2.4. Automobiles must be able to sustain a frontal impact. The automobile design must allow low speed impacts with little sustained damage, while allowing the vehicle front end structure to deform and absorb impact energy at higher speeds. Consider a frontal impact test of a 1000 kg mass vehicle.

(a) For a low speed test at 2.5 m/s, compute the energy in the vehicle just prior to impact. If the bumper is a pure elastic element, what is the effective design stiffness required to limit the bumper maximum deflection during impact to 4 cm?

(b) At a higher speed impact of 25 m/s, considerable deformation occurs. To absorb the energy the front end of a vehicle is designed to deform while providing a nearly constant force. For this condition, what is the amount of energy that must be absorbed by the deformation [neglecting the energy stored in the elastic deformation in (a)]? If it is desired to limit the deformation to 10 cm, what level of resistance force is required? What is the deacceleration of the vehicle in this condition?

a)

$$E = \frac{1}{2} m v^2 = \frac{1}{2} 1000 (2.5)^2 = \boxed{3125 \text{ N-m}}$$

$$F = Kx \qquad E = \frac{1}{2} K x^2 \qquad x = 0.04 \text{ m}$$

$$3125 = \frac{1}{2} K x^2$$

$$\frac{2(3125)}{x^2} = K$$

$$\frac{2(3125)}{(0.04)^2} = \boxed{K = 390.6 \times 10^4 \ \frac{N}{m}}$$

b)

$$E = \frac{1}{2} m V^2 = \frac{1}{2} 1000 (25)^2 = 3125 \times 10^2 \text{ N-m}$$

$$E = \int_0^x F \, dx = Fx \qquad X = 10 \text{ cm} = 0.1 \text{ m}$$

$$\frac{E}{x} = F$$

$$\frac{3125 \times 10^2}{0.1} = \boxed{3.125 \times 10^6 \text{ N}} = F$$

$$F = ma$$

$$\frac{F}{m} = a$$

$$\frac{3.125 \times 10^6}{1000} = \boxed{a = 3125 \text{ m/s}^2}$$