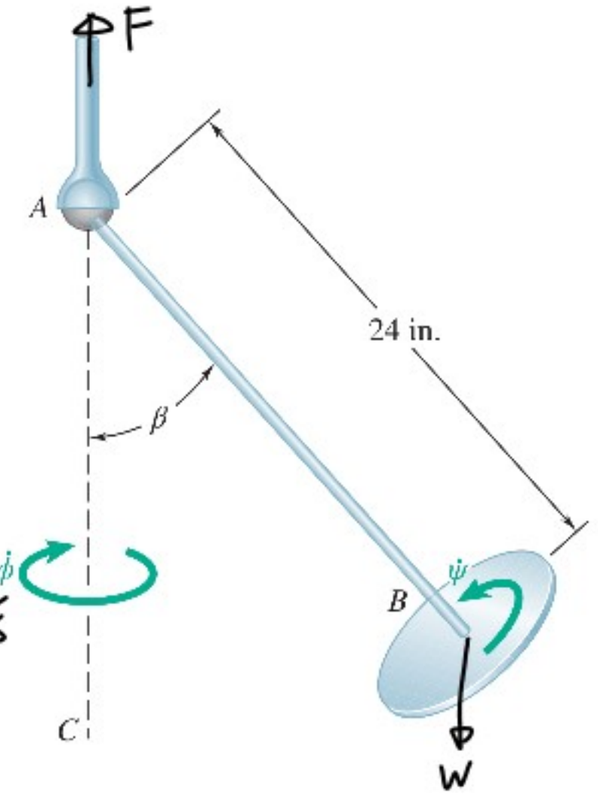
18.107 A uniform thin disk with a 6-in. diameter is attached to the end of a rod AB of negligible mass that is supported by a ball-and-socket joint at point A. Knowing that the disk is observed to precess about the vertical axis AC at the constant rate of 36 rpm in the sense indicated and that its axis of symmetry AB forms an angle $\beta = 60°$ with AC, determine the rate at which the disk spins about rod AB.



$$\Sigma M_O = (I\omega_z - I'\dot{\phi}(\cos\theta)\dot{\phi}\cos\theta$$

$\theta = \beta$

$\dot{\phi} = 36 \text{ rpm} \frac{1 \text{ min}}{60 \text{ s}} \frac{2\pi \text{ rad}}{1 \text{ rev}} = 3.77 \text{ rad/s}$

$I = \frac{1}{2}mr^2$

$I' = \frac{1}{4}mr^2$

$20.8 \text{ in} \cdot mg = \left(\frac{1}{2}mr^2\omega_z - \frac{1}{4}mr^2 3.77 \text{ rad/s} \cos\beta\right) 3.77 \text{ rad/s} \cos\beta$

$20.8 \text{ in} \cdot 32.2 \text{ ft/s}^2 = \left(\frac{1}{2}(3 \text{ in})^2\omega_z - \frac{1}{4}(3 \text{ in})^2 3.77 \text{ rad/s} \frac{1}{2}\right) 3.77 \text{ rad/s} \frac{1}{2}$

$2\,\frac{20.8 \text{ in} \cdot 32.2 \text{ ft/s}^2}{3.77 \text{ rad/s}} \frac{12 \text{ in}}{1 \text{ ft}} = 4264 \frac{\text{in}^2}{\text{s}} = \frac{1}{2}(3 \text{ in})^2\omega_z - \frac{1}{4}(3 \text{ in})^2 3.77 \text{ rad/s} \frac{1}{2}$

$\Sigma M_A = 24 \text{ in} \sin\beta \, W$

$= 20.8 \text{ in} \cdot mg$

$$4264 \frac{in^2}{s} + \frac{1}{4}(3\,in)^2\, 3.77 \frac{rad}{s} \frac{1}{2} = \frac{1}{2}(3\,in)^2 \omega_z$$

$$4268 \frac{in^2}{s} = \frac{1}{2}(3\,in)^2 \omega_z$$

$$\frac{2\left(4268 \frac{in^2}{s}\right)}{(3\,in)^2} = 948 \frac{rad}{s} = \omega_z$$

$$\dot{\psi} + \dot{\phi}\cos\theta = \omega_z$$

$$\dot{\psi} = \omega_z - \dot{\phi}\cos\theta = 948 \frac{rad}{s} - 3.77 \frac{rad}{s} \frac{1}{2} = \boxed{947 \frac{rad}{s}}$$