$$\frac{d}{du}(\vec{P}+\vec{Q}) = \frac{d\vec{P}}{du} + \frac{d\vec{Q}}{du}$$

$\vec{P}(u)$ $f(u)$

$$\frac{d}{du} f(u)\vec{P}(u) = \frac{df(u)}{du}\vec{P}(u) + f(u)\frac{d\vec{P}(u)}{du}$$

$$\frac{d}{du}\vec{P}\cdot\vec{Q} = \frac{d\vec{P}}{du}\cdot\vec{Q} + \vec{P}\cdot\frac{d\vec{Q}}{du}$$

$$\frac{d}{du}\vec{P}\times\vec{Q} = \frac{d\vec{P}}{du}\times\vec{Q} + \vec{P}\times\frac{d\vec{Q}}{du}$$

$$\vec{P} = P_x i + P_y j + P_z K$$

$$\vec{Q} = Q_x i + Q_y j + Q_z K$$

$$\vec{P}\cdot\vec{Q} = P_x Q_x + P_y Q_y + P_z Q_z$$

Work $W = F\cdot d$

$$\frac{d}{du}\vec{P} = \frac{dP_x}{du} i + \frac{dP_y}{du} j + \frac{dP_z}{du} K$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dr_x}{dt} i + \frac{dr_y}{dt} j + \frac{dr_z}{dt} K$$

$$= \dot{x} i + \dot{y} j + \dot{z} K$$

$$\vec{a} = \ddot{x} i + \ddot{y} j + \ddot{z} K$$

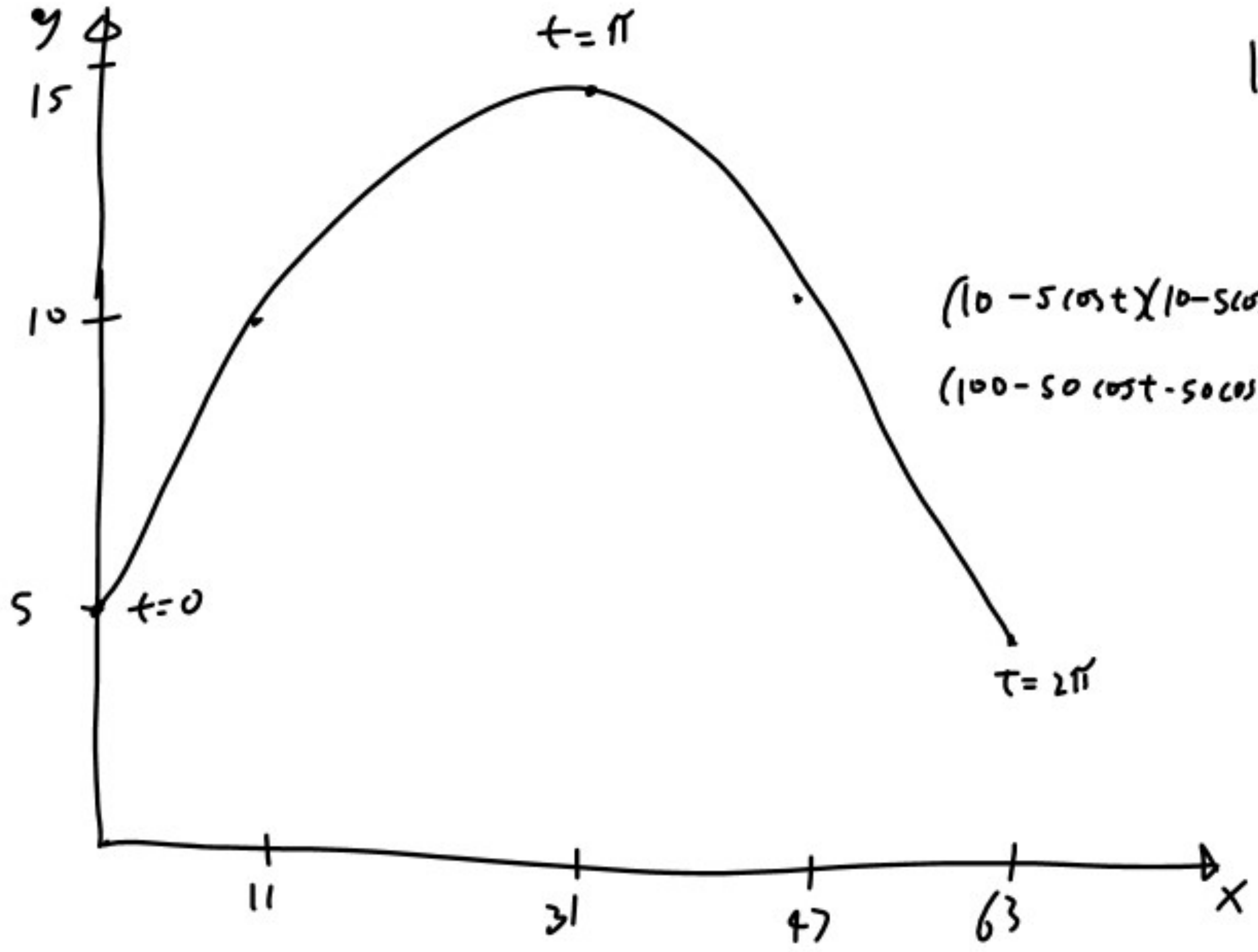
The motion of a particle is defined by the equations $x = 10t - 5\sin t$ and $y = 10 - 5\cos t$, where x and y are expressed in feet and t is expressed in seconds. Sketch the path of the particle for the time interval $0 \le t \le 2\pi$, and determine (*a*) the magnitudes of the smallest and largest velocities reached by the particle, (*b*) the corresponding times, positions, and directions of the velocities.

t	0	$\pi/2$	π	$3\pi/2$	2π
x	0	11	31	47	63
y	5	10	15	10	5

$\vec{r} = (10t - 5\sin t)\,i + (10 - 5\cos t)\,j$

$\vec{v} = (10 - 5\cos t)\,i + (5\sin t)\,j$

$|\vec{v}| = \sqrt{(10 - 5\cos t)^2 + 5^2\sin^2 t}$



$$|\vec{V}| = \sqrt{(10-5\cos t)^2 + (5\sin t)^2}$$

$(10-5\cos t)(10-5\cos t)$

$(100-50\cos t - 50\cos t + 25\cos^2 t$

$$= \sqrt{100 - 100\cos t + 25\cos^2 t + 25\sin^2 t}$$

$$= \sqrt{100 - 100\cos t + 25}$$

$$= \sqrt{125 - 100\cos t}$$

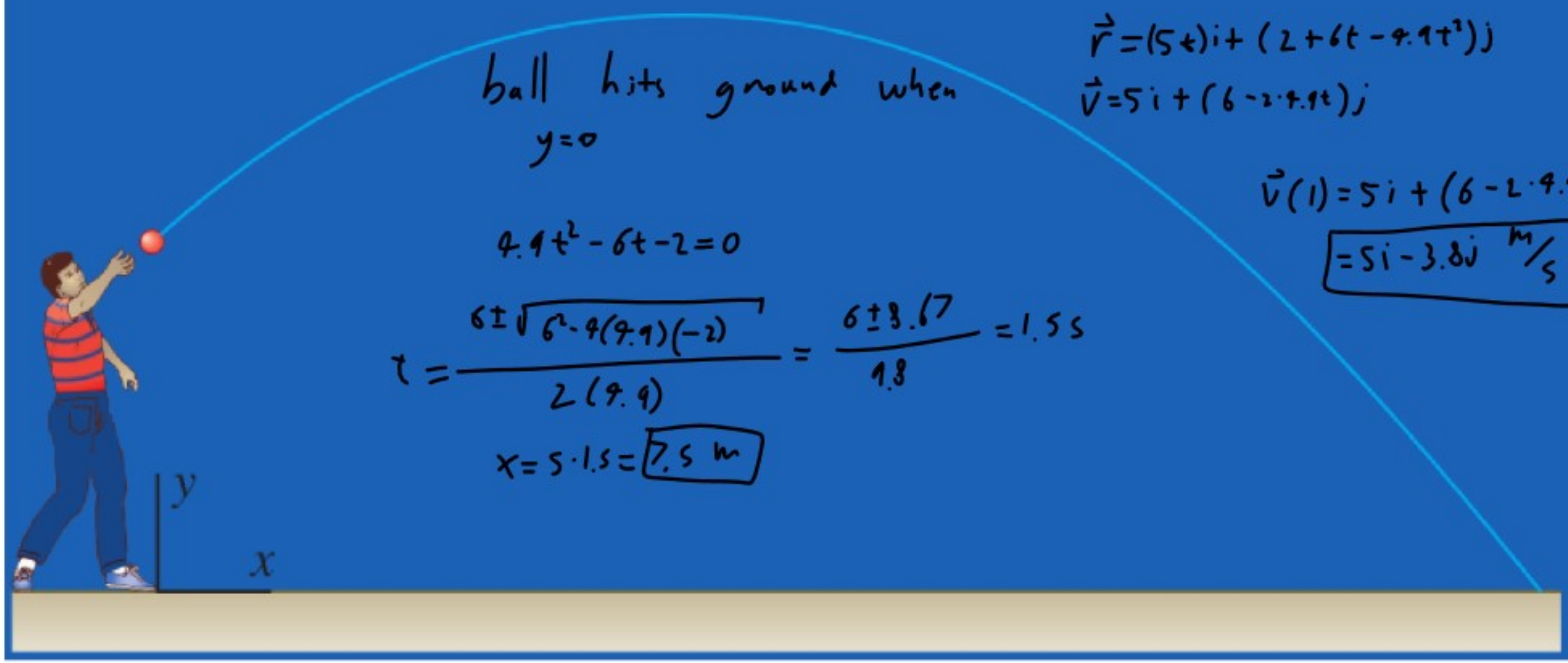
Max when $\cos(t) = -1$ $\quad t = \pi$ s

$$V_{max} = \sqrt{125+100} = 15 \text{ ft/s}$$

min when $\cos(t) = 1$ $\quad t = 0, 2\pi$ s

$$V_{min} = \sqrt{125-100} = \sqrt{25} = 5 \text{ ft/s}$$

A ball is thrown so that the motion is defined by the equations $x = 5t$ and $y = 2 + 6t - 4.9t^2$, where x and y are expressed in meters and t is expressed in seconds. Determine (*a*) the velocity at $t = 1$ s, (*b*) the horizontal distance the ball travels before hitting the ground.



$\vec{r} = (5t)i + (2 + 6t - 4.9t^2)j$

$\vec{v} = 5i + (6 - 2\cdot 4.9t)j$

$\vec{v}(1) = 5i + (6 - 2\cdot 4.9)j$

$= 5i - 3.8j$ m/s

ball hits ground when $y = 0$

$4.9t^2 - 6t - 2 = 0$

$t = \frac{6 \pm \sqrt{6^2 - 4(4.9)(-2)}}{2(4.9)} = \frac{6 \pm 8.67}{9.8} = 1.5$ s

$x = 5\cdot 1.5 = 7.5$ m