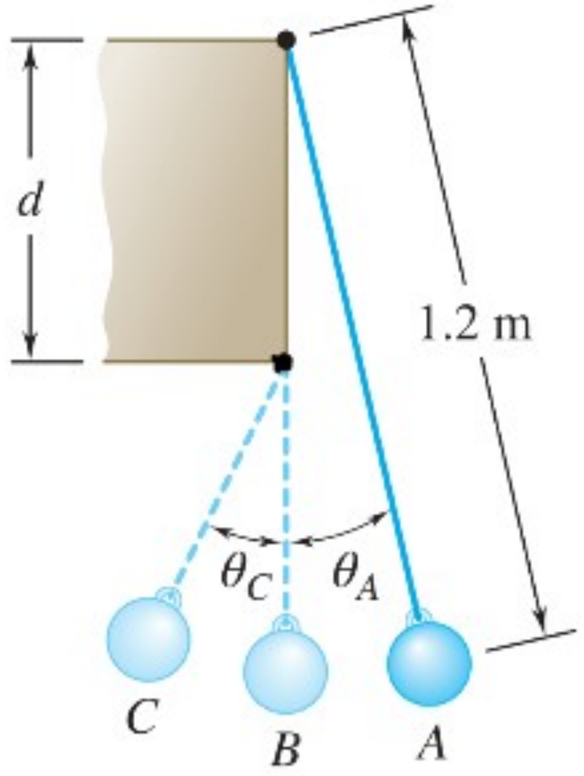
19.16 A small bob is attached to a cord of length 1.2 m and is released from rest when $\theta_A = 5°$. Knowing that $d = 0.6$ m, determine (*a*) the time required for the bob to return to point *A*, (*b*) the amplitude θ_C.



$$\tau_n = 2\pi\sqrt{\frac{l}{g}}$$

$$\tau_A = 2\pi\sqrt{\frac{1.2\,m}{9.8\,m/s^2}} = 2.2\ s$$

$$t_{A\to B} = \frac{\tau_A}{4}$$

$$t_{B\to A} = \frac{\tau_A}{4}$$

$$\tau_C = 2\pi\sqrt{\frac{0.6\,m}{9.8\,m/s^2}} = 1.55\ s$$

$$t_{B\to C} = \frac{\tau_C}{4}$$

$$t_{C\to B} = \frac{\tau_C}{4}$$

$$t = t_{A\to B} + t_{B\to C} + t_{C\to B} + t_{B\to A} = \frac{\tau_A}{2} + \frac{\tau_C}{2}$$

$$= \frac{2.2\,s}{2} + \frac{1.55}{2} = \boxed{1.83\ s}$$

$$\theta_A(t) = \theta_A \cos(\omega_{nA} t)$$

$$\dot{\theta}_A(t) = -\omega_{nA}\theta_A \sin(\omega_{nA} t)$$

$$\dot{\theta}_{BA} = -\omega_{nA}\theta_A$$

$$V_B = l_A \dot{\theta}_{BA}$$

$$\dot{\theta}_{BC} = \frac{V_B}{l_C}$$

$$\dot{\theta}_C(t) = -\omega_{nC}\theta_C \sin(\omega_{nC} t)$$

$$-\omega_{nC}\,\theta_C = \dot{\theta}_{BC} = \frac{V_B}{l_C} = \frac{l_A \dot{\theta}_{BA}}{l_C} = \frac{l_A(-\omega_{nA}\theta_A)}{l_C}$$

$$\theta_C = \frac{l_A\,\omega_{nA}\,\theta_A}{l_C\,\omega_{nC}} = \frac{l_A \sqrt{g/l_A}\,\theta_A}{l_C\sqrt{g/l_C}} = \frac{l_A/\sqrt{l_A}\;\theta_A}{l_C/\sqrt{l_C}} = \frac{\sqrt{l_A}\;\theta_A}{\sqrt{l_C}}$$

$$= \frac{\sqrt{1.2}\;\;5^\circ}{\sqrt{0.6}} = \boxed{7.071^\circ}$$

$$\cancelto{0}{T_A} + U_A = \cancelto{0}{T_C} + U_C$$

$$mgh_A = mgh_c$$

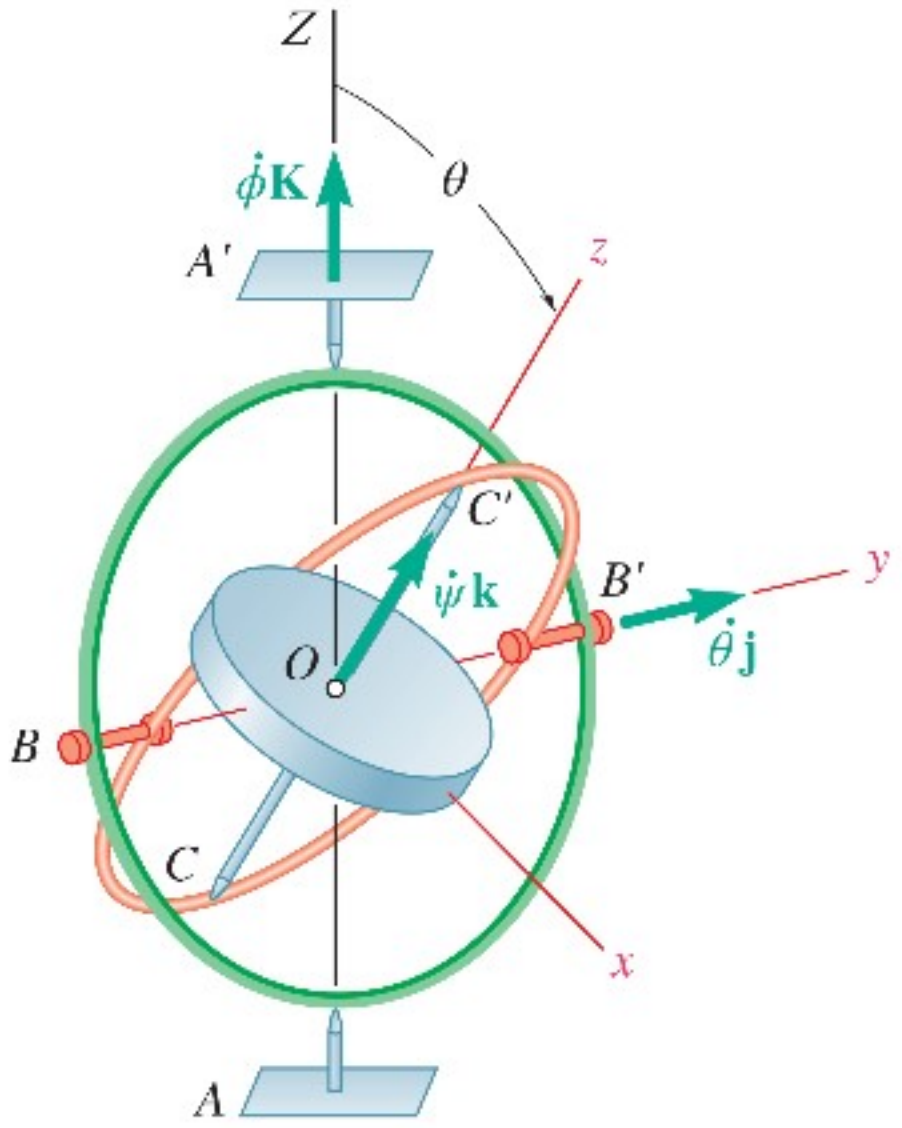
$$h_A = h_c$$

$$-1.2\,m\cos 5^\circ = -0.6\,m - 0.6\,m\cos\theta_c$$

$$0.6\,m\cos\theta_c = 1.2\,m\cos 5^\circ - 0.6\,m$$

$$\cos\theta_c = \frac{1.2\cos 5^\circ - 0.6}{0.6} \qquad \boxed{\theta_c = 7.073^\circ}$$

Gyroscopes



$$\omega = \dot{\phi}\,\mathbf{K} + \dot{\theta}\,j + \dot{\psi}\,k$$

$$\mathbf{K} = -\sin\theta\, i + \cos\theta\, k$$

$$\omega = -\dot{\phi}\sin\theta\, i + \dot{\theta}\, j + (\dot{\psi} + \dot{\phi}\cos\theta)\, k$$

$$H_o = -I'\dot{\phi}\sin\theta\, i + I'\dot{\theta}\, j + I(\dot{\psi} + \dot{\phi}\cos\theta)\, k$$

$$\Omega = \dot{\phi}\,\mathbf{K} + \dot{\theta}\, j$$

$$= \dot{\phi}\sin\theta\, i + \dot{\theta}\, j + \dot{\phi}\cos\theta\, k$$

Stead Procession

θ constant

$$\omega = -\dot{\phi}\sin\theta\, i + \omega_z k$$

$$H_o = -I'\dot{\phi}\sin\theta\, i + I\omega_z\, k$$

$$\Omega = -\dot{\phi}\sin\theta\, i + \dot{\phi}\cos\theta\, k$$

$$\Sigma M_O = \Omega \times H_O$$

$$= (I\omega_z - I'\dot{\phi}\cos\theta)\,\dot{\phi}\sin\theta\,\mathbf{j}$$

if $\theta = 90^\circ$

$$\Sigma M_O = I\dot{\psi}\dot{\phi}\,\mathbf{j} = \dot{\phi} \times I\dot{\psi}$$