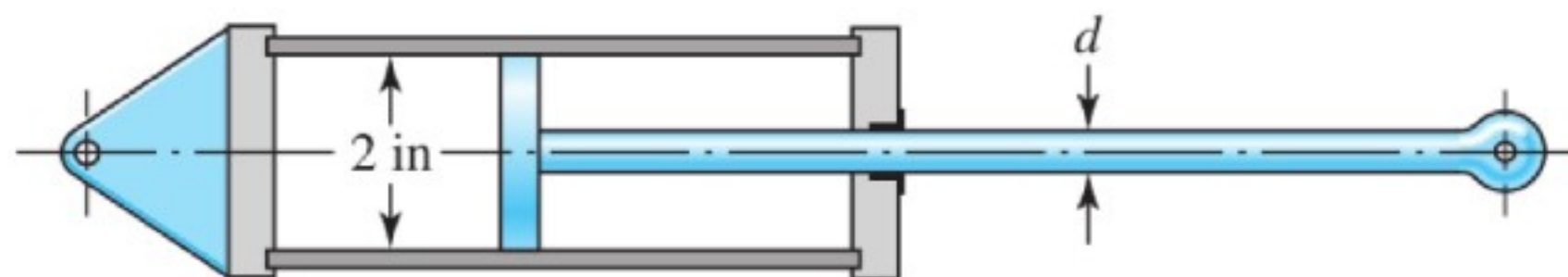


**4-131** The hydraulic cylinder shown in the figure has a 2-in bore and is to operate at a pressure of 1500 psi. With the clevis mount shown, the piston rod should be sized as a column with both ends rounded for any plane of buckling. The rod is to be made of forged AISI 1030 steel without further heat treatment.

$$S_y = 37.5 \text{ kpsi}$$

$$E = 30 \text{ Mpsi}$$

Problem 4-131



- (a) Use a design factor  $n_d = 2.5$  and select a preferred size for the rod diameter if the column length is 50 in.
- (b) Repeat part (a) but for a column length of 16 in.
- (c) What factor of safety actually results for each of the cases above?

$$(1500 \text{ psi}) \left( \frac{\pi 2^2}{4} \right) = 4712 \text{ lb}$$

$$(4712 \text{ lb})(2.5) = 11781 \text{ lb} = P_{cr}$$

$$P_{cr} = \frac{C \pi^2 EI}{l^2}$$

$$11781 = \frac{\pi^2 30 \times 10^6 I}{50^2}$$

$$\frac{(11781)(50^2)}{\pi^2 30 \times 10^6} = \frac{\pi D^4}{64}$$

$$\frac{(11781)(50^2)(64)}{\pi^3 30 \times 10^6} = D^4$$

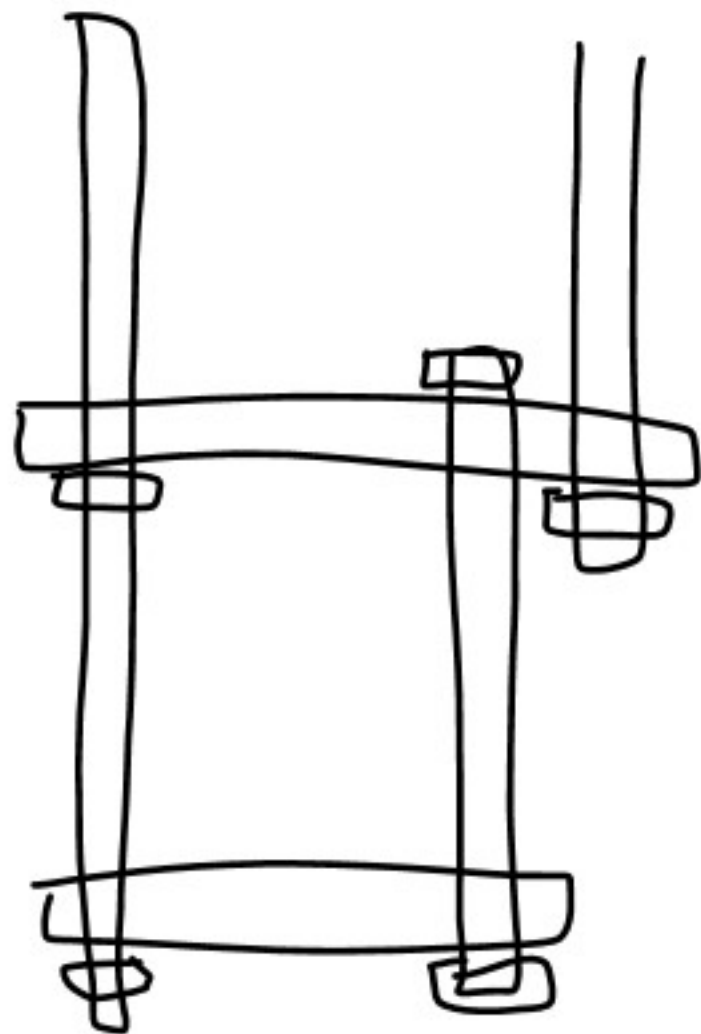
$$D = 1.193 \text{ in}$$

$$\frac{(11781)(16^2)(64)}{\pi^3 30 \times 10^6} = D^4$$

$$D = 0.675 \text{ in}$$

$$\frac{11}{16} = \boxed{0.6875 \text{ in} = D}$$

$$\boxed{D = 1.25 \text{ in}}$$



$$\frac{P_{cr} (l^2) (64)}{\pi^2 30 \times 10^6} = D^4$$

$$\frac{D^4 \pi^3 30 \times 10^6}{64 l^2} = P_{cr}$$

$$\frac{1.25^4 \pi^3 30 \times 10^6}{64 (50^2)} = 19199 \text{ lb}$$

$$\frac{19199}{9712} = \boxed{3.01}$$

$$\frac{0.6875^4 \pi^3 30 \times 10^6}{64 (16)^2} = 12684 \text{ lb}$$

$$\frac{12684}{9712} = \boxed{2.69}$$

# Failure Theories

Ductile (yield)

MSS Max shear stress  
DE Distortion Energy  
DCM Ductile Coulomb-Mohr

Brittle (fracture)

MNS Max normal stress  
BCM Brittle Coulomb-Mohr  
MM Modified-Mohr

MSS

$$\sigma_1 \geq \sigma_2 \geq \sigma_3$$

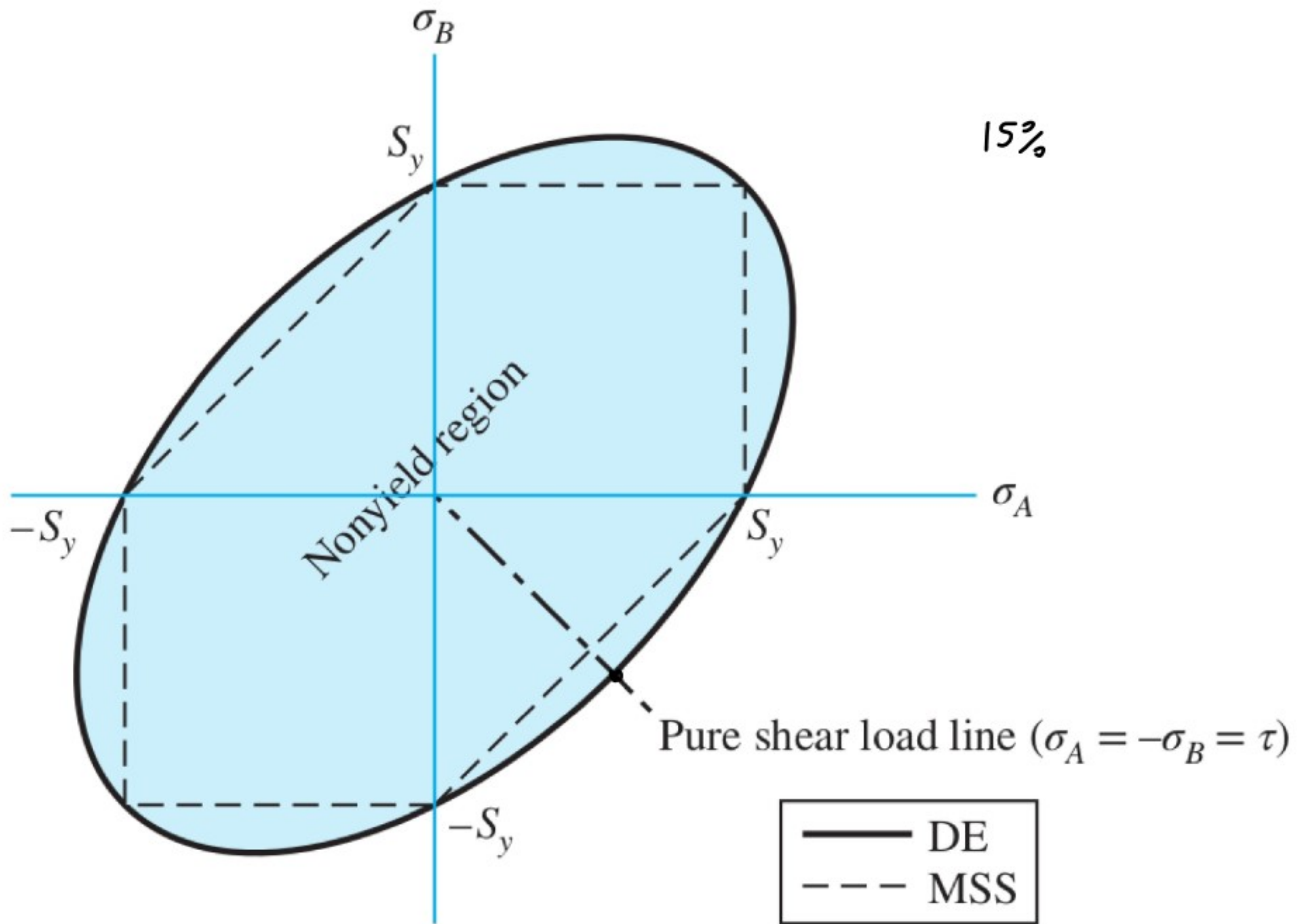
$S_y$  yield strength

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} \geq \frac{S_y}{2} \quad \sigma_1 - \sigma_3 \geq S_y$$

DE

$$\sqrt{\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2}} \geq S_y$$

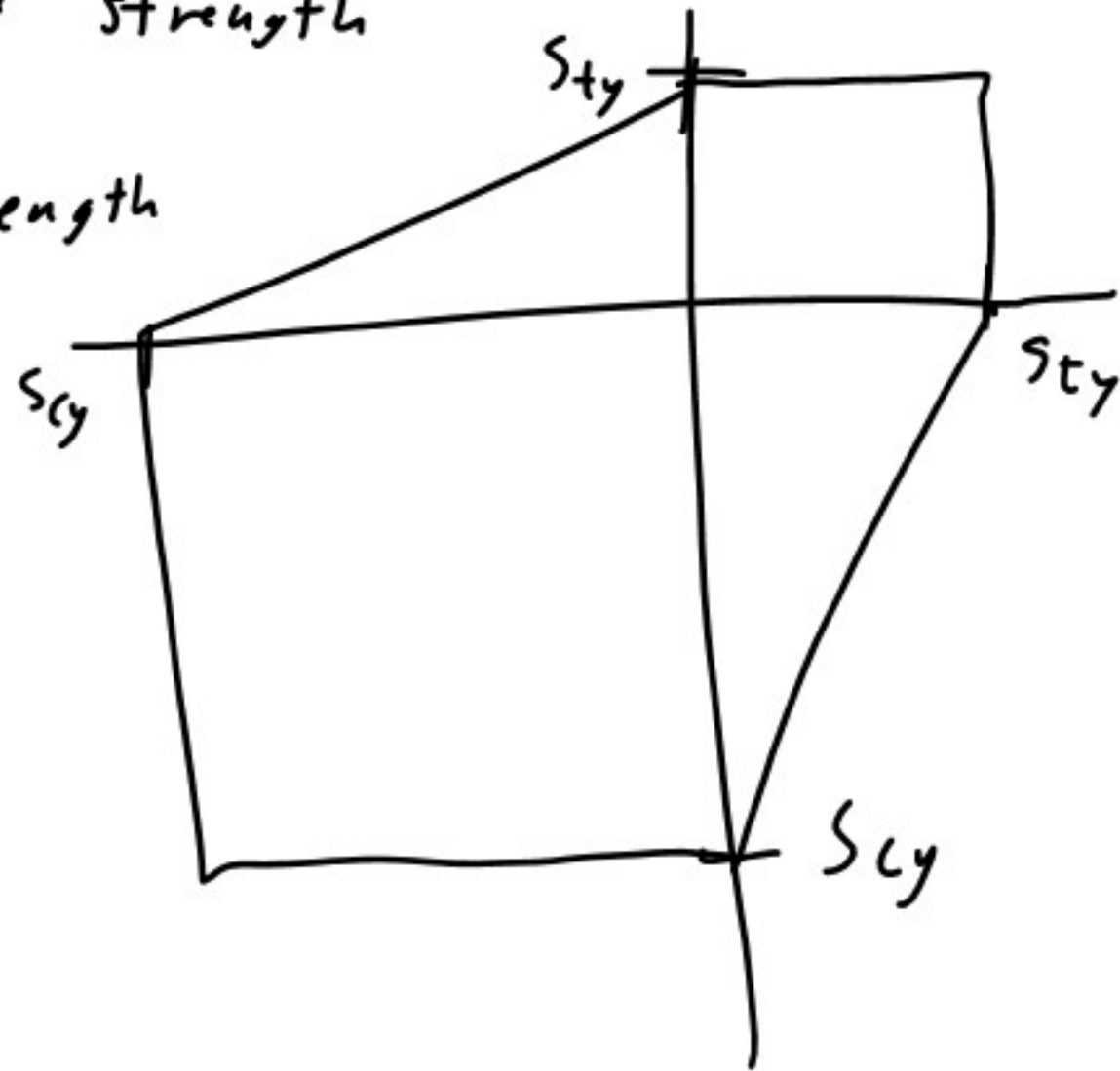
Von Mises Stress  
 $\sigma' \geq S_y$



DCM

$S_{cy}$  compressive yield strength

$S_{ty}$  tensile yield strength



MNS

$$\sigma_1 \geq S_{ut}$$

$$\sigma_3 \leq -S_{uc}$$

BCM

$$\sigma_A = \frac{S_{ut}}{n}$$

$$\sigma_A \geq \sigma_B \geq 0$$

$n$  factor of safety

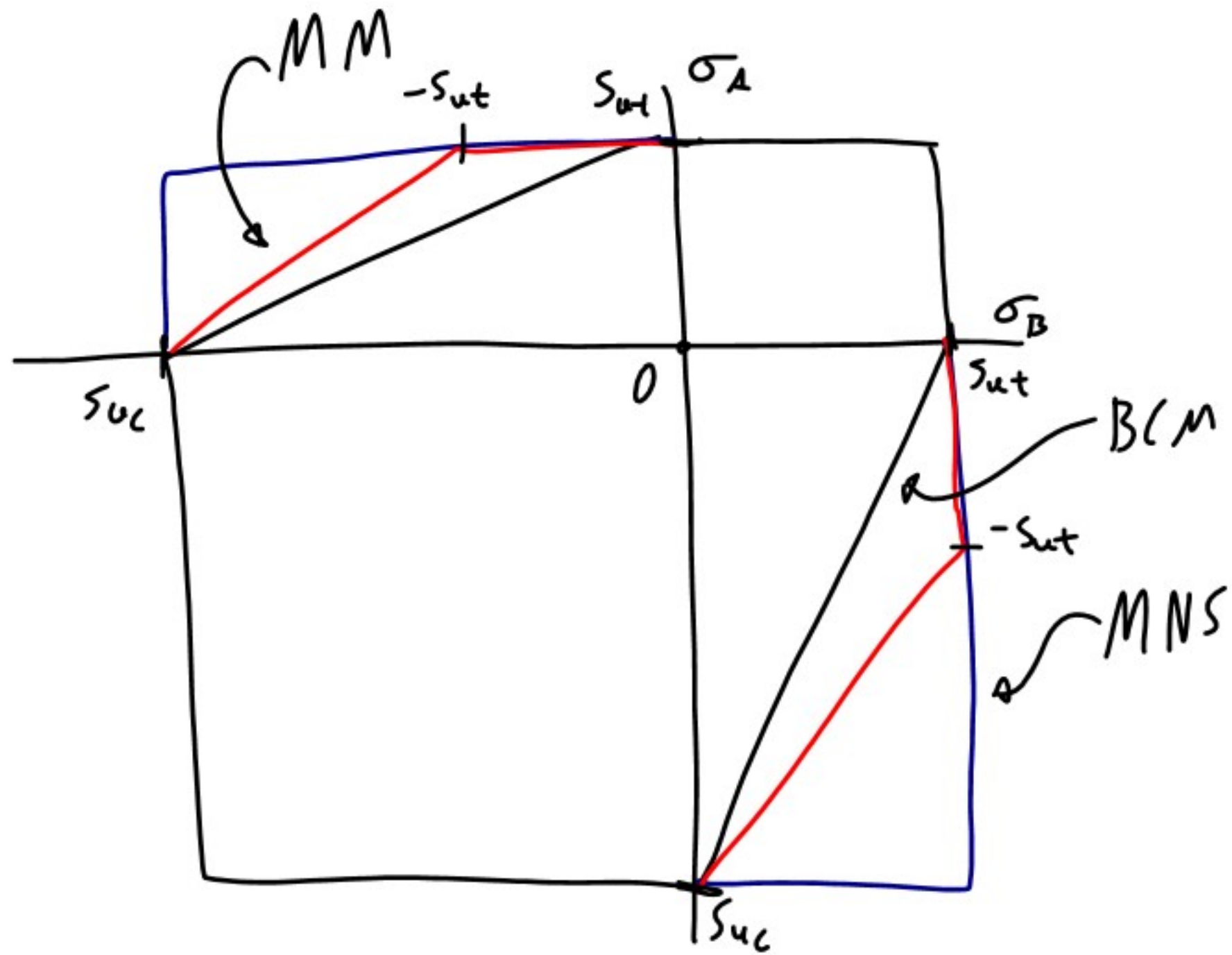
$$\frac{\sigma_A}{S_{ut}} - \frac{\sigma_B}{S_{uc}} = \frac{1}{n}$$

$$\sigma_A \geq 0 \geq \sigma_B$$

$$\sigma_B = -\frac{S_{uc}}{n}$$

$$0 \geq \sigma_A \geq \sigma_B$$





▶ Something Fun

(<https://www.tiktok.com/@engineeringverse.io/video/7192195590426725678>)