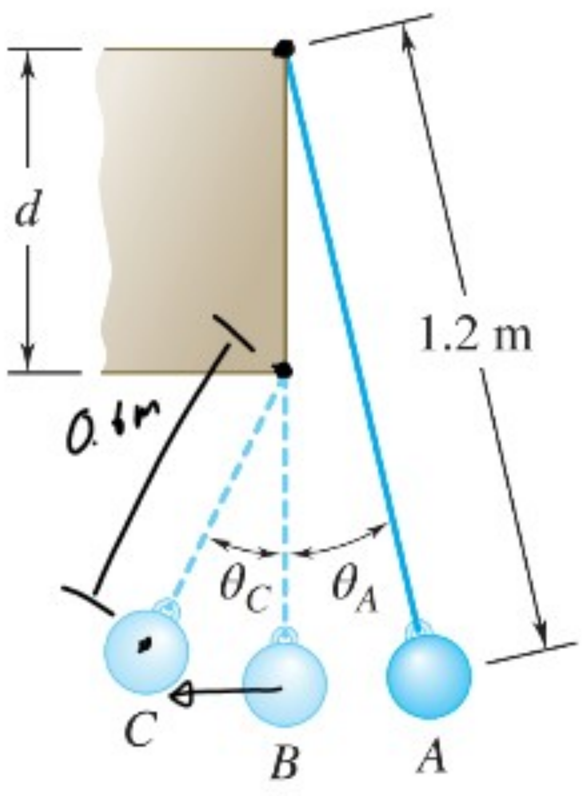
19.16 A small bob is attached to a cord of length 1.2 m and is released from rest when $\theta_A = 5°$. Knowing that $d = 0.6$ m, determine (*a*) the time required for the bob to return to point *A*, (*b*) the amplitude θ_C.



$$\tau_A = 2\pi\sqrt{\frac{l_A}{g}}$$

$$\tau_C = 2\pi\sqrt{\frac{l_C}{g}}$$

$$\tau = \frac{1}{2}\tau_A + \frac{1}{2}\tau_C$$

$$= \pi\sqrt{\frac{l_A}{g}} + \pi\sqrt{\frac{l_C}{g}} = \frac{\pi}{\sqrt{g}}\left(\sqrt{l_A} + \sqrt{l_C}\right)$$

$$= \frac{\pi}{\sqrt{9.8}}\left(\sqrt{1.2} + \sqrt{0.6}\right) = \boxed{1.88 \text{ s}}$$

$$\theta_A(t) = 5° \cos(\omega_n t)$$

$$\omega_n = \sqrt{\frac{g}{l}} = \sqrt{\frac{9.8}{1.2}} = 2.86 \ 1/s$$

$$\dot{\theta}_A(t) = -5° \omega_n \sin(\omega_n t)$$

$$\dot{\theta}_B = -5° \omega_n = -5° \cdot 2.86 \frac{1}{s} = -14.3 °/s$$

$$\omega_n = \sqrt{\frac{g}{l}} = \sqrt{\frac{9.8}{0.6}} = 4 \ 1/s$$

$$\theta_C(t) = \theta_m \cos(\omega_n t)$$

$$\dot{\theta}_C(t) = -\theta_m \omega_n \sin(\omega_n t)$$

$$\dot{\theta}_B = -\theta_m \omega_n = -\theta_m \ 4 \ 1/s$$

$$-14.3 °/s = -\theta_m \ 4 \ 1/s$$

$$\frac{-14.3 °/s}{-4 \ 1/s} = \boxed{3.57°} = \theta_m = \theta_C$$

$$\dot{\theta}_B = -14.3\ ^\circ/s\ \frac{\pi\ rad}{180^\circ} = 0.25\ rad/s$$

$$V_B = l_A \theta_B = 1.2\,m\ 0.25\ rad/s = 0.3\ m/s$$

$$\dot{\theta}_B = \frac{V_B}{l_C} = \frac{0.3\ m/s}{0.6\,m} = 0.5\ rad/s$$

$$\dot{\theta}_B = \theta_m w_n$$

$$0.5\ rad/s = \theta_m\ 4\ 1/s$$

$$\frac{0.5\ rad/s}{4\ 1/s} = 0.125\ rad\ \frac{180^\circ}{\pi\ rad} = \boxed{7.15^\circ}$$

$$\cancel{T_{A_0}} + V_A = \cancel{T_{c_0}} + V_c$$

$$V_A = V_c$$

$$m g h_A = m g h_c$$

$$h_A = h_c$$

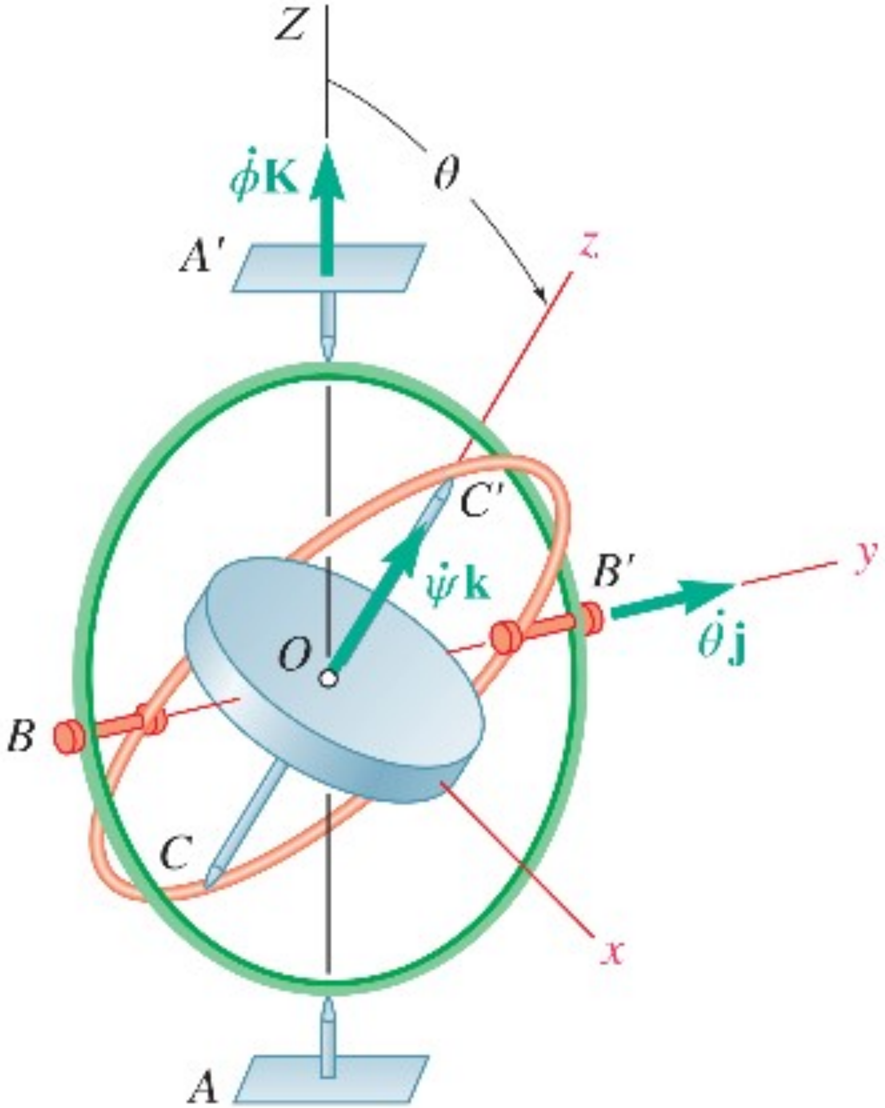
$$-1.2\cos\theta_A = -0.6 - 0.6\cos\theta_c$$

$$-1.2\cos 5^\circ = -0.6 - 0.6\cos\theta_c$$

$$0.6\cos\theta_c = 1.2\cos 5^\circ - 0.6$$

$$\cos\theta_c = \frac{1.2\cos 5^\circ - 0.6}{0.6}$$

$$\boxed{\theta_c = 7.07^\circ}$$



Gyroscopes

$$\omega = \dot\phi \mathbf{K} + \dot\theta j + \dot\psi k$$

$$\mathbf{K} = -\sin\theta i + \cos\theta k$$

$$\omega = -\dot\phi \sin\theta i + \dot\theta j + (\dot\psi + \dot\phi\cos\theta) k$$

$$H_0 = -I'\dot\phi\sin\theta i + I'\dot\theta j + I(\dot\psi + \dot\phi\cos\theta) k$$

$$\Omega = \dot\phi K + \dot\theta j = -\dot\phi\sin\theta i + \dot\theta j + \dot\phi\cos\theta k$$

Steady procession

θ constant

$$\omega = -\dot\phi\sin\theta i + \omega_z k$$

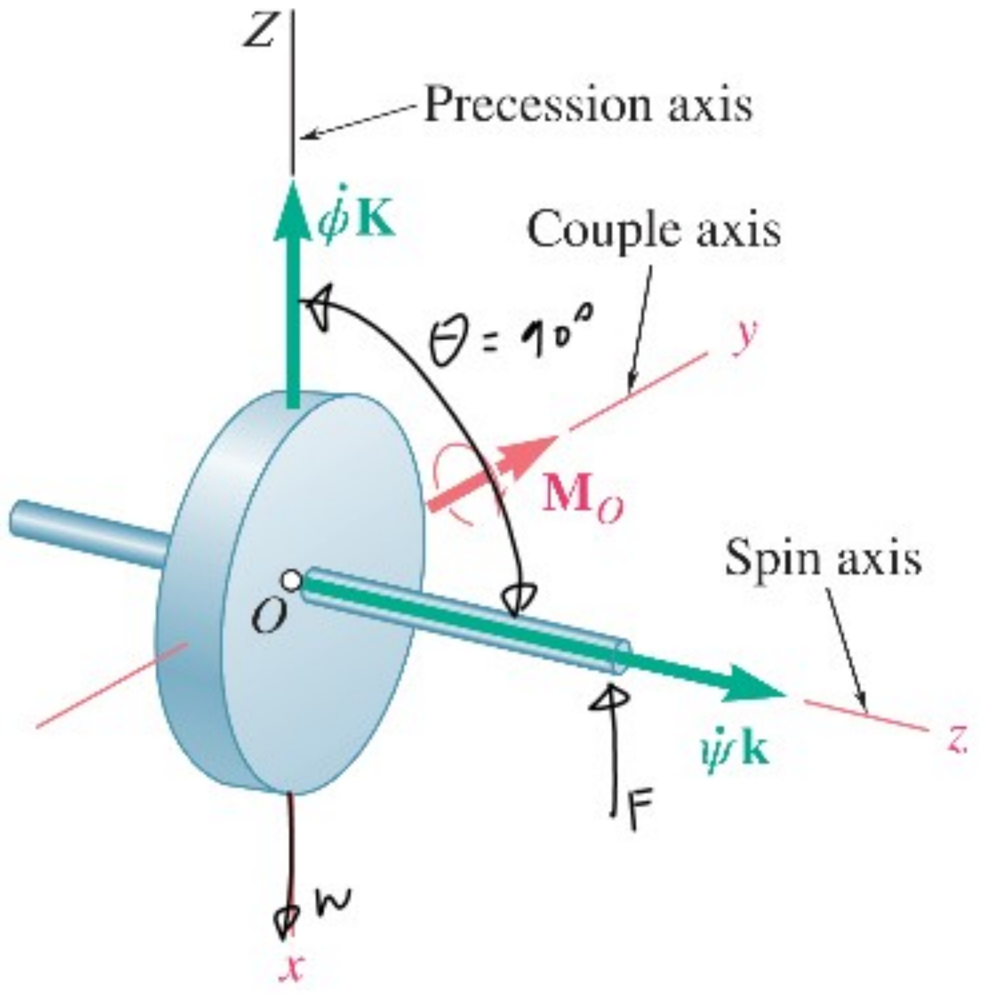
$$H_0 = -I'\dot\phi\sin\theta i + I\omega_z k$$

$$\Omega = -\dot\phi\sin\theta i + \dot\phi\cos\theta k$$

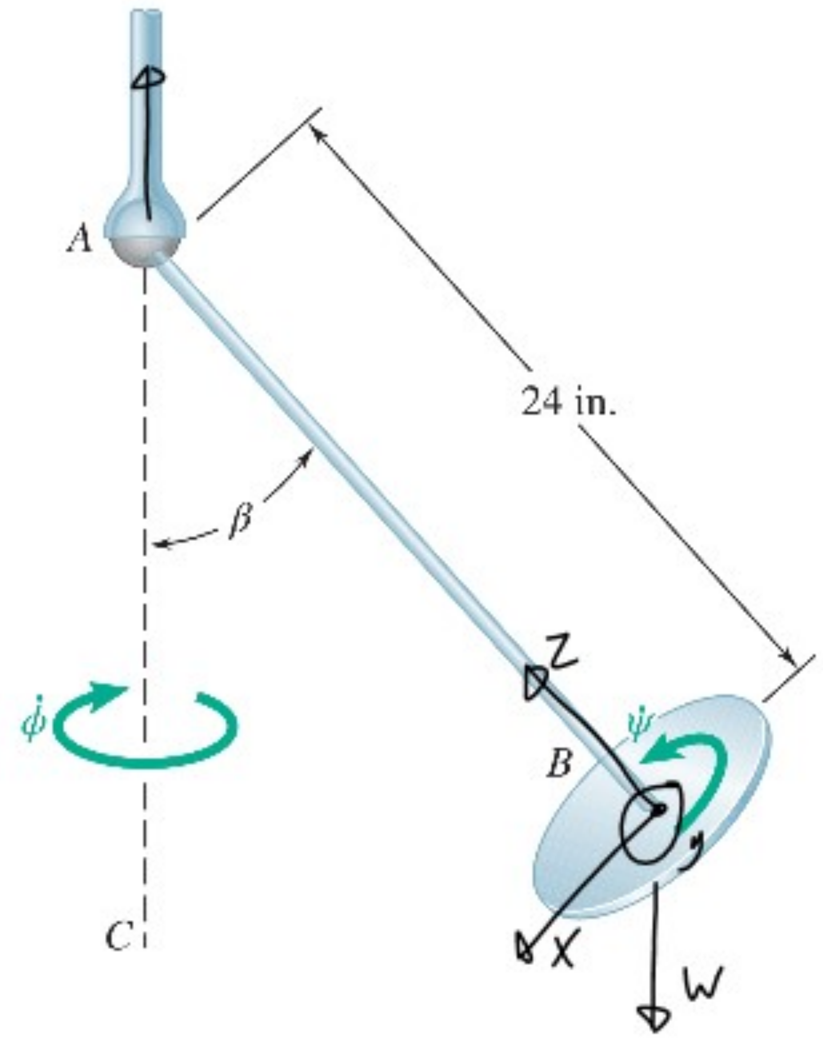
$$\Sigma M_0 = \Omega \times H_0 = (I\omega_z - I'\dot\phi\cos\theta)\dot\phi\sin\theta j$$

if $\theta = 90°$

$$\Sigma M_O = I \dot{\psi} \cancel{\phi} j = \cancel{\phi} \times I \dot{\psi}$$



18.107 A uniform thin disk with a 6-in. diameter is attached to the end of a rod AB of negligible mass that is supported by a ball-and-socket joint at point A. Knowing that the disk is observed to precess about the vertical axis AC at the constant rate of 36 rpm in the sense indicated and that its axis of symmetry AB forms an angle $\beta = 60°$ with AC, determine the rate at which the disk spins about rod AB.



$$\Sigma M_A = (I\omega_z - I'\dot{\phi}\cos\theta)\dot{\phi}\sin\theta\, j$$

$\theta = \beta$

$$\cancel{m} g\, 24\text{ in} \sin\theta = \left(\tfrac{1}{2}\cancel{m} r^2 \omega_z - \tfrac{1}{4}\cancel{m} r^2\, 36\text{ rpm}\cos 60\right)$$

$36\text{ rpm} \sin 60$

$I = \frac{1}{2} m r^2$

$I' = \frac{1}{4} m r^2$

$$g\,\frac{24\text{ in}\,\cancel{\sin 60}}{36\text{ rpm}\,\cancel{\sin 60}} = \frac{1}{2} r^2 \omega_z - \frac{1}{4} r^2\, 36\text{ rpm}\cos 60$$

$\dot{\phi} = 36\text{ rpm}$

$\Sigma M_A = W\, 24 \sin\beta$

$$g\,\frac{24\text{ in}}{36\text{ rpm}} + \frac{1}{4} r^2\, 36\text{ rpm}\cos 60 = \frac{1}{2} r^2 \omega_z$$

$$g \frac{2}{r^2} \frac{24\text{ in}}{36\text{ rpm}} + \frac{1}{2} 36\text{ rpm} \cos 60 = \omega_z$$

$$32.2\,\text{ft}/\text{s}^2 \frac{2}{(3\text{ in})^2} \frac{24\text{ in}}{36\text{ rpm}} + \frac{1}{2} 36\text{ rpm} \cos 60 = \omega_z$$

$$32.2\,\text{ft}/\text{s}^2\; 0.148 \frac{\text{min}}{\text{in rev}} + 9 \frac{\text{rev}}{\text{min}} = \omega_z$$