5.5. Seismometers are used to measure the motion of the earth's surface. A schematic drawing of a simple seismometer is shown in Fig. 5.26. A proof mass is suspended in springs and slides horizontally on a viscous friction material. The relative displacement of the proof mass with respect to the instrument case is used as a measure of the severity of an earthquake.

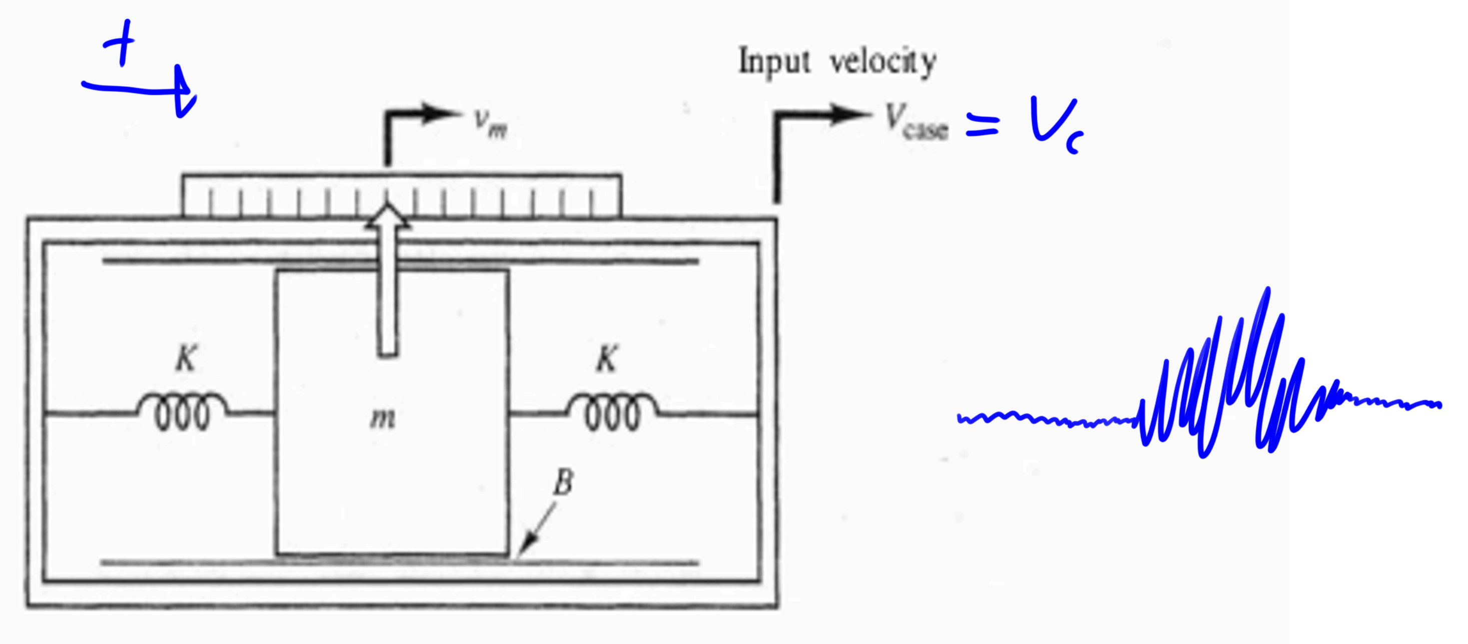


Figure 5.26: A seismometer.

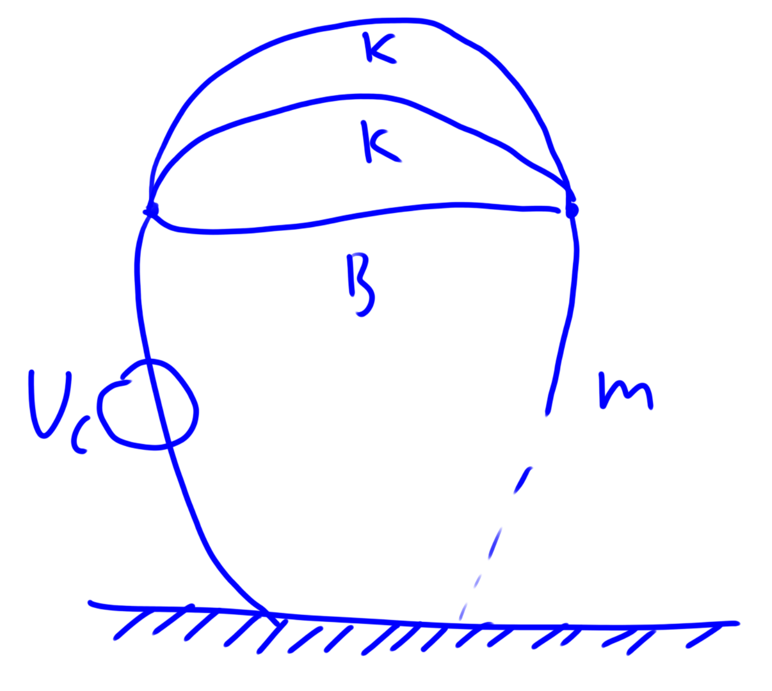
(a) Construct a linear graph model of the system.

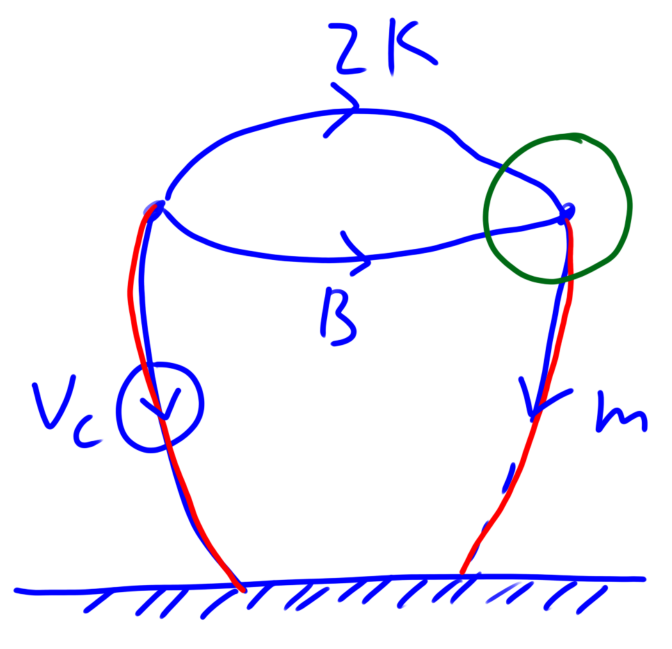
(b) How many independent energy storage elements are there? What are the system state variables?

(c) Derive the system state equations and express them in matrix form.

(d) Derive an output equation for the instrument reading, that is, the relative displacement of the proof mass with respect to the instrument case.

a.





$E = 4$
$N = 3$
$S_A = 1$
$S_T = 0$
$S = 1$

c. primary: V_c V_m F_K F_B

secondary: F_c F_m V_K V_B

state: V_m F_K $n = 2$

$$x = \begin{bmatrix} V_m \\ F_K \end{bmatrix} \quad u = [V_c] \quad y = [V_m]$$

$E - S = 3$

$$\frac{dV_m}{dt} = \frac{1}{m} F_m$$

$$\frac{dF_K}{dt} = 2K\, V_K$$

$$F_B = B\, V_B$$

$N - 1 - S_A = 1$

$$F_m = F_K + F_B$$

$E - N + 1 - S_T = 2$

$$V_K = V_c - V_m$$
$$V_B = V_c - V_m$$

State M, nt → $\dot{x} = Ax + Bu$

$$\frac{dV_m}{dt} = \frac{1}{m}(F_K + F_B)$$

$$\frac{dF_K}{dt} = 2K(V_c - V_m)$$

$$F_B = B(V_c - V_m)$$

$$\frac{dV_m}{dt} = \frac{1}{m}(F_K + B(V_c - V_m))$$

$$\frac{d}{dt}\begin{bmatrix} V_m \\ F_K \end{bmatrix} = \underbrace{\begin{bmatrix} -B/m & 1/m \\ -2K & 0 \end{bmatrix}}_{A} \begin{bmatrix} V_m \\ F_K \end{bmatrix} + \underbrace{\begin{bmatrix} B/m \\ 2K \end{bmatrix}}_{B} [V_c]$$

$$\dot{x} = Ax + Bu$$

$$y = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C} \begin{bmatrix} V_m \\ F_K \end{bmatrix} + \underbrace{[0]}_{D} [V_c]$$