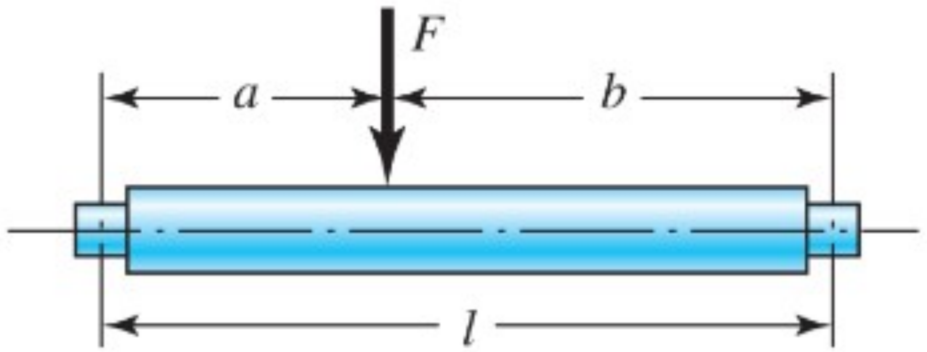
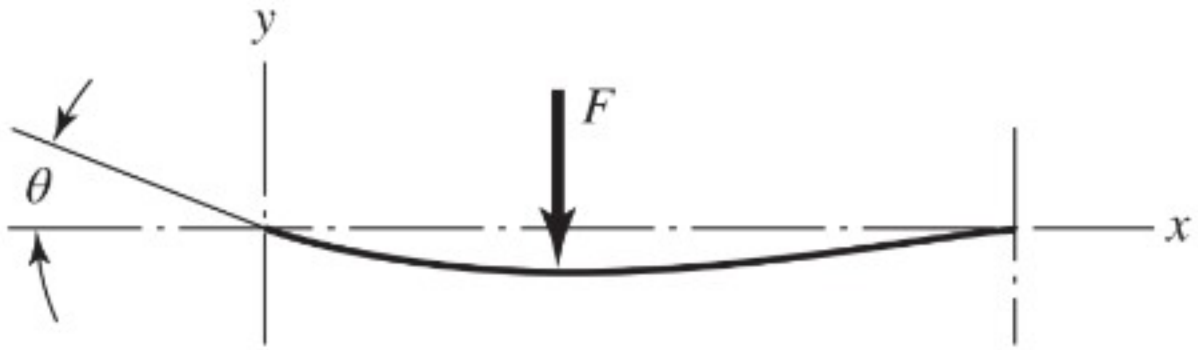
4–45 The designer of a shaft usually has a slope constraint imposed by the bearings used. This limit will be denoted as ξ. If the shaft shown in the figure is to have a uniform diameter d except in the locality of the bearing mounting, it can be approximated as a uniform beam with simple supports. Show that the minimum diameters to meet the slope constraints at the left and right bearings are, respectively,

$$d_L = \left|\frac{32Fb(l^2 - b^2)}{3\pi El\xi}\right|^{1/4} \qquad d_R = \left|\frac{32Fa(l^2 - a^2)}{3\pi El\xi}\right|^{1/4}$$



em 4–45



$$y_{AB} = \frac{Fbx}{6EIl}(x^2+b^2-l^2) = \frac{Fb}{6EIl}(x^3+x(b^2-l^2))$$

$$I = \frac{\pi d^4}{64}$$

$$\frac{d}{dx}y_{AB} = \frac{Fb}{6EIl}(3x^2+b^2-l^2)$$

$$\left.\frac{d}{dx}y_{AB}\right|_{x=l} = \frac{Fb}{6EIl}(3(0)^2+b^2-l^2) = \frac{Fb}{6EIl}(b^2-l^2)$$

$$= \frac{Fb}{6E\frac{\pi d^4}{64}l}(b^2-l^2) = \int$$

$$\frac{64Fb}{6E\pi S_l}(b^2-l^2) = d^4$$

$$\sqrt[4]{\frac{32Fb}{3E\pi S l}(b^2-l^2)} = d$$

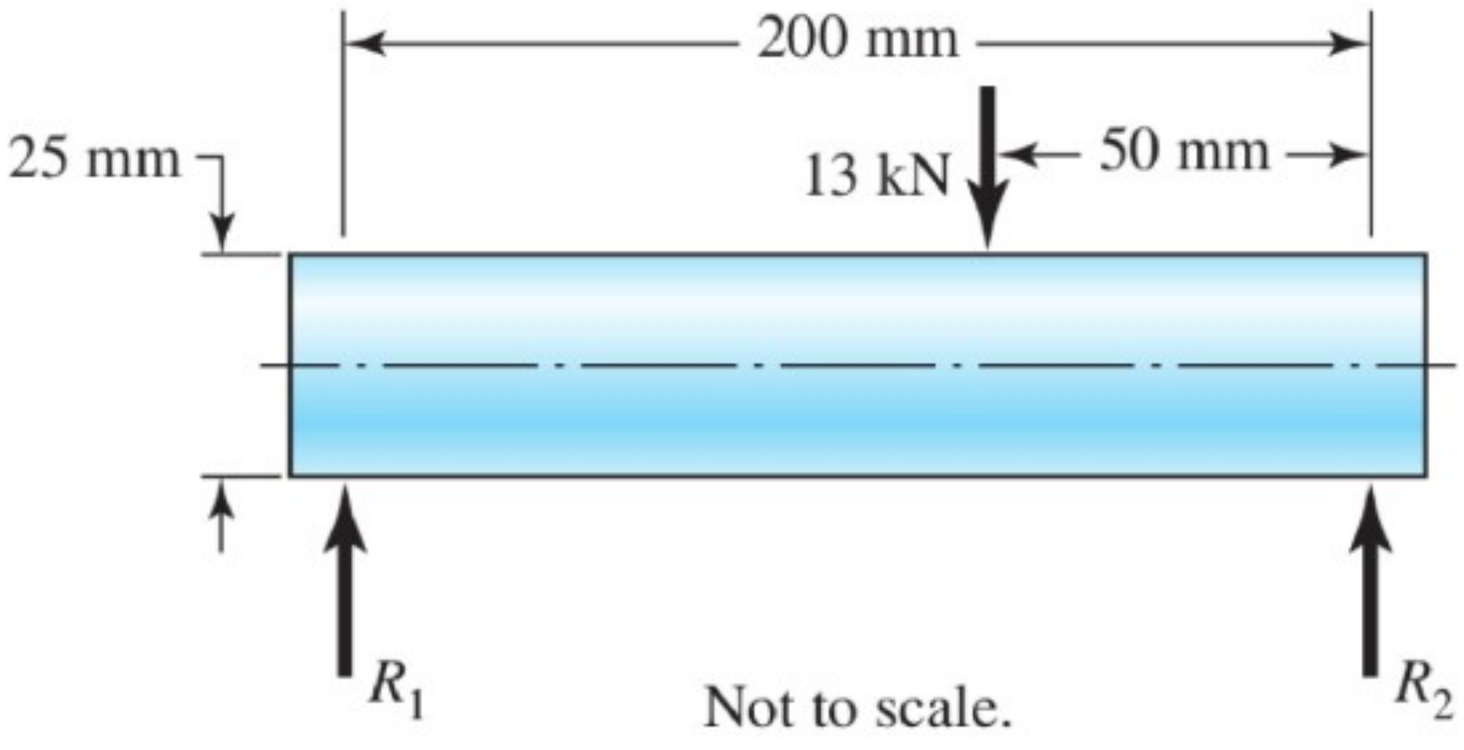
A *rotating* shaft of 25-mm diameter is simply supported by bearing reaction forces R_1 and R_2. The shaft is loaded with a transverse load of 13 kN as shown in the figure. The shaft is made from AISI 1045 hot-rolled steel. The surface has been machined. Determine

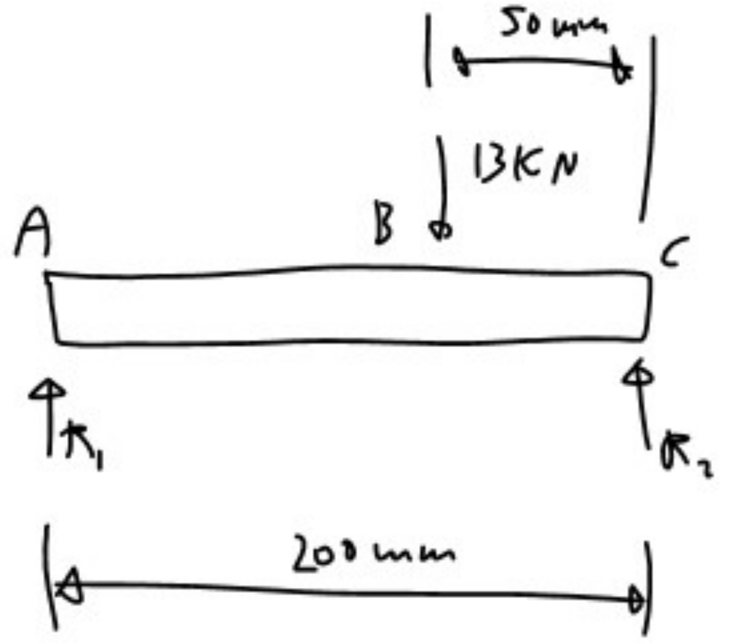
✓(*a*) the minimum static factor of safety based on yielding.

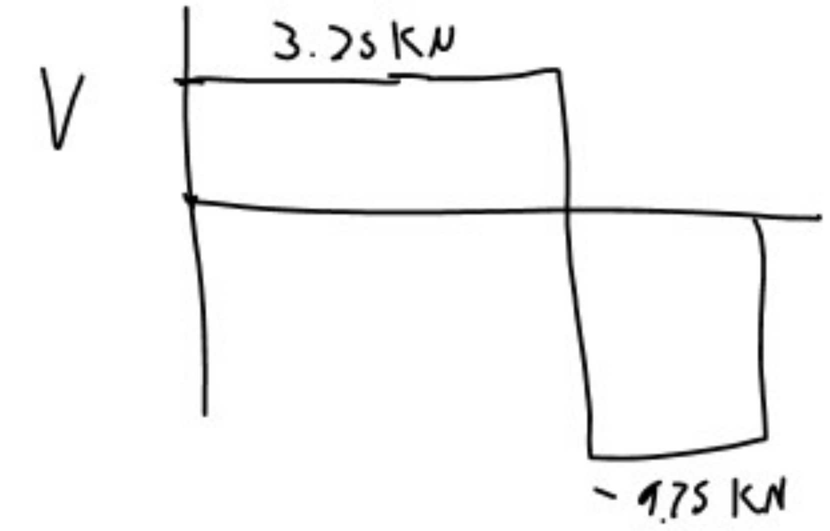
✓(*b*) the endurance limit, adjusted as necessary with Marin factors.

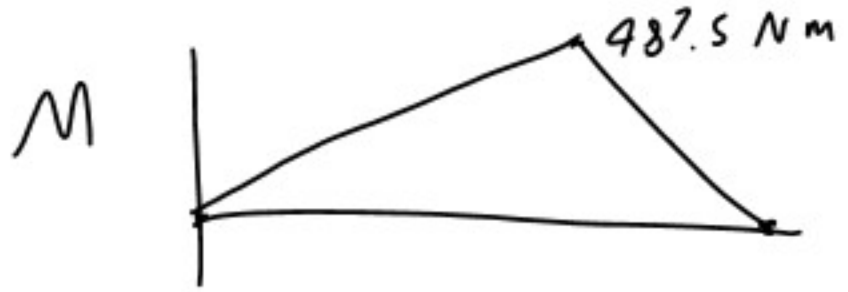
✓(*c*) the minimum fatigue factor of safety based on achieving infinite life.

(*d*) If the fatigue factor of safety is less than 1 (*hint:* it should be for this problem), then estimate the life of the part in number of rotations.









$$\Sigma M_A = -150\text{ mm}\,(13\text{ KN}) + 200\text{ mm}\; R_2 = 0$$

$$150\,(13) = 200\,R_2$$

$$\frac{150\,(13)}{200} = R_2 = \boxed{9.75\text{ KN}}$$

$$I = \frac{\pi d^4}{64} = \frac{\pi\,(0.025)^4}{64}$$

$$= 1.92 \times 10^{-8}$$

$$\Sigma F_y = R_1 + R_2 - 13\text{ KN} = R_1 + 9.75 - 13 = 0$$

$$R_1 = 13 - 9.75 = \boxed{3.25\text{ KN}}$$

$$\sigma = \frac{Mc}{I} = \frac{487.5\left(0.025/2\right)}{1.92\times10^{-8}} = 318\text{ MPa}$$

$$S_y = 310\text{ MPa}$$

$$N = \frac{S_y}{\sigma} = \frac{310}{318} = \boxed{0.975}$$

$$S_e' = 0.5\, S_{ut}$$

$$S_{ut} = 570\ \text{MPa}$$

$$= 0.5(570)$$

$$= 285\ \text{MPa}$$

$$S_e = K_a K_b K_c K_d K_e S_e'$$

$$K_b = 1.24 d^{-0.107}$$

$$= 1.24(25)^{-0.107} = 0.879$$

$$= 0.78 \cdot 0.879 \cdot 1 \cdot 1 \cdot 1 \cdot 285$$

$$\boxed{= 195\ \text{MPa}}$$

$$N = \frac{S_e}{\sigma} = \frac{195\ \text{MPa}}{318\ \text{MPa}} = \boxed{0.61}$$

fig 6-23 $\quad f = 0.87$

$$N = \left(\frac{\sigma_{ar}}{a}\right)^{1/b} \qquad 6-13 \qquad a = \frac{(f S_{ut})^2}{S_e} = \frac{(0.87 \cdot 570)^2}{195} = 1261 \text{ MPa}$$

$$= \left(\frac{318}{1261}\right)^{1/-0.135} \qquad 6-14 \qquad b = -\frac{1}{3}\log_{10}\left(\frac{f S_{ut}}{S_e}\right) = -\frac{1}{3}\log_{10}\left(\frac{0.87 \cdot 570}{195}\right) = -0.135$$

$$\boxed{= 26.8 \times 10^3 \text{ revolutions}}$$

A pin in a knuckle joint is shown in part (*a*) of the figure. The joint is subject to a repeatedly applied and released load of 6000 N in tension. Assume the loading on the pin is modeled as concentrated forces as shown in part (*b*) of the figure. The shaft is made from AISI 1018 hot-rolled steel that has been machined to its final diameter. Based on a stress element on the outer surface at the cross-section *A*, determine a suitable diameter of the pin, rounded up to the next mm increment, to provide at least a factor of safety of 1.5 for both infinite fatigue life and for yielding.