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A ductile hot-rolled steel bar has a minimum yield strength in tension and compression of 350 MPa. Using the distortion-energy and maximum-shear-stress theories determine the factors of safety for the following plane stress states:

(a)  $\sigma_x = 100 \text{ MPa}$ ,  $\sigma_y = 100 \text{ MPa}$        $\sigma_z = 0$

(b)  $\sigma_x = 100 \text{ MPa}$ ,  $\sigma_y = 50 \text{ MPa}$

$$S_y = 350 \text{ MPa}$$

$$\sigma_1 - \sigma_3 \geq S_y$$

$$\sigma_1 - \sigma_3 = \frac{S_y}{N}$$

$$100 - 0 = \frac{350}{N} \Rightarrow N = \frac{350}{100} = \boxed{3.5} \quad \text{MSS}$$

$$\frac{S_y}{N} = \sqrt{\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2}} \geq S_y$$

$$\sqrt{\frac{(100 - 100)^2 + (100 - 0)^2 + (100 - 0)^2}{2}} = \sqrt{\frac{10000 + 10000}{2}} = 100$$

$$\frac{S_y}{N} = 100$$

$$N = \frac{350}{100} = \boxed{3.5} \quad \text{DE}$$

A brittle material has the properties  $S_{ut} = 30$  kpsi and  $S_{uc} = 90$  kpsi. Using the brittle Coulomb-Mohr and modified-Mohr theories, determine the factor of safety for the following states of plane stress.

(a)  $\sigma_x = 25$  kpsi,  $\sigma_y = 15$  kpsi  $\sigma_z = 0$

(b)  $\sigma_x = 15$  kpsi,  $\sigma_y = -15$  kpsi

$$\sigma_A = \frac{S_{ut}}{N}$$
$$25 = \frac{30}{N} \Rightarrow N = \frac{30}{25} = \boxed{1.2}$$

BCM    MM

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A ductile hot-rolled steel bar has a minimum yield strength in tension and compression of 350 MPa. Using the distortion-energy and maximum-shear-stress theories determine the factors of safety for the following plane stress states:

(a)  $\sigma_x = 100$  MPa,  $\sigma_y = 100$  MPa

(b)  $\sigma_x = 100$  MPa,  $\sigma_y = 50$  MPa

A brittle material has the properties  $S_{ut} = 30$  kpsi and  $S_{uc} = 90$  kpsi. Using the brittle Coulomb-Mohr and modified-Mohr theories, determine the factor of safety for the following states of plane stress.

(a)  $\sigma_x = 25$  kpsi,  $\sigma_y = 15$  kpsi

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MM: Pg 264

A ductile hot-rolled steel bar has a minimum yield strength in tension and compression of 350 MPa. Using the distortion-energy and maximum-shear-stress theories determine the factors of safety for the following plane stress states:

(a)  $\sigma_x = 100 \text{ MPa}$ ,  $\sigma_y = 100 \text{ MPa}$

(b)  $\sigma_x = 100 \text{ MPa}$ ,  $\sigma_y = 50 \text{ MPa}$      $\sigma_z = 0$      $\sigma_1 = 100 \text{ MPa}$      $\sigma_2 = 50 \text{ MPa}$      $\sigma_3 = 0$

$$\frac{S_y}{N} = \sigma_1 - \sigma_3 \quad \frac{350 \text{ MPa}}{N} = 100 \text{ MPa} \quad \Rightarrow \quad \frac{350}{100} = N = \boxed{3.5} \quad \text{MSS}$$

$$\frac{S_y}{N} = \sqrt{\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2}} = \sqrt{\frac{(100 - 50)^2 + (50 - 0)^2 + (0 - 100)^2}{2}} = \sqrt{\frac{2500 + 2500 + 10000}{2}} = \sqrt{7500} = 86.6$$

$$\frac{S_y}{86.6} = \frac{350}{86.6} = \boxed{4.04} = N \quad \text{DE}$$

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(a)  $\sigma_x = 25$  kpsi,  $\sigma_y = 15$  kpsi

(b)  $\sigma_x = 15$  kpsi,  $\sigma_y = -15$  kpsi

$$\sigma_A = 15 \text{ kpsi} \quad \sigma_D = -15 \text{ kpsi}$$

$$\frac{\sigma_A}{S_{ut}} - \frac{\sigma_B}{S_{uc}} = \frac{1}{N}$$

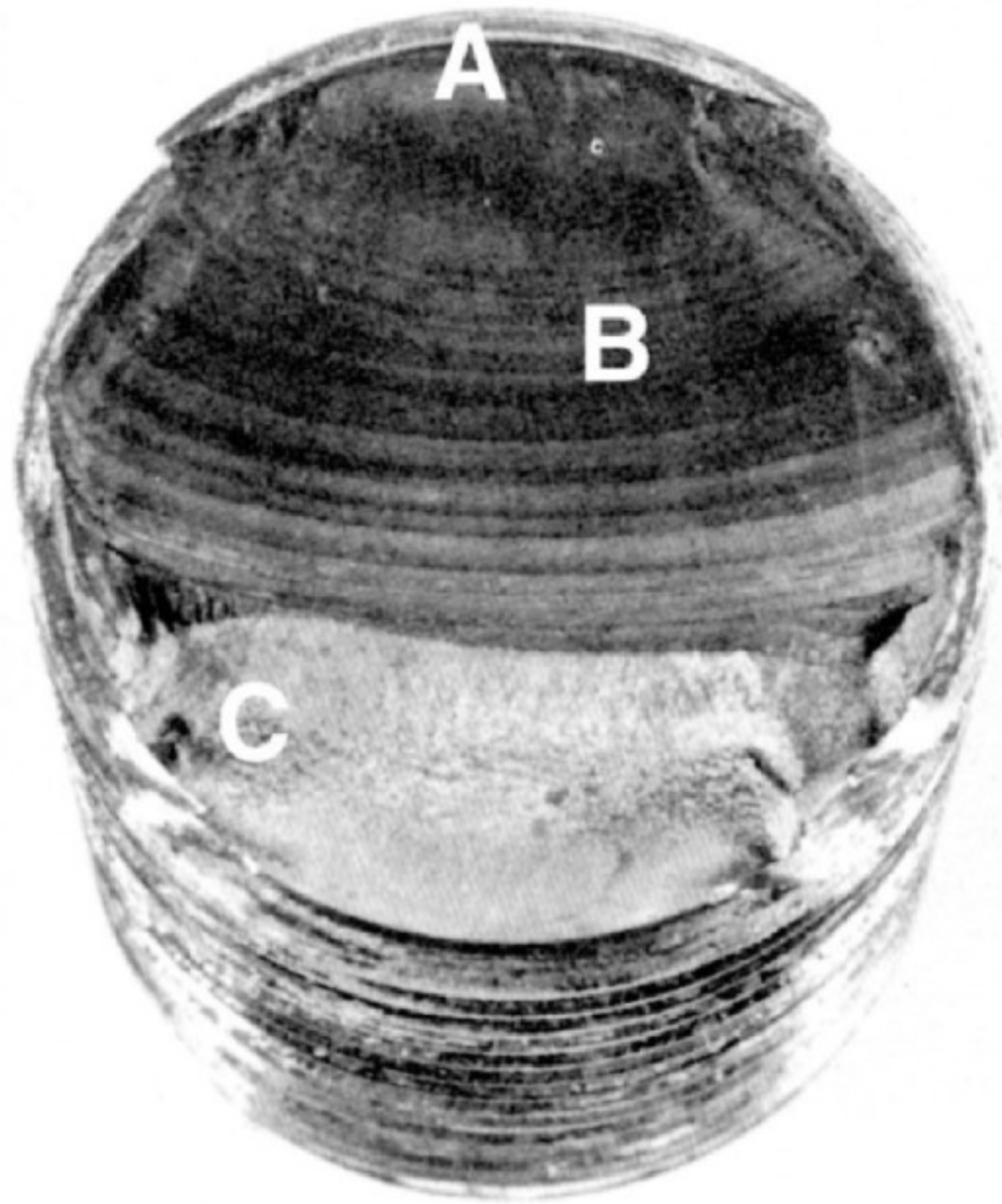
$$\frac{15}{30} - \frac{-15}{90} = 0.67 = \frac{1}{N} \Rightarrow N = \frac{1}{0.67} = \boxed{1.5} \text{ BCM}$$

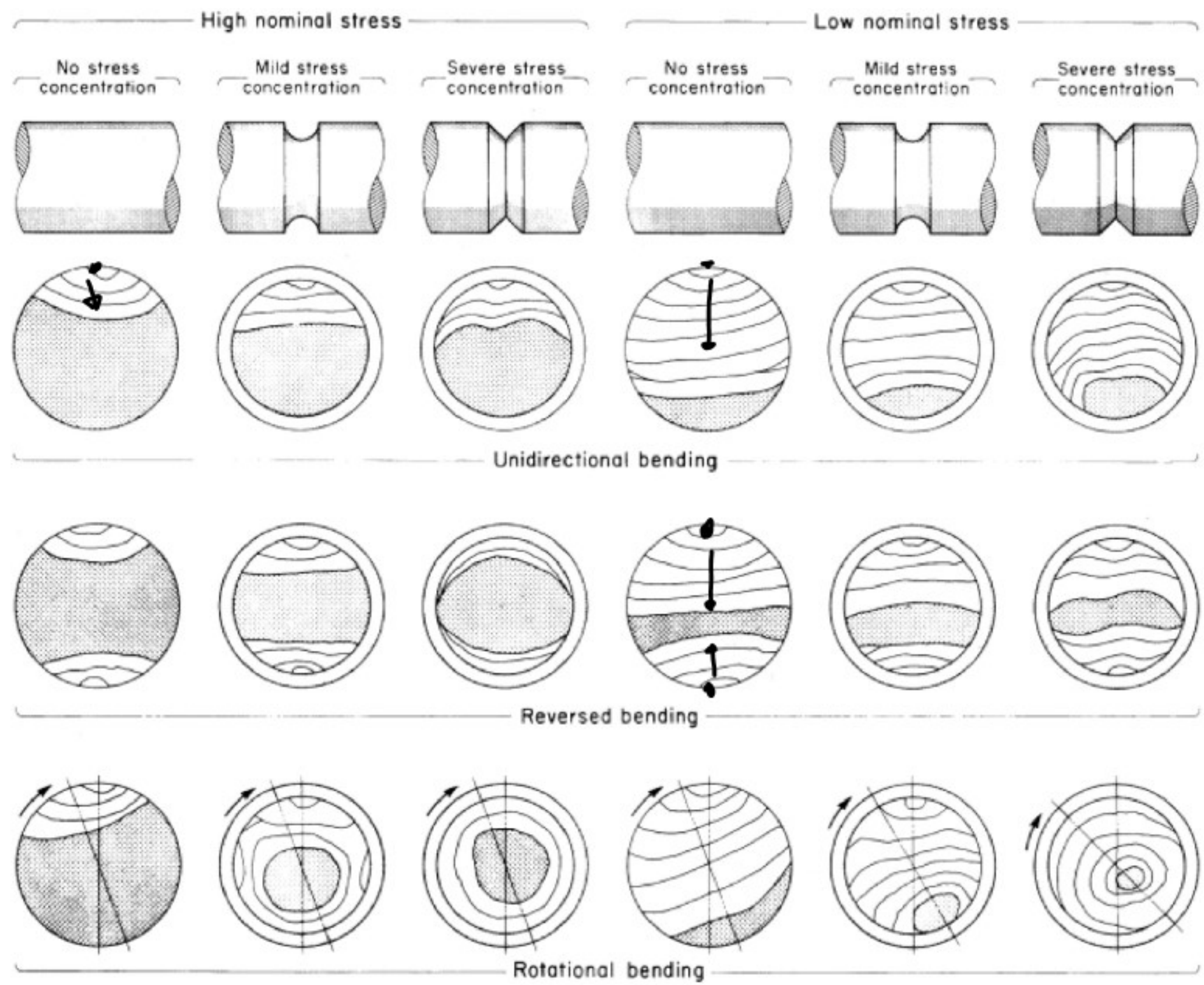
$$\left| \frac{\sigma_B}{\sigma_A} \right| = \left| \frac{-15}{15} \right| = 1$$

$$\sigma_A = \frac{S_{ut}}{N}$$

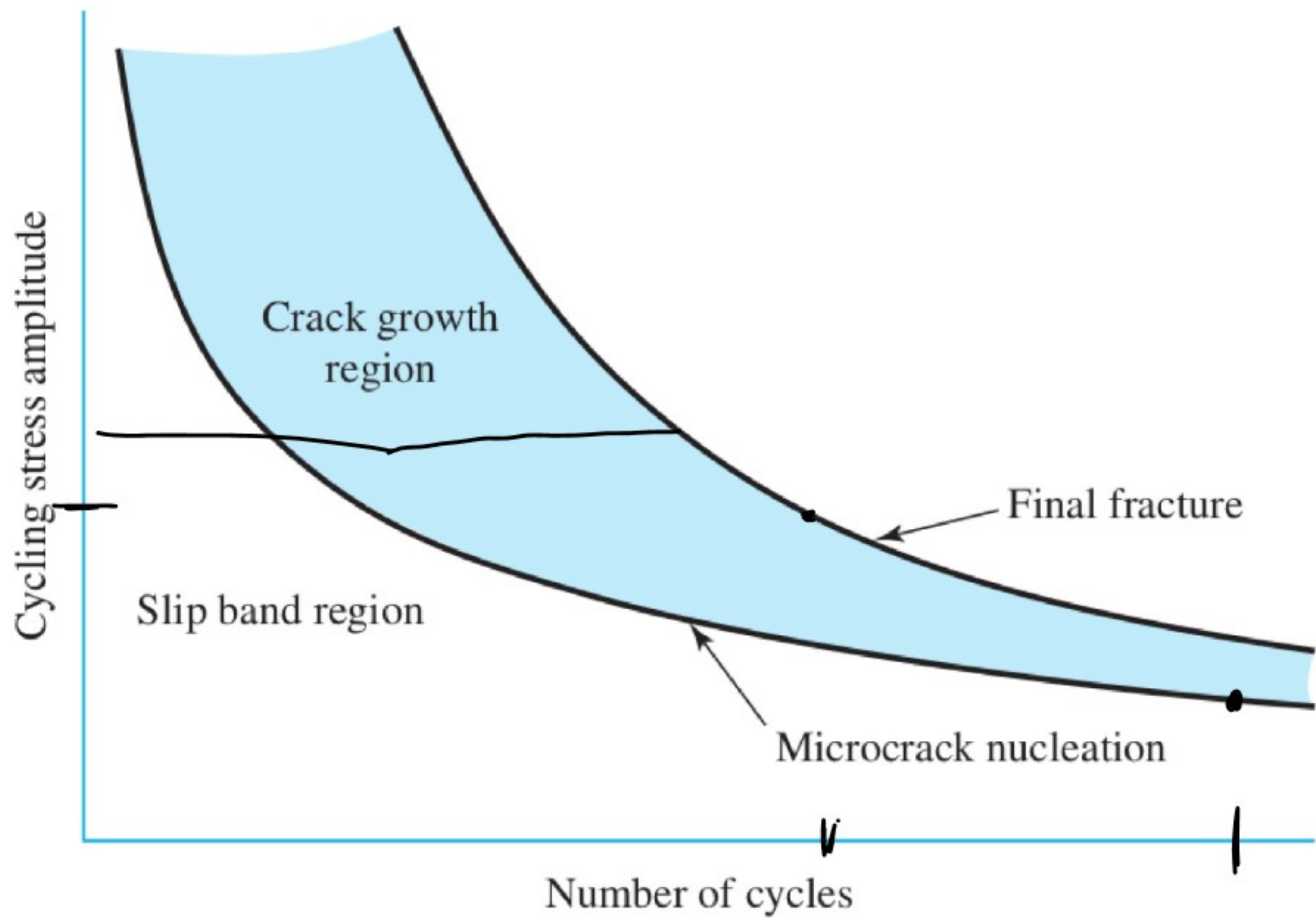
$$15 = \frac{30}{N} \Rightarrow N = \frac{30}{15} = \boxed{2} \text{ MM}$$

Fatigue









## Methods

Stress-life Method

Strain-life Method

LEFM linear elastic fracture mechanics

## Design Criteria

Infinite life design

Safe life design

Fail-safe design

Damage tolerant design

# LEFM

Stress fluctuating from  $\sigma_{min}$  to  $\sigma_{max}$

$$\Delta\sigma = \sigma_{max} - \sigma_{min}$$

$$\Delta K_I = \beta \Delta\sigma \sqrt{\pi a}$$

$a$  crack size

$\beta$  stress intensity modification factor (Pg. 271)

$K_I$  stress intensity

Paris eqn

$$\frac{da}{dN} = C (\Delta K_I)^m$$

$C, m$  constants

$N$  number of cycles

$$\int_0^{N_f} dN = N_f = \frac{1}{C} \int_{a_i}^{a_f} \frac{dA}{(\beta \Delta \sigma \sqrt{\pi a})^m}$$

$N_f$  number of cycles

$a_i$  initial crack size

$a_f$  final crack size