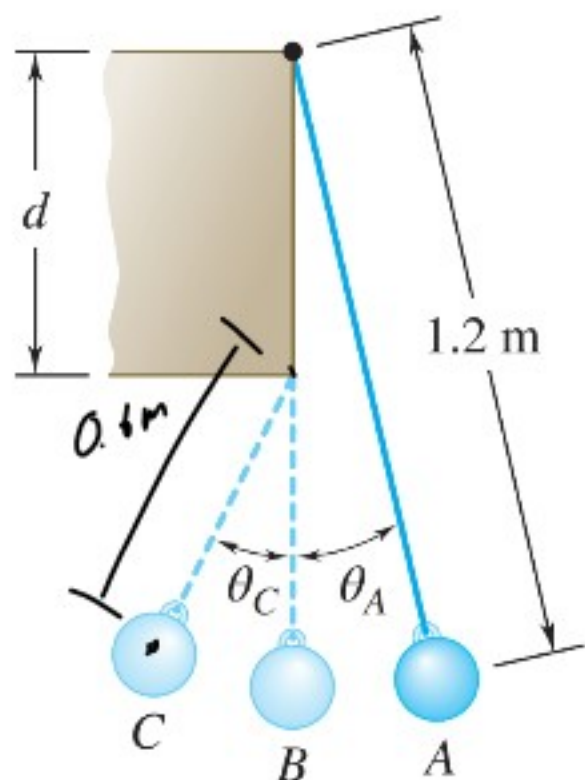
$$\tau_n = \frac{2\pi}{\omega_n} = 2\pi \sqrt{\frac{l}{g}}$$

Rigid Body Vibration

$$\sum \vec{F} = m\vec{a}$$

$$\sum M_o = \dot{H}_o$$

19.16 A small bob is attached to a cord of length 1.2 m and is released from rest when $\theta_A = 5^\circ$. Knowing that $d = 0.6$ m, determine (a) the time required for the bob to return to point A, (b) the amplitude θ_C .



$$\tau_A = 2\pi\sqrt{\frac{l_A}{g}}$$

$$\tau_C = 2\pi\sqrt{\frac{l_C}{g}}$$

$$\begin{aligned}\tau &= \frac{1}{2}\tau_A + \frac{1}{2}\tau_C \\ &= \pi\sqrt{\frac{l_A}{g}} + \pi\sqrt{\frac{l_C}{g}} = \frac{\pi}{\sqrt{g}}(\sqrt{l_A} + \sqrt{l_C}) \\ &= \frac{\pi}{\sqrt{9.8}}(\sqrt{1.2} + \sqrt{0.6}) = \boxed{1.88 \text{ s}}\end{aligned}$$

$$\theta_A(t) = 5^\circ \cos(\omega_n t)$$

$$\dot{\theta}_A(t) = -5^\circ \omega_n \sin(\omega_n t)$$

$$\dot{\theta}_B = -5^\circ \omega_n = -5^\circ \cdot 2.36 \frac{1}{\text{s}} = -11.8^\circ/\text{s}$$

$$\theta_C(t) = \theta_m \cos(\omega_n t)$$

$$\dot{\theta}_C(t) = -\theta_m \omega_n \sin(\omega_n t)$$

$$\dot{\theta}_B = -\theta_m \omega_n = -\theta_m 9 \frac{1}{\text{s}}$$

$$\omega_n = \sqrt{\frac{g}{l}} = \sqrt{\frac{9.8}{1.2}} = 2.86 \frac{1}{\text{s}}$$

$$\omega_n = \sqrt{\frac{g}{l}} = \sqrt{\frac{9.8}{0.6}} = 4 \frac{1}{\text{s}}$$

$$-11.8 \frac{1}{\text{s}} = -\theta_m 4 \frac{1}{\text{s}}$$

$$\frac{-11.8 \frac{1}{\text{s}}}{-4 \frac{1}{\text{s}}} = \boxed{3.57^\circ} = \theta_m = \theta_C$$