5.4. A one-quarter car model is useful for studying the effects of the road surface on the vertical motion of a car body. The model includes the tire stiffness as well as the suspension damping and stiffness, and supports a mass of one quarter of that of the car, as shown in Fig. 5.25. The roadway is modeled as providing a vertical velocity input to the tire as the car travels along the road. (Because the gravitational force is constant, it may be omitted when modeling incremental motions about an equilibrium point.)

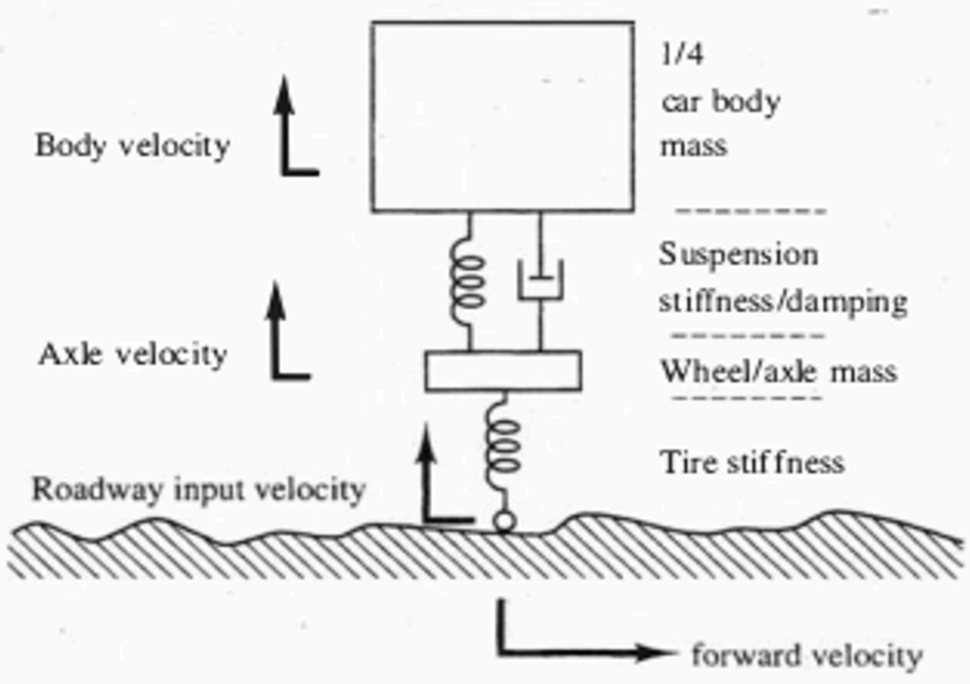
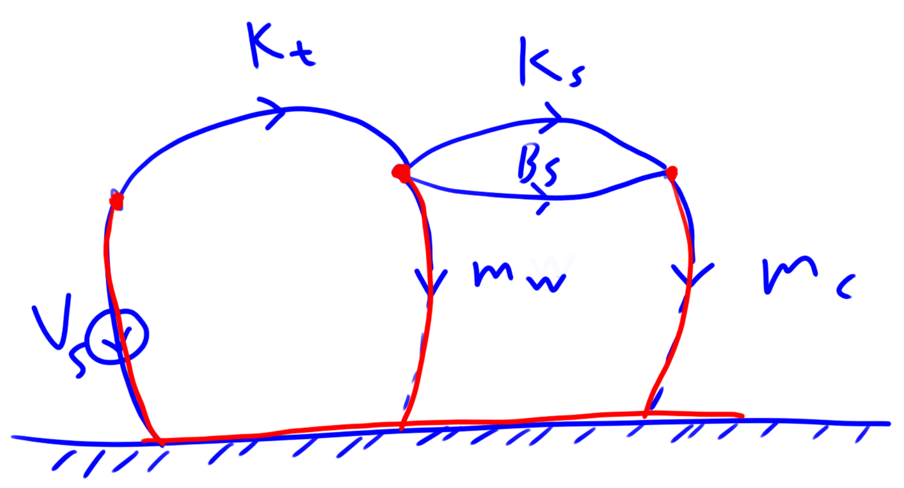


Figure 5.25: A quarter-car model of an automobile suspension.

(a) Construct the system linear graph.
(b) Identify the system state variables.
(c) Derive the state equations and express them in matrix form.
(d) Derive output equations for the total force acting on the car body mass from the suspension spring and damper.



State Space Models

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

$$x = \begin{bmatrix} V_{m_c} \\ V_{m_w} \\ F_{k_s} \\ F_{k_t} \end{bmatrix} \qquad u = [V_s]$$

$$A = \begin{bmatrix} -B_s/m_c & B_s/m_c & 1/m_c & 0 \\ B_s/m_w & -B_s/m_w & -1/m_w & 1/m_w \\ -k_s & k_s & 0 & 0 \\ 0 & -k_t & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ k_t \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \qquad D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\frac{d}{dt} f(t) = \lim_{\Delta t \to 0} \frac{f(t+\Delta t) - f(t)}{\Delta t}$$

$$\Delta t \neq 0$$

$$\frac{d}{dt} f(t) \simeq \frac{f(t+\Delta t) - f(t)}{\Delta t}$$

$$f(t) + \Delta t \left(\frac{d}{dt} f(t) \right) = f(t + \Delta t)$$