$$H_x = \bar{I}_x w_x - \bar{I}_{xy} w_y - \bar{I}_{xz} w_z$$

$$H_y = -\bar{I}_{yx} w_x + \bar{I}_y w_y - \bar{I}_{yz} w_z$$

$$H_z = -\bar{I}_{zx} w_x - \bar{I}_{zy} w_y + \bar{I}_z w_z$$

$$\Sigma M = \dot{H}_G$$

$$\Sigma M = I\alpha$$

Inertia tensor

$$\begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix} = \begin{bmatrix} \bar{I}_x & -\bar{I}_{xy} & -\bar{I}_{xz} \\ -\bar{I}_{yx} & \bar{I}_y & -\bar{I}_{yz} \\ -\bar{I}_{zx} & -\bar{I}_{zy} & \bar{I}_z \end{bmatrix} \begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix}$$

$$\begin{bmatrix} \bar{I}_{x'} & 0 & 0 \\ 0 & \bar{I}_{y'} & 0 \\ 0 & 0 & \bar{I}_{z'} \end{bmatrix}$$

Mass products of inertia
are zero if part is symetric

A homogeneous disk of mass $m = 8$ kg rotates at the constant rate $\omega_1 = 12$ rad/s with respect to arm OA, which itself rotates at the constant rate $\omega_2 = 4$ rad/s about the y axis. Determine the angular momentum $\mathbf{H}_A$ of the disk about its center A.

$\bar{I}_x = \frac{1}{4} m r^2 = \frac{1}{4} 8\,\text{kg}\,(0.1\,\text{m})^2 = 0.02\ \text{Kg m}^2$

$\bar{I}_y = \frac{1}{4} m r^2 = 0.02\ \text{Kg m}^2$

$\bar{I}_z = \frac{1}{2} m r^2 = \frac{1}{2} 8\,\text{Kg}\,(0.1\,\text{m})^2 = 0.04\ \text{Kg m}^2$

$w_x = 0$

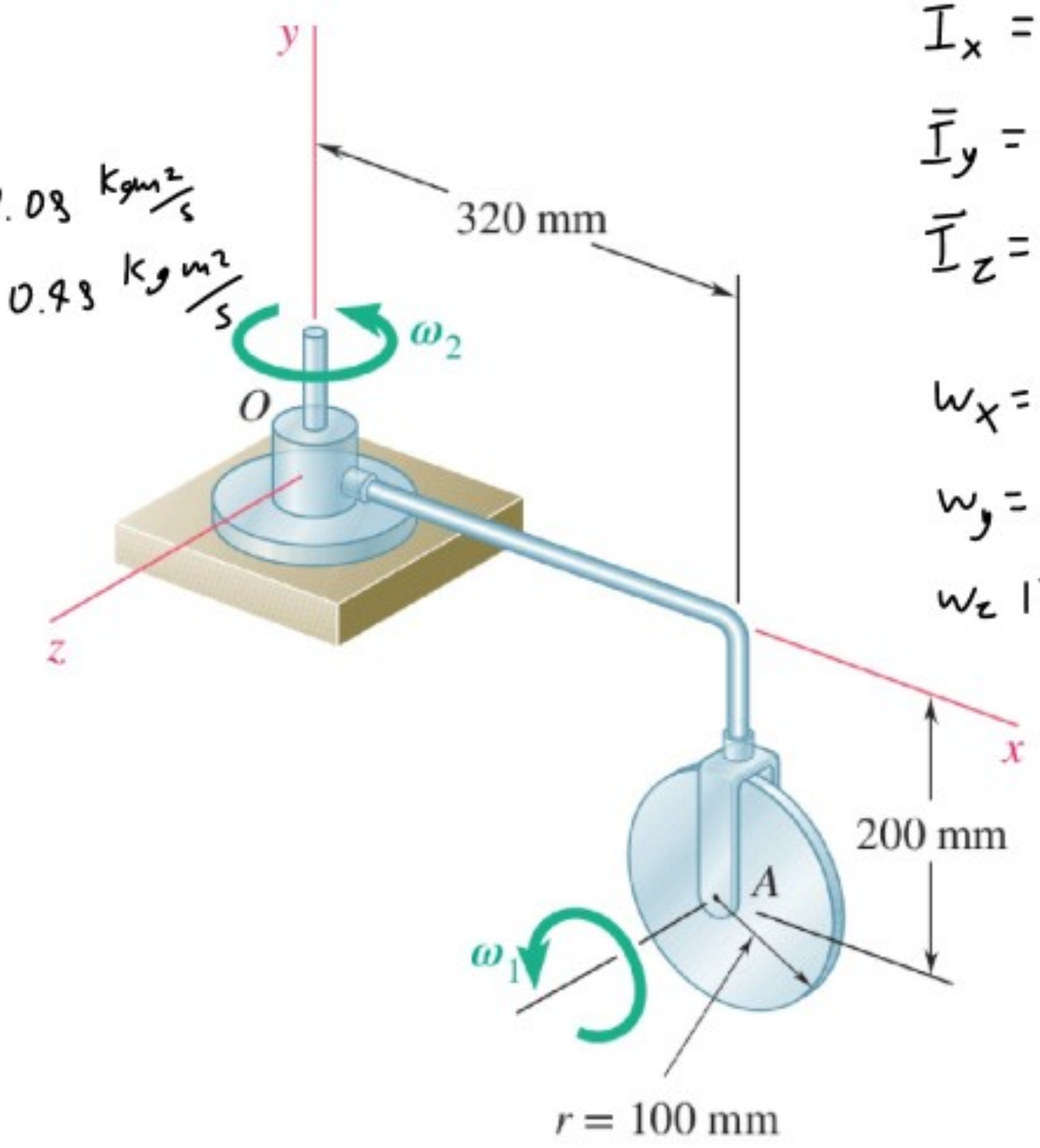
$w_y = 4\ \text{rad/s}$

$w_z\ 12\ \text{rad/s}$

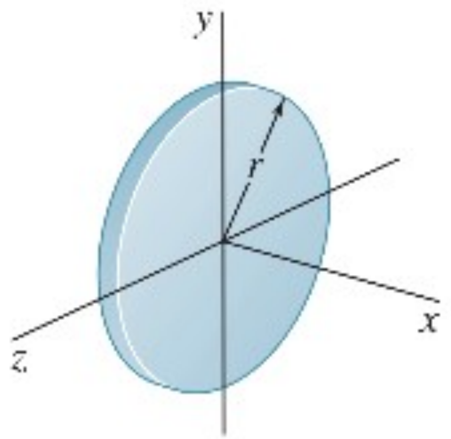
$H_x = \bar{I}_x w_x = 0.02\ \text{Kg m}^2\ 0 = 0$

$H_y = \bar{I}_y w_y = 0.02\ \text{Kg m}^2\ 4\ \text{rad/s} = 0.08\ \text{Kg m}^2/\text{s}$

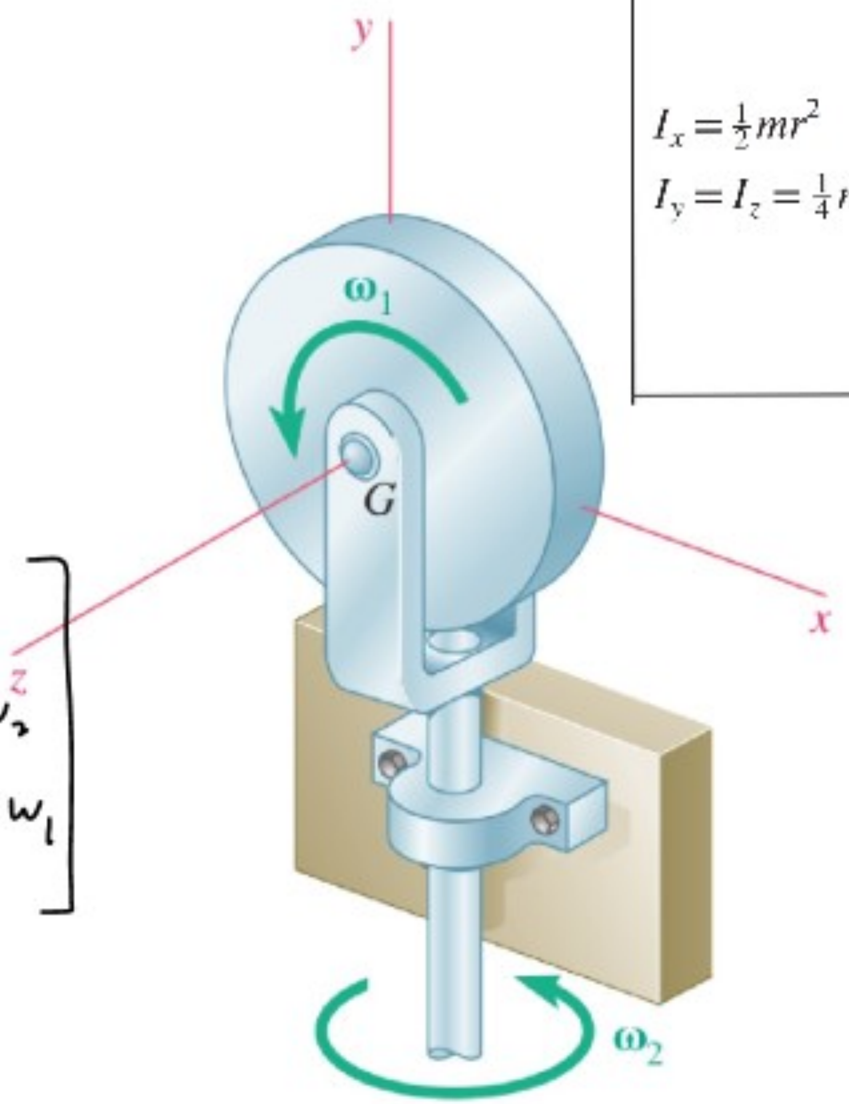
$H_z = \bar{I}_z w_z = 0.04\ \text{Kg m}^2\ 12\ \text{rad/s} = 0.48\ \text{Kg m}^2/\text{s}$



A thin, homogeneous disk of mass m and radius r spins at the constant rate ω_1 about an axle held by a fork-ended vertical rod that rotates at the constant rate ω_2. Determine the angular momentum $\mathbf{H}_G$ of the disk about its mass center G.

Thin disk $I_x = \frac{1}{2}mr^2$ $I_y = I_z = \frac{1}{4}mr^2$	

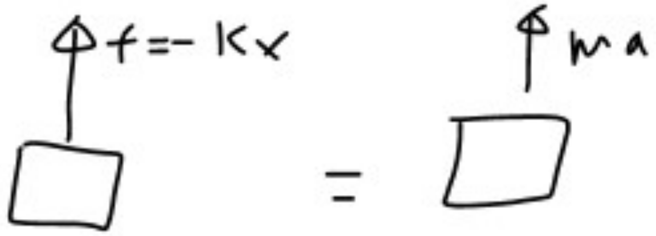
$$\begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix} = \begin{bmatrix} \bar{I}_x & 0 & 0 \\ 0 & \bar{I}_y & 0 \\ 0 & 0 & \bar{I}_z \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$



$$\begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix} = \begin{bmatrix} \bar{I}_x \omega_x \\ \bar{I}_y \omega_y \\ \bar{I}_z \omega_z \end{bmatrix} = \begin{bmatrix} \frac{1}{4}mr^2 \, 0 \\ \frac{1}{4}mr^2 \omega_2 \\ \frac{1}{2}mr^2 \omega_1 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{1}{4}mr^2 \omega_2 \\ \frac{1}{2}mr^2 \omega_1 \end{bmatrix}$$

Free Vibration

$f = -kx$



$$f = ma$$

$$-Kx = ma$$

$$-Kx = m\ddot{x}$$

$$m\ddot{x} + Kx = 0$$

$$x = \sin\left(\sqrt{\frac{K}{m}}\, t\right)$$

$$\dot{x} = \sqrt{\frac{K}{m}} \cos\left(\sqrt{\frac{K}{m}}\, t\right)$$

$$\ddot{x} = \frac{-K}{m} \sin\left(\sqrt{\frac{K}{m}}\, t\right)$$

$$\cancel{m}\left(\frac{-K}{\cancel{m}}\right)\sin\left(\sqrt{\frac{K}{m}}\, t\right) + k\sin\left(\sqrt{\frac{K}{m}}\, t\right) = 0$$

$$0 = 0 \checkmark$$