2.6. A computer hard disk stores data on a rotating cylindrical disk. Consider such a disk with a radius of 6 cm and a mass of 0.02 kg.

(a) What is the moment of inertia of the disk?

(b) If the disk drive motor provides a torque of 0.1 N-m during spin-up, what is the rotational speed of the disk after 5 seconds?

(c) A new disk design uses composite materials to make a disk of the same mass but with a radius 1.5 times that of the original disk. What is the inertia of the new disk? What motor torque would be required to spin the new disk to the same speed as the original in 5 seconds?

a) $r = 6 cm = 0.06 m$

$$J = \frac{1}{2} m r^2 = \frac{1}{2} \, 0.02 \, (0.06)^2 = \boxed{36 \times 10^{-6} \; kg \; m^2}$$

b)

$$\frac{dw}{dt} = \frac{T}{J} = \frac{0.1}{36 \times 10^{-6}} = 2.78 \times 10^3 \; \frac{rad}{s^2}$$

$$\int_0^5 \frac{dw}{dt} \, dt = \int_0^5 2.78 \times 10^3 \, dt = t \, 2.78 \times 10^3 \Big|_0^5$$

$$= 5 \, (2.78 \times 10^3) = \boxed{13.89 \times 10^3 \; \frac{rad}{s}}$$

c) $$J = \frac{1}{2} m r^2 = \frac{1}{2} \, 0.02 \, (1.5 \, (0.06))^2 = \boxed{81 \times 10^{-6} \; kg \; m^2}$$

$$w_s = 5 \frac{T}{J} \implies T = \frac{w_s \, J}{5} = \frac{13.89 \times 10^3 \; 81 \times 10^{-6}}{5}$$

$$= \boxed{0.225 \; N \; m}$$