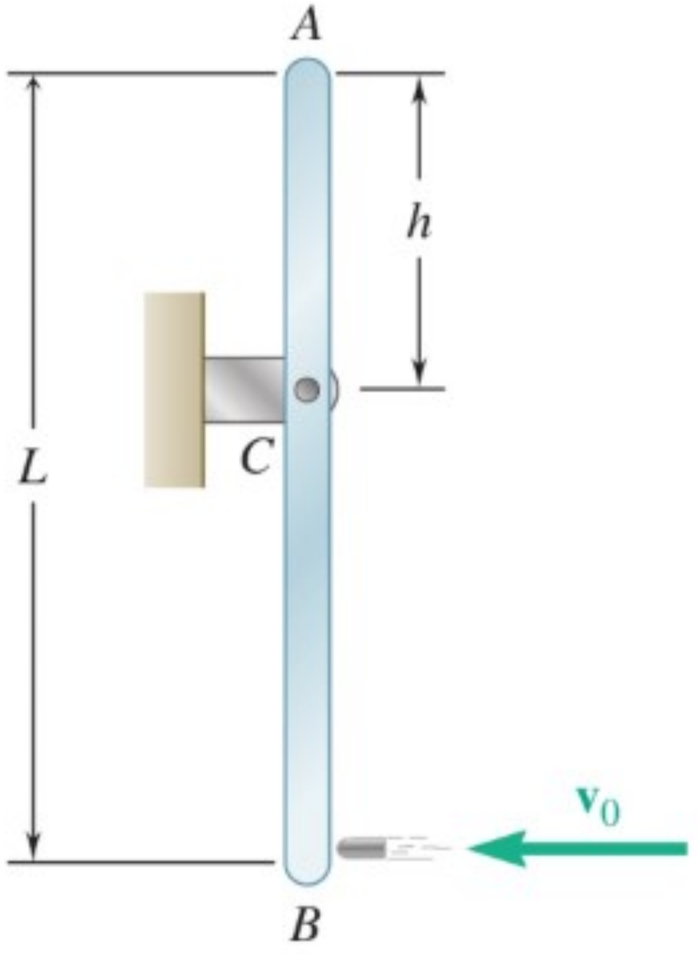
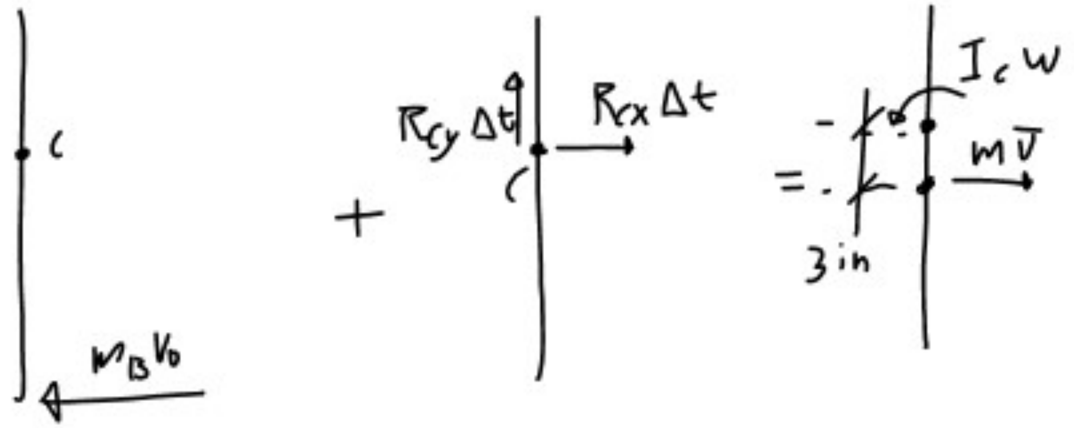
A bullet weighing 0.08 lb is fired with a horizontal velocity of 1800 ft/s into the lower end of a slender 15-lb bar of length $L = 30$ in. Knowing that $h = 12$ in. and that the bar is initially at rest, determine (*a*) the angular velocity of the bar immediately after the bullet becomes embedded, (*b*) the impulsive reaction at *C*, assuming that the bullet becomes embedded in 0.001 s.

$$m_B = \frac{W_B}{g} = \frac{0.08\text{ lb}}{32.2\text{ ft/s}^2} = 0.0025\text{ slug}$$





$$H_{c1} + \cancel{M_c \Delta t}^{\,0} = H_{c2}$$

$$H_{c1} = m_B V_0 (\overline{BC})$$
$$= m_B V_0 (L - h)$$
$$= 0.0025\text{ slug } 1800\,\tfrac{\text{ft}}{\text{s}}\,(30\text{ in} - 12\text{ in})$$
$$= 80.5\,\frac{\text{lb s}^2}{\cancel{\text{ft}}}\,\frac{\cancel{\text{ft}}}{\cancel{\text{s}}}\,\text{in} = 80.5\text{ lb s in}$$

$$H_{c2} = I_c w$$

$$\bar{I} = \frac{1}{12} m L^2$$

$$I_c = \bar{I} + m d^2 = \frac{1}{12} m L^2 + m\left(\frac{L}{2} - h\right)^2$$
$$= \frac{1}{12}\,\frac{15\text{ lb }(30\text{ in})^2}{32.2\text{ ft/s}^2} + \frac{15\text{ lb}}{32.2\text{ ft/s}^2}\left(\frac{30\text{ in}}{2} - 12\text{ in}\right)^2 = 39.1\,\frac{\text{lb s}^2\text{ in}^2}{\text{ft}}$$

$$80.5\ \text{lb s in} = I_c\,\omega$$

$$= 39.1\ \frac{\text{lb s}^2\text{in}^2}{\text{ft}}\ \omega$$

$$\frac{80.5\ \cancel{\text{lb}}\ \cancel{\text{s}}\ \cancel{\text{in}}}{39.1\ \cancel{\text{lb}}\ \text{s}^{\cancel{2}}\text{in}^{\cancel{2}}/\text{ft}} = 2.05\ \frac{\cancel{\text{ft}}}{\cancel{\text{in}}\ \text{s}}\ \frac{12\ \cancel{\text{in}}}{1\ \cancel{\text{ft}}} = \boxed{24.7\ \text{rad/s}}$$

$$\cancel{L_{1y}}^{\,0} + Imp_{1\to 2} = \cancel{L_{2y}}^{\,0}$$

$$Imp_{1\to 2} = 0$$

$$R_{cy}\,\Delta t = 0$$

$$\boxed{R_{cy} = 0}$$

$$L_{1x} + Imp_{1\to 2} = L_{2x}$$

$$-m_B v_0 + Imp_{1\to 2} = m\bar{v} = -m\,\omega\, 3\,\text{in}$$

$$-0.0025\ \text{slug}\ 1800\ \text{ft/s} + Imp_{1\to 2} = \frac{-15\ \text{lb}}{32.2\ \text{ft/s}^2}\ 29.7\ \text{rad/s}\ 3\ \text{in}$$

$$Imp_{1\to 2} = 0.0025\ \text{slug}\ 1800\ \text{ft/s} - \frac{15\ \text{lb}}{32.2\ \text{ft/s}^2}\ 29.7\ \frac{\text{rad}}{\text{s}}\ 3\ \text{in}\ \frac{1\ \text{ft}}{12\ \text{in}}$$

$$= 1.625\ \text{lb s}$$

$$Imp_{1\to 2} = R_{cx}\,\Delta t$$

$$\frac{Imp_{1\to 2}}{\Delta t} = R_{cx}$$

$$\frac{1.625\ \text{lbs}}{0.001\ \text{s}} = \boxed{1625\ \text{lb}}$$

3D Rigid Body Kinematics

$$\Sigma F_x = m\bar{a}_x$$

$$\Sigma F_y = m\bar{a}_y$$

$$\Sigma F_z = m\bar{a}_z$$

$$\Sigma M_G = \dot{H}_G$$

$$H_G = \sum_{i=1}^{n} (r_i' \times V_i' \Delta m_i)$$

$$H_G = \sum_{i=1}^{n} (r_i' \times (w \times r_i') \Delta m_i)$$

$$H_x = \sum_{i=1}^{n} (y_i (w \times r_i')_z - z_i (w \times r_i')_y \Delta m_i)$$

$$= w_x \sum_i (y_i^2 + z_i^2) \Delta m_i - w_y \sum_i x_i y_i \Delta m_i - w_z \sum z_i x_i \Delta m_i$$

$$= w_x \int (x^2 + y^2)\, dm - w_y \int xy\, dm - w_z \int zx\, dm$$

$$= \bar{I}_x w_x - \bar{I}_{xy} w_y - \bar{I}_{zx} w_z$$

$\bar{I}_x$ Mass moment of Inertia

$\bar{I}_{xy}$ Mass product of Inertia