$W_n = \sqrt{\frac{K}{m}}$ Natural Frequency rad/s

$X = \sin(w_n t)$

Example

Spring

Known Mass = 0.2 kg

Unknown Mass

Stopwatch

0.475 s/bounce

$2.1 \frac{\text{bounces}}{s} \cdot \frac{2\pi \text{ rad}}{1 \text{ bounce}} = 13.2 \frac{\text{rad}}{s}$

$13.2 = \sqrt{\frac{K}{0.2 kg}}$

$(13.2 \text{ rad/s})^2 = \frac{K}{0.2 kg}$

$0.2 kg (13.2 \text{ rad/s}^2)^2 = 350 \frac{kg}{s^2} = K$

$350 \frac{kg}{s^2} \cdot \frac{1 N}{1 \frac{kg \cdot m}{s^2}} = 350 \frac{N}{m} = K$

Unknown Mass

$0.315 \, \frac{s}{bounce}$

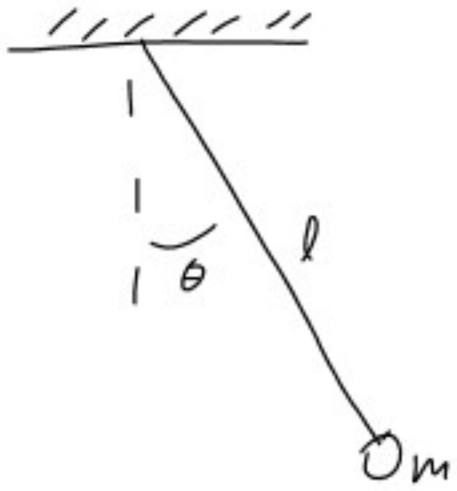
$3.18 \, \frac{bounces}{s} \cdot \frac{2\pi \, rad}{1 \, bounce} = 20 \, \frac{rad}{s}$

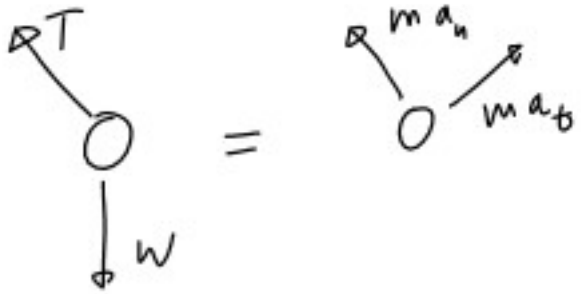
$20 \, \frac{rad}{s} = \sqrt{\frac{350 \, N/m}{m}}$

$\left(20 \, \frac{rad}{s}\right)^2 = \frac{350 \, N/m}{m}$

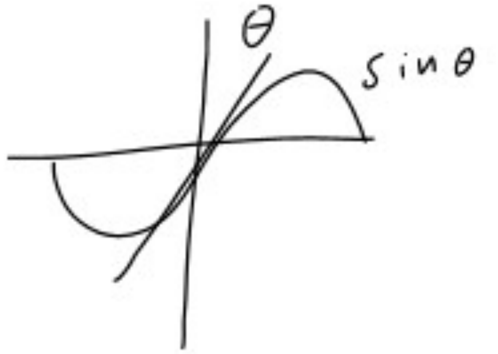
$m = \frac{350 \, N/m}{\left(20 \, rad/s\right)^2} = 0.087 \, \frac{N s^2}{m} \cdot \frac{1 \, \frac{kg\,m}{s^2}}{1 \, N} = \boxed{0.087 \, kg}$

Pendulums





$$\Sigma F_t = m a_t$$



$$-W\sin\theta = m l \ddot{\theta}$$

$$-mg\sin\theta = ml\ddot{\theta}$$

$$-g\sin\theta = l\ddot{\theta}$$

$$l\ddot{\theta} + g\sin\theta = 0$$

$\sin\theta \approx \theta$ if θ small

Small angle theorem

$$l\ddot{\theta} + g\theta = 0$$

$$l\lambda^2 + g = 0$$

$$\lambda_1, \lambda_2 = ?$$

$$c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t}$$

Eulers formula

$$\theta = \theta_m \sin(\omega_n t)$$

$$\omega_n = \sqrt{g/l}$$

$$\tau_n = \frac{2\pi}{\omega_n} = 2\pi\sqrt{l/g}$$

$$\tau_n = \frac{2K}{\pi}\left(2\pi\sqrt{l/g}\right)$$