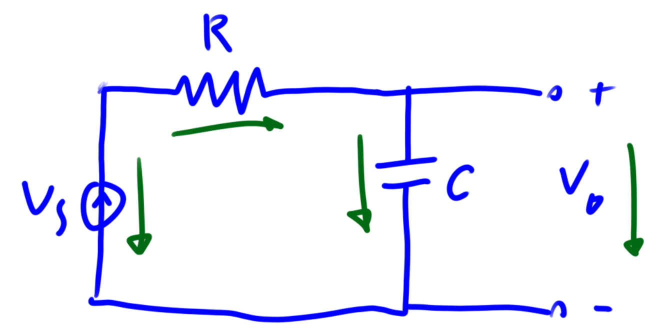
State Space Representation to ODE

RC circuit



solve via
1. Differential eq'n
2. Normal tree to SS State space to ODE

$0 = V_c - V_o \implies V_c = V_o$

1. Differential eq'n

Elemental $\quad \dfrac{dV_c}{dt} = \dfrac{1}{C} i_c$

$V_R = R i_R \implies i_R = \dfrac{V_R}{R}$

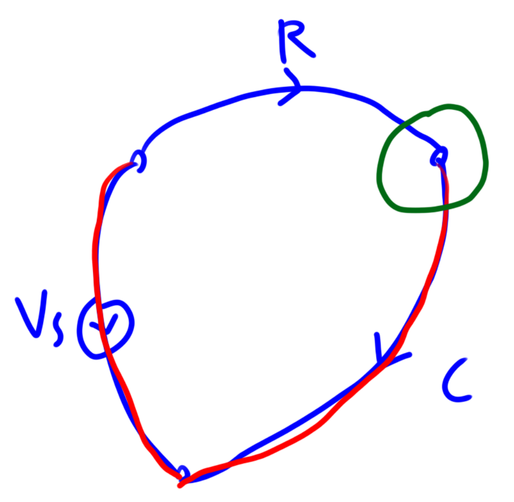
KVL $\quad V_s = V_R + V_c \qquad V_c = V_o$

KCL $\quad i_R = i_c \qquad V_R = V_s - V_c$

$$\frac{dV_c}{dt} = \frac{1}{C} i_c = \frac{1}{C} i_R = \frac{1}{RC} V_R = \frac{V_s - V_c}{RC}$$

$$\frac{dV_c}{dt} + \frac{1}{RC} V_c = \frac{1}{RC} V_s$$

2. Linear graph to SS



Primary: $V_s \quad V_c \quad i_R$
Secondary: $I_s \quad i_c \quad V_R$
State: $V_c \qquad n=1$

$x = [V_c] \qquad y = [V_c] \qquad u = [V_s]$

Elemental

$$\frac{dV_c}{dt} = \frac{1}{C} i_c = \frac{1}{C} i_R = \frac{V_s - V_c}{RC}$$

$i_R = \frac{1}{R} V_R = \dfrac{V_s - V_c}{R}$

Continuity
$i_c = i_R$

Compadability
$V_R = V_s - V_c$

$$\dot{x} = \underbrace{\left[-1/RC\right]}_{A} x + \underbrace{\left[1/RC\right]}_{B} u$$

$$y = \underbrace{[1]}_{C} x + \underbrace{[0]}_{D} u$$

SS to TF

$$H(s) = C(sI - A)^{-1}B + D = 1\left(s - \frac{-1}{RC}\right)^{-1}\frac{1}{RC} + 0$$

$$= \left(s + \frac{1}{RC}\right)^{-1}\frac{1}{RC}$$

$$= \frac{1}{s + 1/RC}\,\frac{1}{RC}$$

$$= \frac{1}{RCs + 1} = \frac{Y(s)}{U(s)}$$

$$\frac{RC\dfrac{dV_c}{dt} + V_c = V_s}{RC} \qquad \frac{\ }{RC}$$

$$\frac{dV_c}{dt} + \frac{1}{RC} V_c = \frac{1}{RC} V_s \quad \checkmark$$