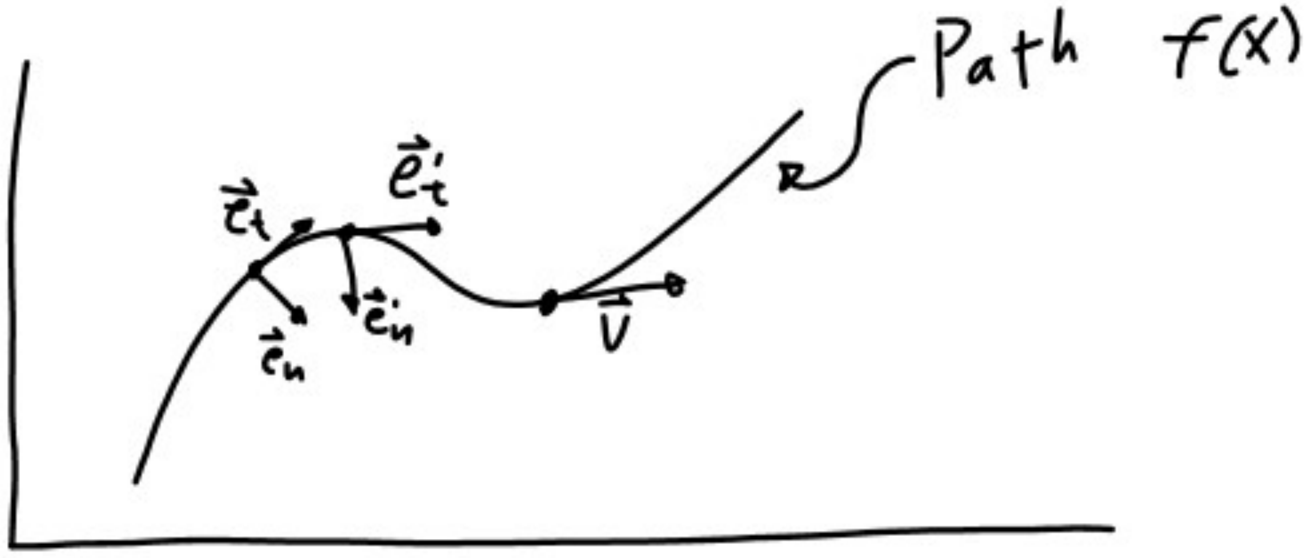
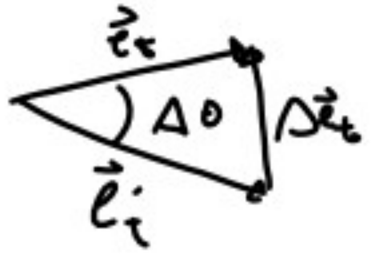
Tangential and Normal Components



$$\Delta \vec{e}_t = \vec{e}_t' - \vec{e}_t$$



$\vec{e}_t$ and $\vec{e}_n$ are unit vectors

$$|\Delta \vec{e}_t| = 2\sin\left(\Delta\theta/2\right)$$

$$\lim_{\Delta\theta \to 0} \frac{\Delta e_t}{\Delta\theta} = \lim_{\Delta\theta \to 0} \frac{2\sin(\Delta\theta/2)}{\Delta\theta} = 1$$

$$\vec{e}_n = \frac{d\vec{e}_t}{d\theta}$$

$$\vec{V} = \vec{e}_t V$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt} v\vec{e}_t = \frac{dv}{dt}\vec{e}_t + v\frac{d\vec{e}_t}{dt}$$

$$= \frac{dv}{dt}\vec{e}_t + \frac{v^2}{\rho}\vec{e}_n$$

$$\frac{d\vec{e}_t}{dt} = \underbrace{\frac{d\vec{e}_t}{d\theta}}_{\vec{e}_n}\ \underbrace{\frac{d\theta}{ds}}_{\frac{1}{\rho}}\ \underbrace{\frac{ds}{dt}}_{v}$$

$$\frac{d\vec{e}_t}{dt} = \vec{e}_n \frac{1}{\rho} v$$

ρ curvature

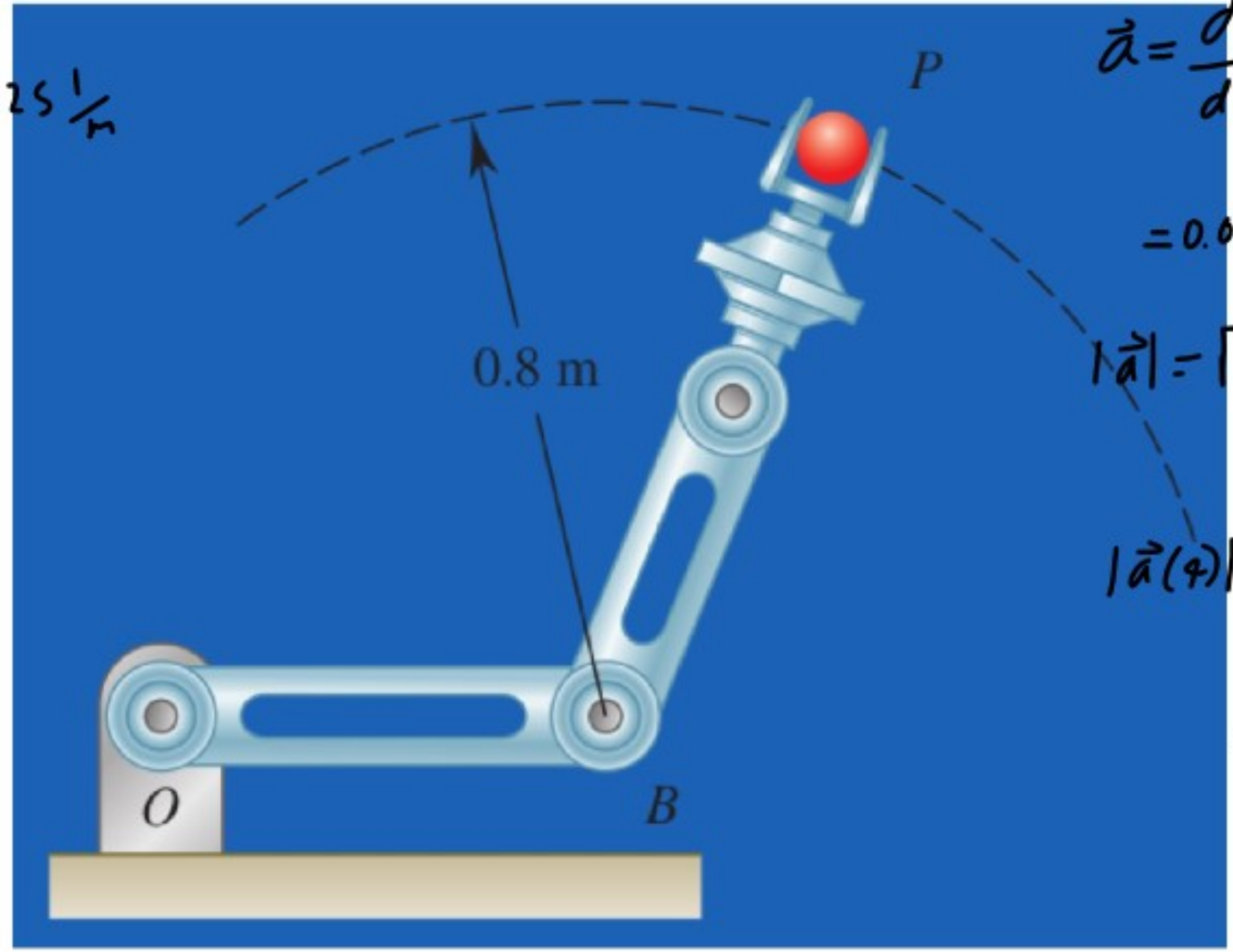
$$\rho = \frac{1}{r}$$

for a straight line

$$r = \infty$$

$$\rho = \frac{1}{r} = \frac{1}{\infty} = 0$$

A robot arm moves so that P travels in a circle about point B, which is not moving. Knowing that P starts from rest, and its speed increases at a constant rate of 10 mm/s^2, determine (*a*) the magnitude of the acceleration when $t = 4$ s, (*b*) the time for the magnitude of the acceleration to be 80 mm/s^2.



$$\rho = \frac{1}{r} = \frac{1}{0.8\,m} = 1.25\ 1/m$$

$$V_p = V_0 + at$$
$$= 0.01\,t$$

$$\vec{a} = \frac{dv}{dt}\vec{e}_t + \frac{v^2}{\rho}\vec{e}_n$$
$$= 0.01\,\vec{e}_t + \frac{(0.01\,t)^2}{1.25}\vec{e}_n$$

$$|\vec{a}| = \sqrt{0.01^2 + \left(\frac{(0.01t)^2}{1.25}\right)^2}$$

$$|\vec{a}(4)| = \sqrt{0.01^2 + \left(\frac{(0.01\cdot 4)^2}{1.25}\right)^2}$$
$$= 0.01008\ m/s^2$$

$$0.08 = \sqrt{0.01^2 + \left(\frac{(0.01t)^2}{1.25}\right)^2}$$

$$0.0064 = 0.01^2 + \left(\frac{(0.01t)^2}{1.25}\right)^2$$

$$0.0064 - 0.01^2 = \left(\frac{(0.01t)^2}{1.25}\right)^2$$

$$0.0063 = \left(\frac{(0.01t)^2}{1.25}\right)^2$$

$$0.079 = \frac{(0.01t)^2}{1.25}$$

$$\frac{0.099}{0.01^2} = \frac{0.01^2 t^2}{0.01^2}$$

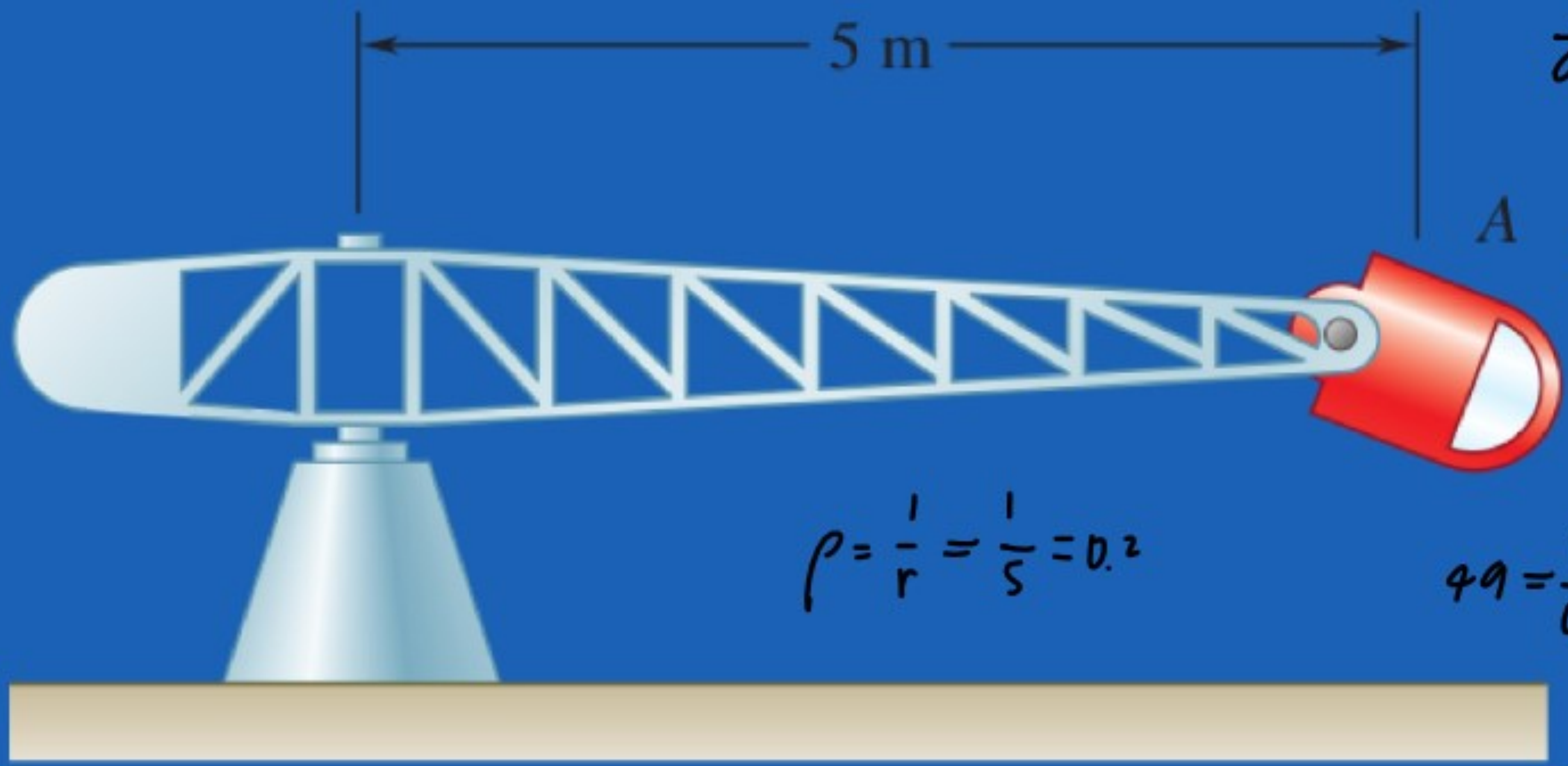
$$992 = t^2$$

$$\boxed{31.5\ s = t}$$

Human centrifuges are often used to simulate different acceleration levels for pilots and astronauts. Pilots typically face inward toward the center of the gondola in order to experience a simulated forward acceleration. Knowing that the pilots sits 5 m from the axis of rotation and experiences 5 g's inward, determine her velocity.

$5g = 5 \cdot 9.8\ \text{m/s}^2$

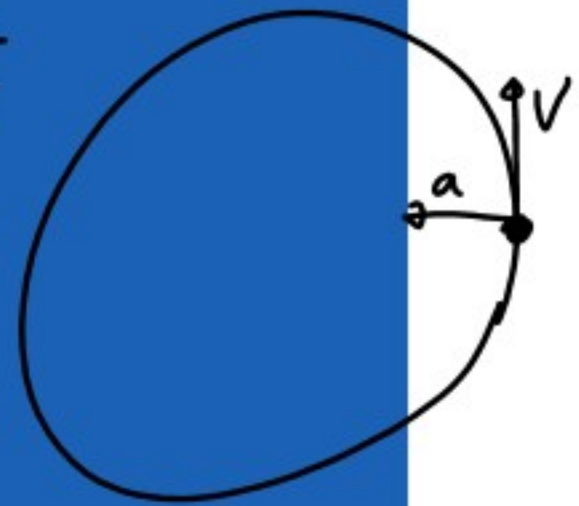
$= 49\ \text{m/s}^2$



$\vec{a} = \frac{dv}{dt}\vec{e}_t + \frac{v^2}{\rho}\vec{e}_n$

$\vec{a} = \frac{v^2}{\rho}\vec{e}_n$

$a = \frac{v^2}{\rho}$



$\rho = \frac{1}{r} = \frac{1}{5} = 0.2$

$49 = \frac{V^2}{0.2}$

$\sqrt{0.2 \cdot 49} = V$

$\boxed{V = 3.13\ \text{m/s}}$