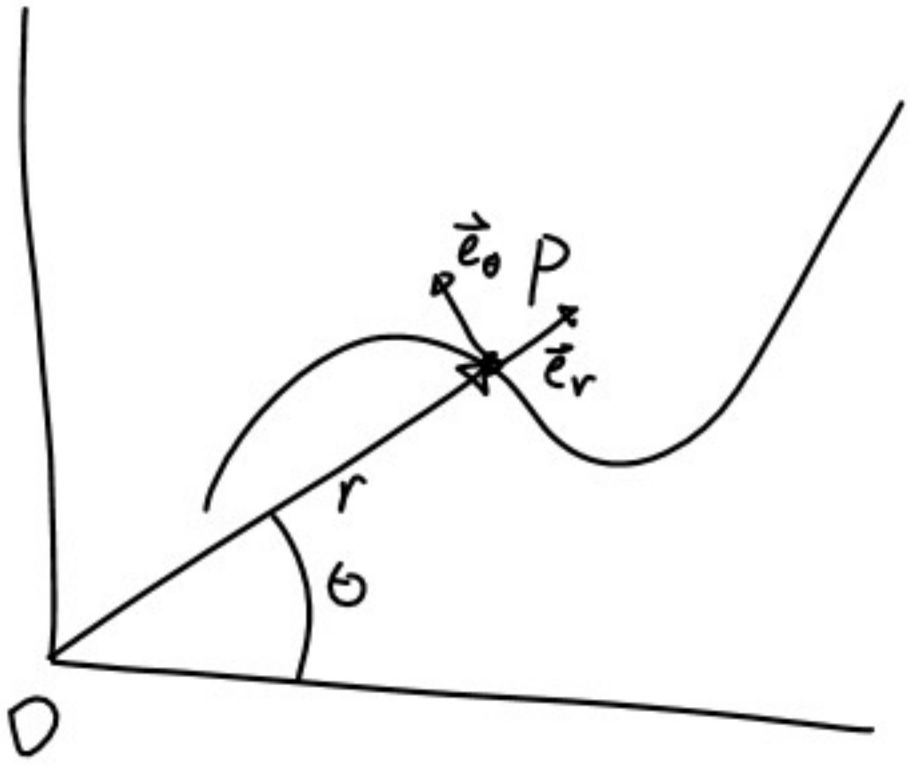
Radial and Transverse Components



$\vec{e}_\theta$ and $\vec{e}_r$ unit vectors

$$\frac{d\vec{e}_r}{d\theta} = \vec{e}_\theta \qquad \frac{d\vec{e}_\theta}{d\theta} = -\vec{e}_r$$

$$\frac{d\vec{e}_r}{dt} = \frac{d\vec{e}_r}{d\theta}\frac{d\theta}{dt} = \vec{e}_\theta\frac{d\theta}{dt} \qquad \frac{d\vec{e}_\theta}{dt} = -\vec{e}_r\frac{d\theta}{dt}$$

$$\dot{\vec{e}}_r = \vec{e}_\theta\dot{\theta} \qquad \dot{\vec{e}}_\theta = -\vec{e}_r\dot{\theta}$$

$$\vec{V} = \frac{d}{dt} r\vec{e}_r = \dot{r}\vec{e}_r + r\dot{\vec{e}}_r = \dot{r}\vec{e}_r + r\dot{\theta}\vec{e}_\theta$$

$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{d}{dt}\dot{r}\vec{e}_r + r\dot{\theta}\vec{e}_\theta = \ddot{r}\vec{e}_r + \dot{r}\dot{\vec{e}}_r + \dot{r}\dot{\theta}\vec{e}_\theta + r\ddot{\theta}\vec{e}_\theta + r\dot{\theta}\dot{\vec{e}}_\theta$$

$$= (\ddot{r} - r\dot{\theta}^2)\vec{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\vec{e}_\theta$$

As rod OA rotates, pin P moves along the parabola BCD. Knowing that the equation of this parabola is $r = 2b/(1 + \cos\theta)$ and that $\theta = kt$, determine the velocity and acceleration of P when (*a*) $\theta = 0$, (*b*) $\theta = 90°$.

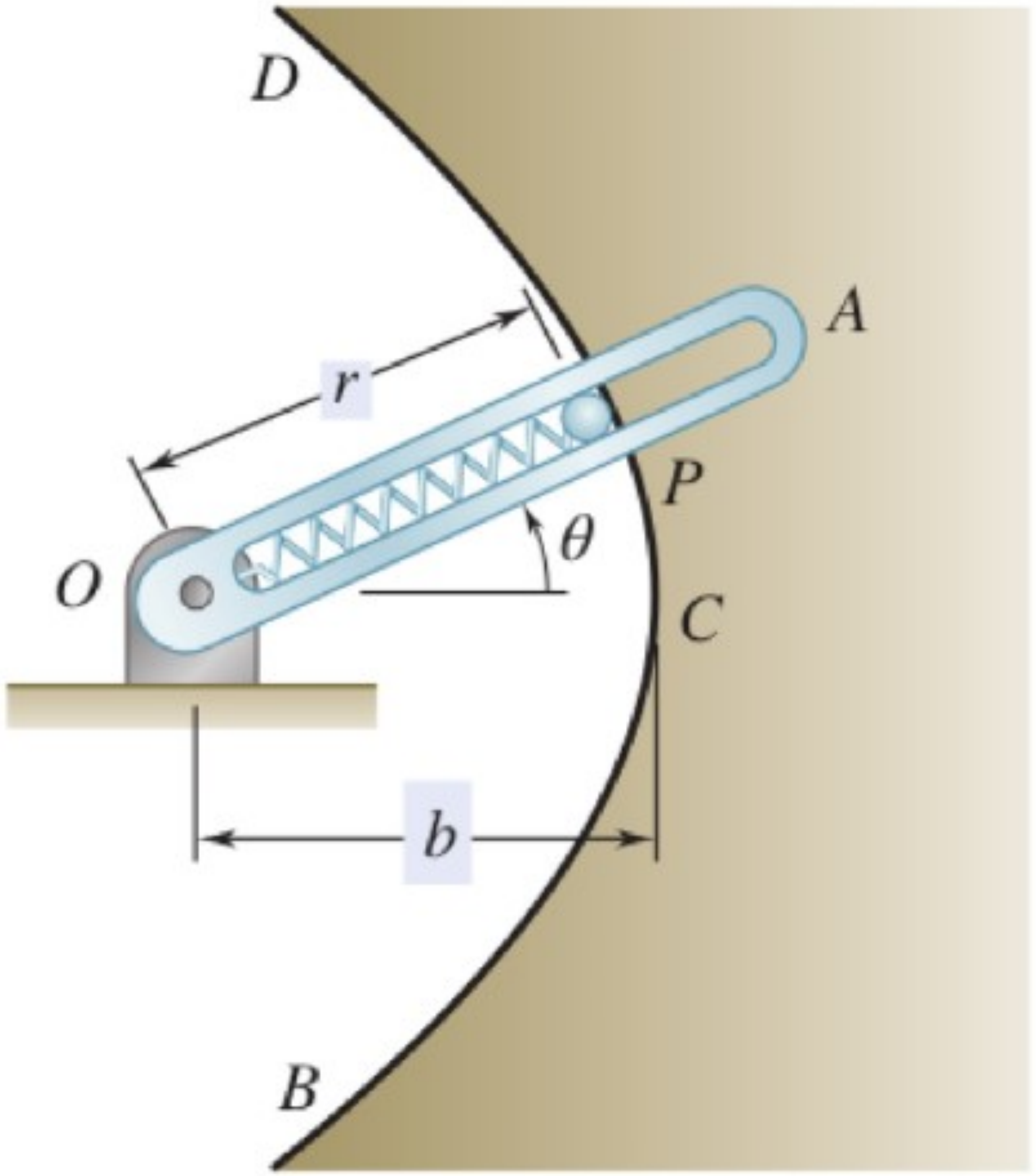
$$\vec{V} = \dot{r}\vec{e}_r + r\dot{\theta}\vec{e}_\theta$$

$$= \frac{2bk\sin kt}{(1+\cos kt)^2}\vec{e}_r + \frac{2b}{1+\cos kt}k\vec{e}_\theta$$

$$\vec{V}(0) = 0\,\vec{e}_r + bk\vec{e}_\theta$$

$$90° = kt$$

$$\vec{V}\left(\frac{90°}{k}\right) = 2bk\,\vec{e}_r + 2bk\,\vec{e}_\theta$$



$$r = \frac{2b}{1+\cos\theta} \qquad \theta = kt$$

$$r = \frac{2b}{1+\cos kt} \qquad \dot{\theta} = k$$

$$\dot{r} = \frac{2b\,k\sin(kt)}{(\cos(kt)+1)^2} \qquad \ddot{\theta} = 0$$

$$\ddot{r} = 2b\left(\frac{k^2\cos kt}{(\cos kt+1)^2} + \frac{2k^2\sin^2 kt}{(\cos kt+1)^3}\right)$$

$$\vec{a} = (\ddot{r} + r\dot{\theta}^2)\vec{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\vec{e}_\theta$$

$$= \left(2b\left(\frac{k^2 \cos kt}{(1+\cos kt)^2} + \frac{2k^2 \sin^2 kt}{(1+\cos kt)^3}\right) + \frac{2b}{1+\cos kt}k^2\right)\vec{e}_r + \left(\frac{2b}{1+\cos kt}0 + 2\frac{2bk\sin kt}{(1+\cos kt)^2}k\right)\vec{e}_\theta$$

$$\vec{a}(0) = \left(2b\left(\frac{k^2}{4}\right) + bk^2\right)\vec{e}_r + 0\,\vec{e}_\theta$$

$$\vec{a}\left(\frac{90°}{k}\right) = (2b(2k) + 2bk^2)\vec{e}_r + 4bk^2\vec{e}_\theta$$

During a parasailing ride, the boat is traveling at a constant 30 km/hr with a 200-m long tow line. At the instant shown, the angle between the line and the water is 30° and is increasing at a constant rate of 2°/s. Determine the velocity and acceleration of the parasailer at this instant.

$V_x = V_\theta \sin\theta$

$V_y = V_\theta \cos\theta$

$V_{x\,tot} = V_x + 30\ \text{km/h}$