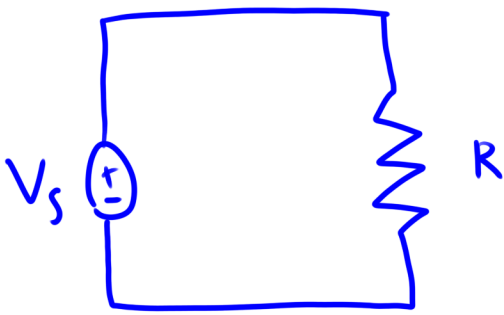


3.7. Consider an electric home heating system. A resistive element  $R$  is connected to the 110-volt, 60-Hz electric supply.

- Determine the current flow through the element as a function of time.
- Determine the average thermal power generated per cycle.
- If the heater is rated at 1000 watts, what is the value of the heater resistance  $R$ ?

a)



$$V_s = \frac{110}{2} \sin(60(2\pi)t)$$

$$V_R = R i_R \Rightarrow i_R = \frac{V_R}{R} = \boxed{\frac{V_s}{R} = i_R}$$

b)

$$P = \frac{1}{T} \int_0^T V_R i_R dt = \frac{1}{60} \int_0^{1/60} \frac{110^2}{4R} \sin^2(60(2\pi)t) dt$$

$$= \frac{110^2}{4R} \frac{-1}{4(60)(2\pi)} \left( \sin(2(60)(2\pi)t) - 2(60)(2\pi)t \right) \Big|_0^{1/60}$$

$$= \frac{110^2}{4R} \frac{-1}{4(60)(2\pi)} \left( -2(60)(2\pi) \frac{1}{60} + 0 \right)$$

$$= \frac{110^2 (2)}{4R (60)} = \boxed{\frac{1512}{R}}$$

c)

$$1000 = \frac{1512}{R} \Rightarrow R = \frac{1512}{1000} = \boxed{1.51 \Omega = R}$$

