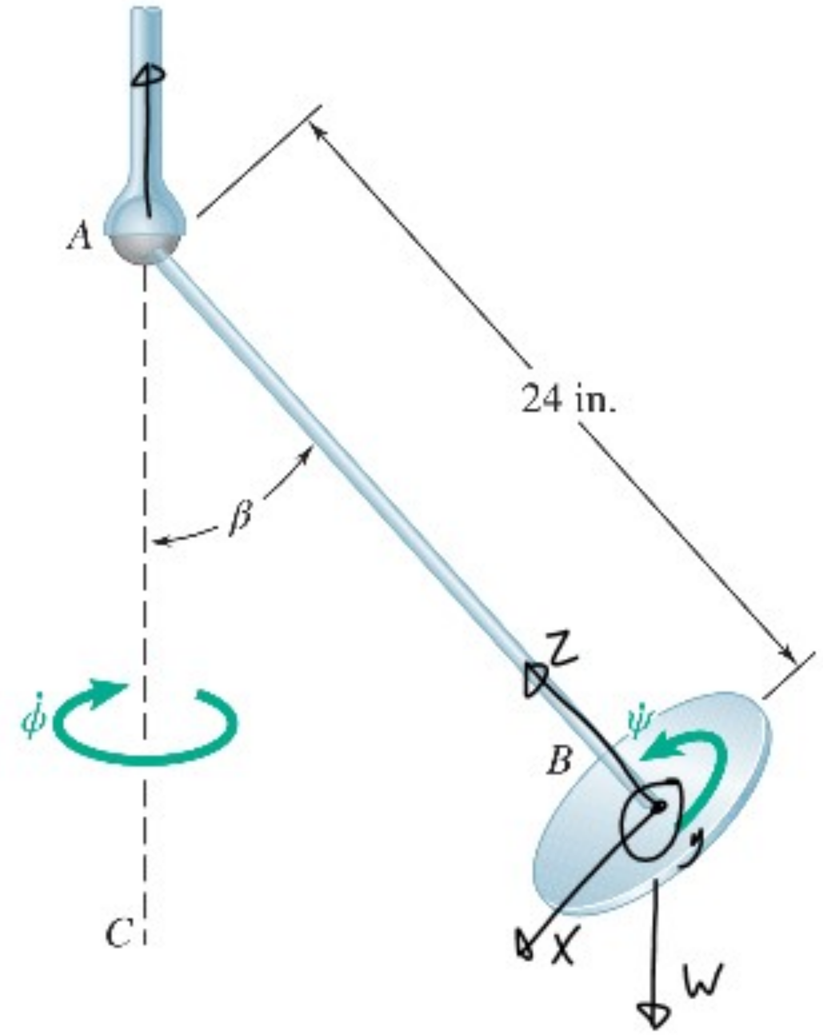
18.107 A uniform thin disk with a 6-in. diameter is attached to the end of a rod AB of negligible mass that is supported by a ball-and-socket joint at point A. Knowing that the disk is observed to precess about the vertical axis AC at the constant rate of 36 rpm in the sense indicated and that its axis of symmetry AB forms an angle $\beta = 60°$ with AC, determine the rate at which the disk spins about rod AB.



$$\Sigma M_A = (I\omega_z - I'\dot\phi\cos\theta)\dot\phi\sin\theta\, j$$

$\theta = \beta$

$$\cancel{m} g\ 24\text{ in}\sin\theta = \left(\tfrac{1}{2}\cancel{m}r^2\omega_z - \tfrac{1}{4}\cancel{m}r^2\, 36\text{ rpm}\cos 60\right)$$

$I = \frac{1}{2}mr^2$

$I' = \frac{1}{4}mr^2$

36 rpm sin 60

$$g\,\frac{24\text{ in}\ \cancel{\sin 60}}{36\text{ rpm}\ \cancel{\sin 60}} = \frac{1}{2}r^2\omega_z - \frac{1}{4}r^2\, 36\text{ rpm}\cos 60$$

$\dot\phi = 36$ rpm

$\Sigma M_A = W\, 24\sin\beta$

$$g\,\frac{24\text{ in}}{36\text{ rpm}} + \frac{1}{4}r^2\, 36\text{ rpm}\cos 60 = \frac{1}{2}r^2\omega_z$$

$$g \frac{2}{r^2} \frac{24 \text{ in}}{36 \text{ rpm}} + \frac{1}{2} 36 \text{ rpm} \cos 60 = \omega_z$$

$$32.2 \frac{\text{ft}}{\text{s}^2} \frac{2}{(3 \text{ in})^2} \frac{24 \text{ in}}{36 \text{ rpm}} + \frac{1}{2} 36 \text{ rpm} \cos 60 = \omega_z$$

$$32.2 \frac{\text{ft}}{\text{s}^2} 0.148 \frac{\text{min}}{\text{in rev}} + 9 \frac{\text{rev}}{\text{min}} = \omega_z$$

$$32.2 \frac{\cancel{\text{ft}}}{\text{s}^2} 0.148 \frac{\cancel{\text{min}}}{\text{in } \cancel{\text{rev}}} \frac{12 \cancel{\text{in}}}{1 \cancel{\text{ft}}} \frac{60 \text{ s}}{1 \cancel{\text{min}}} \frac{1 \cancel{\text{rev}}}{2\pi \text{ rad}} + 9 \frac{\cancel{\text{rev}}}{\cancel{\text{min}}} \frac{2\pi \text{ rad}}{1 \cancel{\text{rev}}} \frac{1 \cancel{\text{min}}}{60 \text{ s}} = 547 \text{ rad/s} = \omega_z$$

$$\omega_z = \dot{\psi} + \dot{\phi} \cos\theta$$

$$547 \text{ rad/s} = \dot{\psi} + 36 \text{ rpm} \cos 60$$

$$\dot{\psi} = 547 \text{ rad/s} - 36 \frac{\text{rev}}{\text{min}} \cos 60 \frac{1 \text{ min}}{60 \text{ s}} \frac{2\pi \text{ rad}}{1 \text{ rev}} = \boxed{545 \text{ rad/s}}$$

$$\Sigma M_A = (I\omega_z - I'\dot{\phi}\cos\theta)\dot{\phi}\sin\theta$$

$$W\ 24\text{ in}\sin\beta = \left(\frac{1}{2}mr^2\ 547\ \text{rad/s} - \frac{1}{4}mr^2\ 36\ \frac{\text{rev}}{\text{min}}\cos\beta\right) 36\ \frac{\text{rev}}{\text{min}}\sin\beta$$

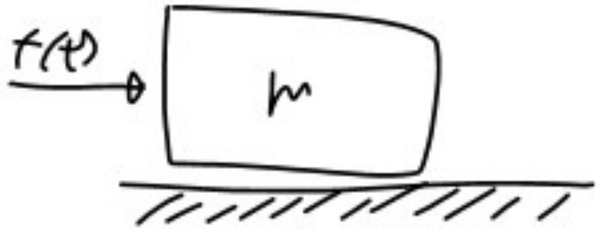
$$mg\ 24\text{ in}\sin 60 = \left(\frac{1}{2}m(3\text{ in})^2\ 547\ \text{rad/s} - \frac{1}{4}m(3\text{ in})^2\ 36\ \frac{\text{rev}}{\text{min}}\cos\right.$$

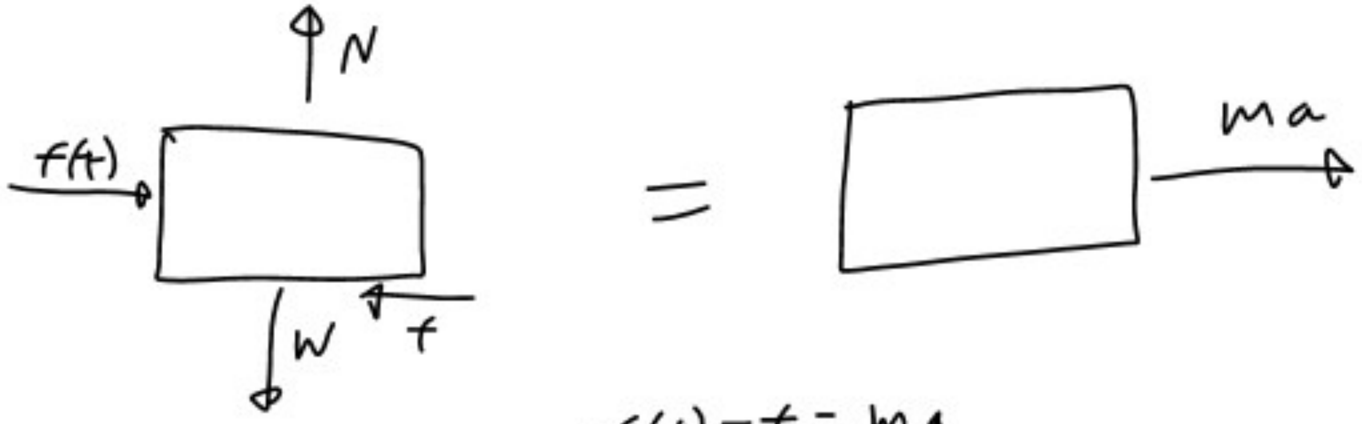
Analytic Solutions vs Numerical Solutions

Analytic	Numerical
+ No Computer Needed	− Needs Computer
− Slower for large problems	+ Faster for large problems
- difficult	+ easy
+ perfect answer	− imperfect
+ all solutions	- one solution

Computer Algebra Systems

Numerical Simulations





$$a = \dot{v}$$

$$f(t) - f = ma$$

$$f(t) - N\mu = ma$$

$$f(t) - W\mu = ma$$

$$f(t) - W\mu = m\dot{v}$$

$$\dot{v} = \frac{f(t) - W\mu}{m}$$

$$\frac{df}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

if Δx small

$$\frac{df}{dx} \approx \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$\dot{V} = \frac{f(t) - W\mu}{m}$$

$$\frac{dV}{dt} = \frac{f(t) - W\mu}{m}$$

$$\frac{dV}{dt} \approx \frac{V(t+\Delta t) - V(t)}{\Delta t}$$

if $V(0)$ known
$f(t)$ known

$$\frac{f(t) - W\mu}{m} = \frac{V(t+\Delta t) - V(t)}{\Delta t}$$

$$\Delta t \, \frac{f(t) - W\mu}{m} = V(t+\Delta t) - V(t)$$

$$V(t) + \Delta t \, \frac{f(t) - W\mu}{m} = V(t+\Delta t)$$