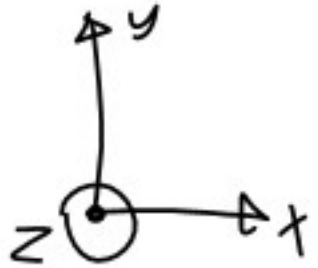
The sprocket wheel and chain shown are initially at rest. If the wheel has a uniform angular acceleration of 90 rad/s^2 counterclockwise, determine (*a*) the acceleration of point *A* of the chain, (*b*) the magnitude of the acceleration of point *B* of the wheel after 3 s.

$\alpha = 90\,k \ \text{rad}/s^2$



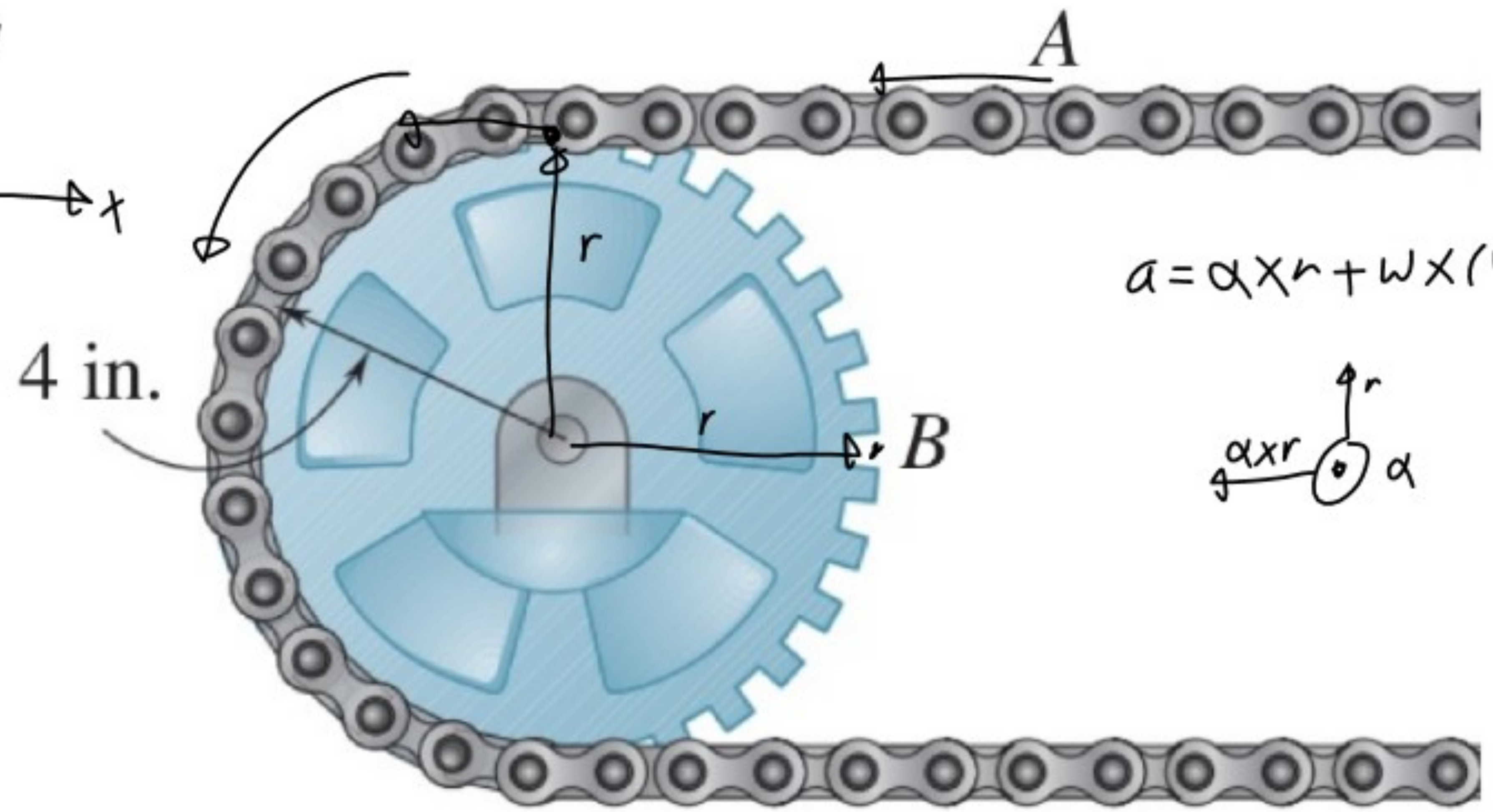
$\omega = \omega_0 + at$

$w = \alpha t$

$W = 90\,k\ 3$

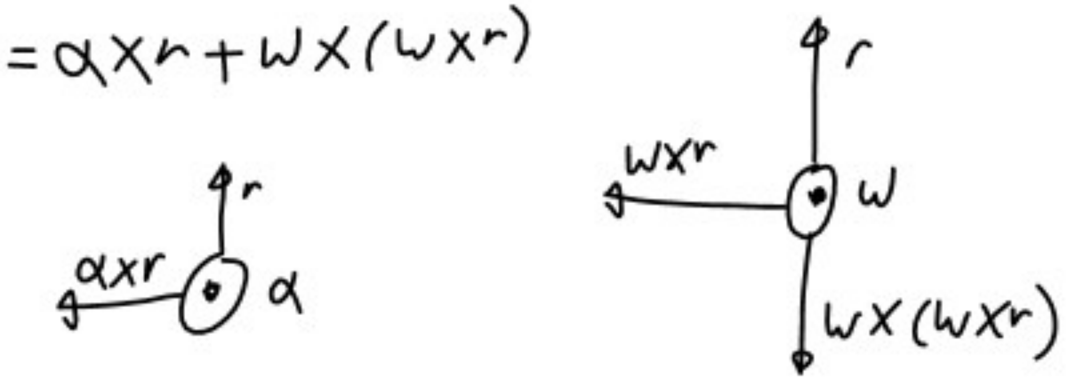
$W = 270\,k \ \text{rad}/s$

$r = 4i \ \text{in}$

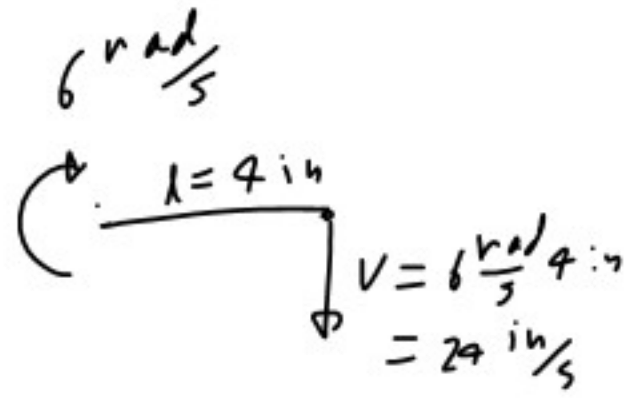


$r = 4j \ \text{in}$

$a = \alpha \times r + \omega \times (\omega \times r)$



$$a = \alpha \times r = \left|\begin{array}{ccc|cc} i & j & k & i & j \\ 0 & 0 & 90 & 0 & 0 \\ 0 & 4 & 0 & 0 & 4 \end{array}\right| = -4 \cdot 90\, i \ \text{in}/s^2 = \boxed{-360 \ \text{in}/s^2}$$



$$a = \alpha \times r + \omega \times (\omega \times r)$$
$$= 360 j - 291.6 \times 10^3\, i \ \text{in}/s^2$$

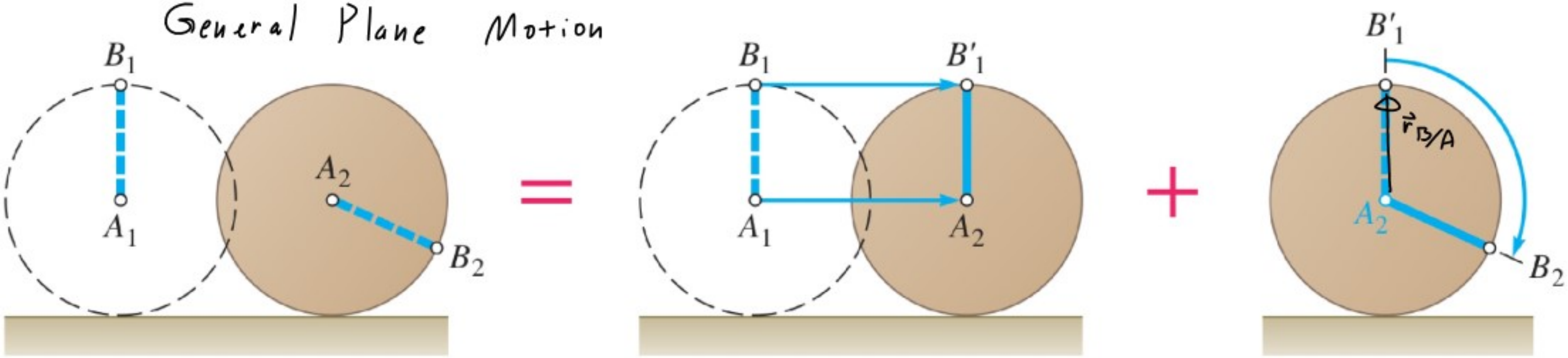
$$\alpha \times r = \left|\begin{array}{ccc|cc} i & j & k & i & j \\ 0 & 0 & 90 & 0 & 0 \\ 4 & 0 & 0 & 4 & 0 \end{array}\right| = 360 j \ \text{in}/s^2$$

$$\sqrt{360^2 + (291.6 \times 10^3)^2} = \boxed{291.6 \times 10^3 \ \text{in}/s^2}$$

$$\omega \times r = \left|\begin{array}{ccc|cc} i & j & k & i & j \\ 0 & 0 & 270 & 0 & 0 \\ 4 & 0 & 0 & 4 & 0 \end{array}\right| = 4 \cdot 270\, j = 1080 j \ \text{in}/s$$

$$\omega \times (\omega \times r) = \left|\begin{array}{ccc|cc} i & j & k & i & j \\ 0 & 0 & 270 & 0 & 0 \\ 0 & 1080 & 0 & 0 & 1080 \end{array}\right| = -270 \cdot 1080\, i = -291.6 \times 10^3\, i \ \text{in}/s^2$$

General Plane Motion



Plane motion = Translation with A + Rotation about A

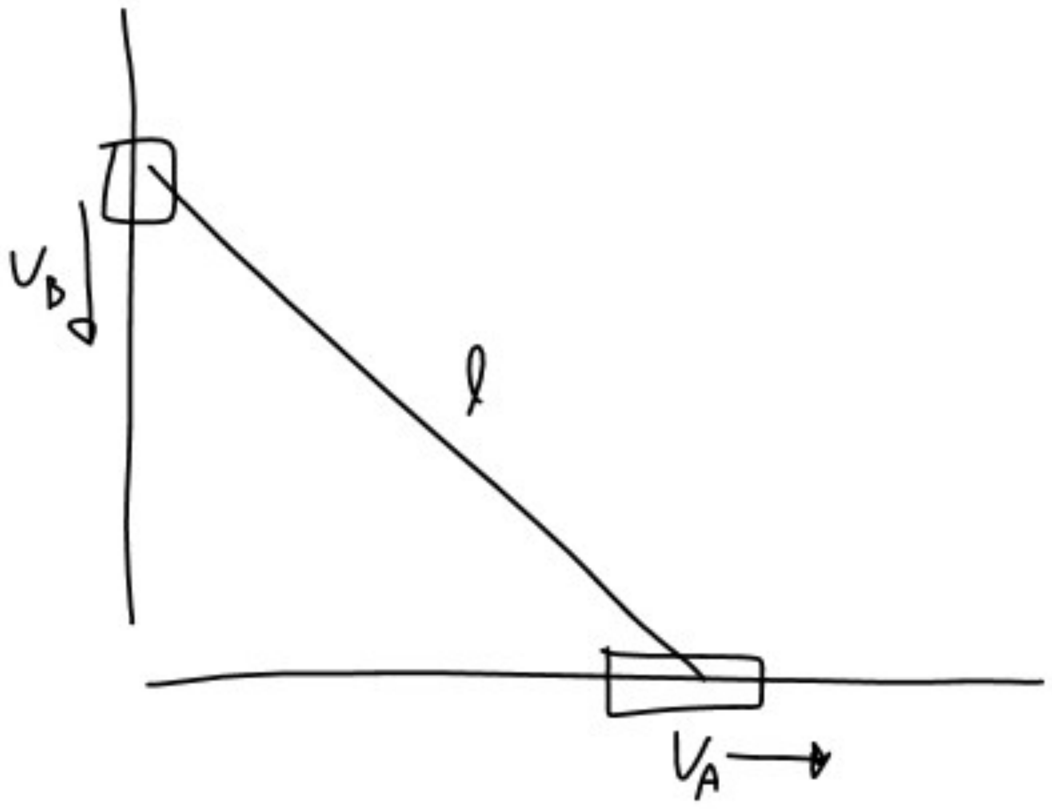
$\vec{V}_B = \vec{V}_A + \vec{V}_{B/A}$

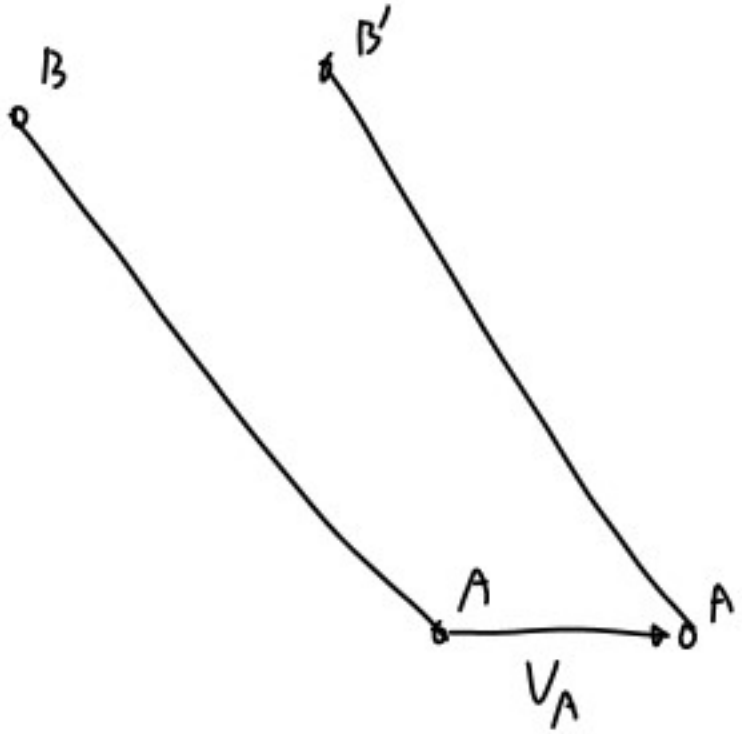
$\vec{V}_B = \vec{V}_A + \omega k \times \vec{r}_{B/A}$

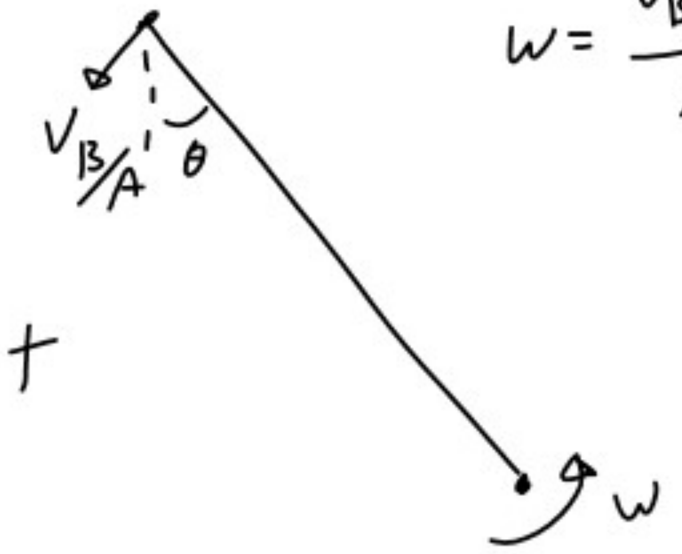
$\vec{V}_{B/A} = \vec{\omega} \times \vec{r}_{B/A}$ 3D

$\vec{V}_{B/A} = \omega k \times \vec{r}_{B/A}$ 2D "plane motion"

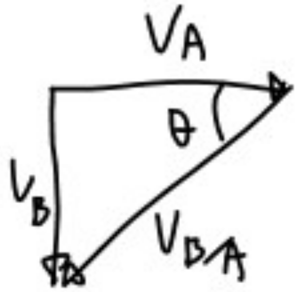
$V_{B/A} = r\omega$







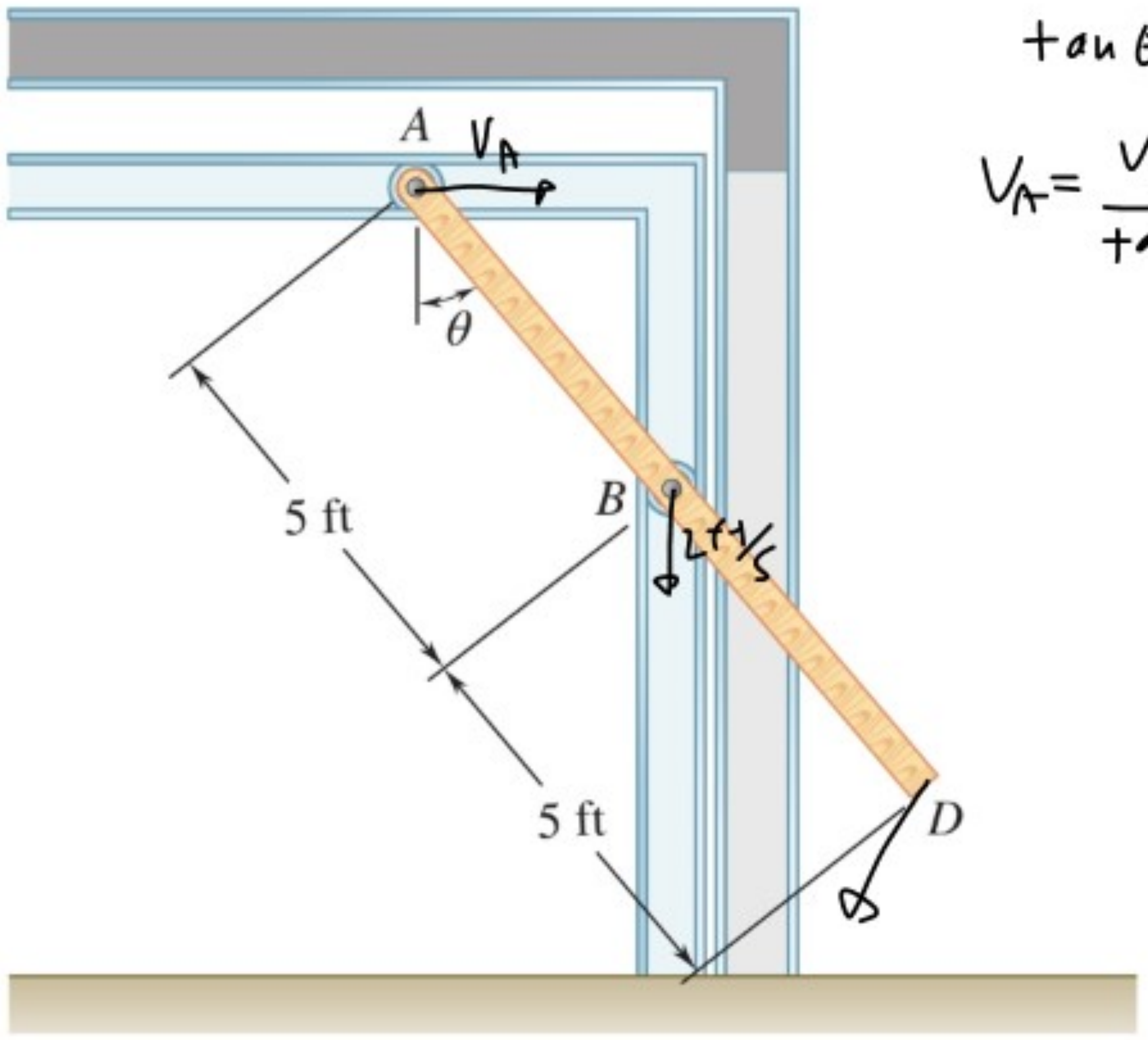
$$\omega = \frac{V_{B/A}}{l} = \frac{V_A}{l\cos\theta}$$



$$\frac{V_B}{V_A} = \tan\theta$$

$$V_{B/A} = \frac{V_A}{\cos\theta}$$

An overhead door is guided by wheels at A and B that roll in horizontal and vertical tracks. Knowing that when $\theta = 30°$ the velocity of wheel B is 2 ft/s downward, determine (a) the angular velocity of the door, (b) the velocity of end D of the door.



$$\tan\theta = \frac{V_B}{V_A}$$

$$V_A = \frac{V_B}{\tan\theta}$$

$$\omega = \frac{V_A}{l\cos\theta}$$

$$\omega = \frac{V_B}{l\cos\theta\tan\theta}$$

$$= \frac{V_B}{l\cos\theta\,\frac{\sin\theta}{\cos\theta}}$$

$$= \frac{V_B}{l\sin\theta} = \frac{2}{5\sin 30}$$

$$\boxed{= 0.8\ \text{rad/s}}$$