6.3. A ball-screw linear actuator is shown in Fig. 6.26. The actuator uses a dc electric motor, with an electromotive coupling constant K_a, armature resistance R, and inductance L, and driven by a voltage source $V_s(t)$. The motor drives a threaded shaft with N threads/cm on which the ball-nut moves linearly. The mechanical load is a mass m which slides on a surface with viscous damping coefficient B.

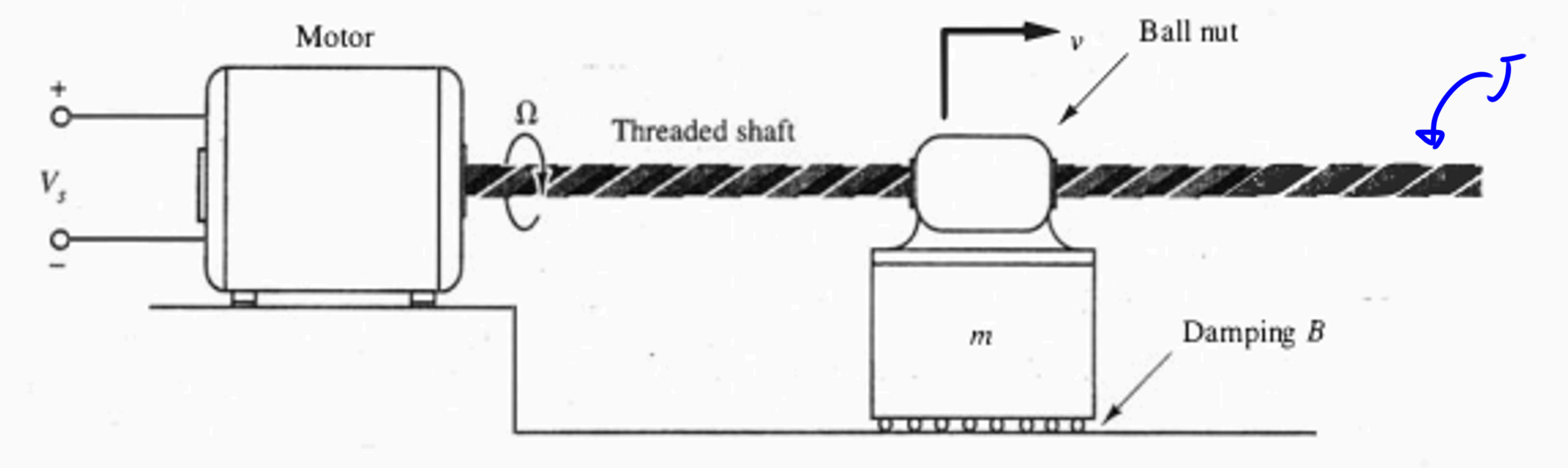
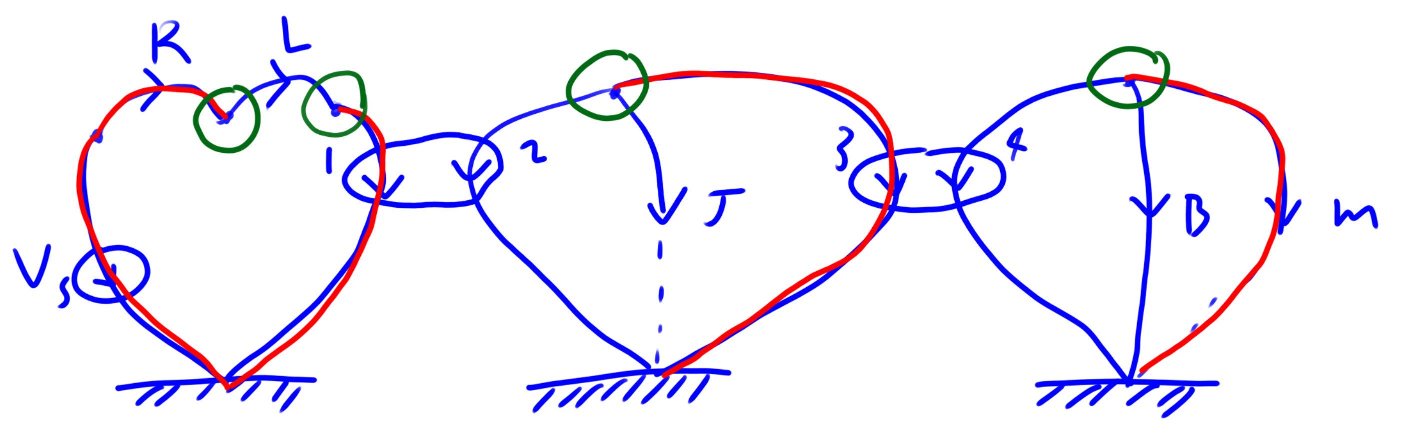


Figure 6.26: A ball-screw drive mechanism.



$$\begin{bmatrix} V_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} K_a & 0 \\ 0 & -1/K_a \end{bmatrix} \begin{bmatrix} \Omega_2 \\ \tau_2 \end{bmatrix} \qquad \frac{\Omega_3}{V_4} = TF$$

$$V_1 = K_a \Omega_2 \qquad \int \Omega_3 = \int TF\, V_4$$

$$i_1 = \frac{-1}{K_a}\tau_2$$

Primory: V_s V_R V_1 i_L Ω_3 τ_2 τ_J V_m F_B F_4

Secondary: I_s i_R i_1 V_L τ_3 Ω_2 Ω_J F_m V_B V_4

State: V_m i_L $n=2$

$$\begin{bmatrix} \Omega_3 \\ \tau_3 \end{bmatrix} = \begin{bmatrix} 2\pi N & 0 \\ 0 & -1/2\pi N \end{bmatrix} \begin{bmatrix} V_4 \\ F_4 \end{bmatrix} \quad \theta_3 = TF\, x_4$$

if $\theta_3 = 2\pi$ $\quad x_4 = \frac{1}{N}$

$$\Omega_3 = 2\pi N V_4$$

$$\tau_3 = \frac{-1}{2\pi N} F_4 \qquad 2\pi = TF \frac{1}{N} \Rightarrow TF = 2\pi N$$

Elemental

$$V_R = R i_R = R\, i_L$$

- $$\frac{di_L}{dt} = \frac{1}{L} V_L = \frac{1}{L}(V_s - V_R - V_1) = \frac{1}{L}(V_s - R i_L - K_a \Omega_3)$$
$$= \frac{1}{L}(V_s - R i_L - K_a 2\pi N V_m)$$

$$V_1 = K_a \Omega_2 = K_a \Omega_3$$

$$\tau_2 = -K_a i_1 = -K_a i_L$$

$$\tau_J = J \frac{d\Omega_J}{dt} = J \frac{d\Omega_3}{dt}$$

$$\Omega_3 = 2\pi N V_4 = 2\pi N V_m$$

$$F_4 = -2\pi N \tau_3 = 2\pi N(\tau_2 + \tau_J)$$

$$F_B = B V_B = B V_m$$

- $$\frac{dV_m}{dt} = \frac{1}{m} F_m = \frac{-1}{m}(F_4 + F_B) = \frac{-1}{m}\left(2\pi N(\tau_2 + \tau_J) + B V_m\right)$$

Continuity

$$= \frac{-1}{m}\left(2\pi N\left(-K_a i_L + J\frac{d\Omega_3}{dt}\right) + B V_m\right)$$

$$i_R = i_L$$

$$= \frac{-1}{m}\left(2\pi N\left(-K_a i_L + J\, 2\pi N \frac{dV_m}{dt}\right) + B V_m\right)$$

$$i_1 = i_L$$

$$\tau_3 = -\tau_2 - \tau_J$$

$$= \frac{2\pi N K_a}{m} i_L - \frac{4\pi^2 N^2 J}{m}\frac{dV_m}{dt} - \frac{B}{m} V_m$$

$$F_m = -F_4 - F_B$$

Compatibility

$$\frac{dV_m}{dt}\left(1 + \frac{4\pi^2 N^2 J}{m}\right) = \frac{2\pi N K_a}{m} i_L - \frac{B}{m} V_m$$

$$V_L = V_s - V_R - V_1$$

$$\Omega_2 = \Omega_3$$

$$\Omega_J = \Omega_3$$

$$V_4 = V_m \qquad \frac{dV_m}{dt} = \frac{2\pi N K_a}{m + 4\pi^2 N^2 J} i_L$$

$$V_B = V_m \qquad - \frac{B}{m + 4\pi^2 N^2 J} V_m$$

$$X = \begin{bmatrix} i_L \\ V_m \end{bmatrix} \qquad u = [V_s] \qquad y = [V_m]$$

$$\dot{X} = \begin{bmatrix} -R/L & \frac{-2\pi N K_a}{L} \\ \frac{2\pi N K_a}{m + 4\pi^2 N^2 J} & \frac{-B}{m + 4\pi^2 N^2 J} \end{bmatrix} X + \begin{bmatrix} 1/L \\ 0 \end{bmatrix} u$$

$$y = [0 \quad 1] X + [0] u$$