Mechanical Engineering
345 – Mechatronics

Midterm Exam 2
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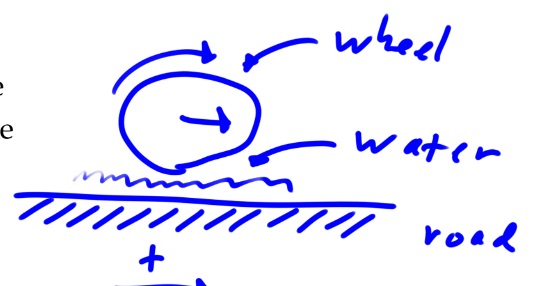
22 November 2021

Directions: take-home, due at 11:59 PM on Tuesday November 23rd, open notes, open book. Calculators, MATLAB, etc. allowed. Use of StateMint is not allowed. Use your own paper, work neatly, and clearly mark your answers. Partial credit may be given.

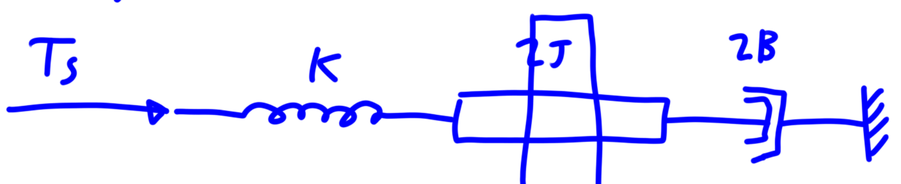
Problem madrid

_____/15 p.

Consider the drivetrain of a standard internal combustion engine vehicle. When accelerating from a stop in wet weather it is common for the wheels to slip due to a film of water between the wheels and the road. Develop a lumped parameter model of this system with the following assumptions,

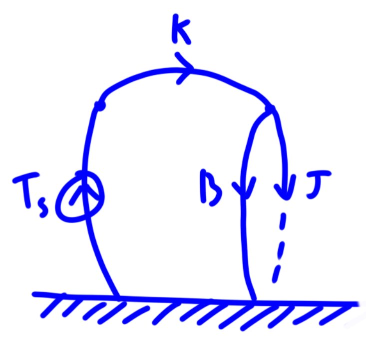


- the engine and transmission together can be simulated as a torque source,
- the transmission and wheels are connected with a drive shaft of finite stiffness, and
- each wheel has equal mass.



From this description please,

a draw a one dimensional lumped parameter model (like the diagrams in problem granda), and
b draw a linear graph of the lumped parameter model.

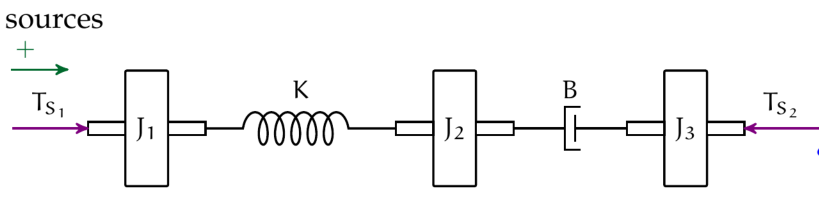


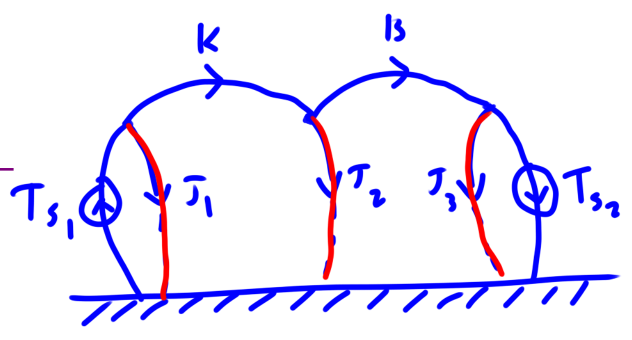
Problem granada

_____/35 p.

Use the assigned coordinate arrows to draw a linear graph, a normal tree, and identify state variables and system order for each of the following systems.

a rotational mechanical system, two torque sources

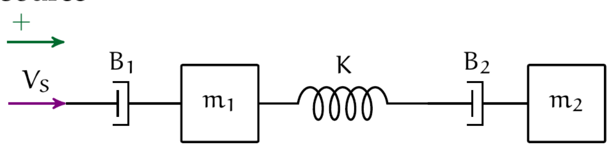


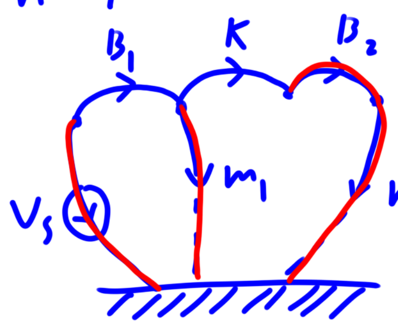


State: Ω_{J_1}, Ω_{J_2}, Ω_{J_3}, τ_K

$n = 4$

b translational mechanical system, velocity source





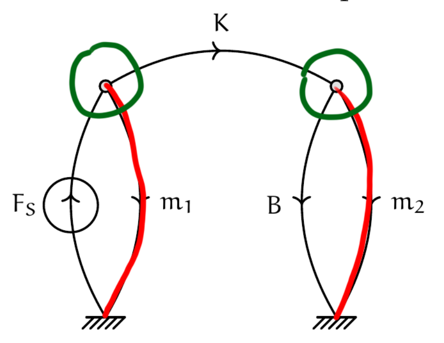
State: V_{m_1}, V_{m_2}, F_K

$n = 3$

Problem valencia

_____/50 p.

Use the following linear graph for a mechanical translational system to answer the questions below, which are the steps to determining a state-space model from the linear graph.
Use the sign convention from the diagram. F_S is a force source. Let the outputs be v_{m_1} and v_{m_2}.



State: V_{m_1}, V_{m_2}, F_K

$n = 3$

$$x = \begin{bmatrix} V_{m_1} \\ V_{m_2} \\ F_K \end{bmatrix} \quad u = [F_s] \quad y = \begin{bmatrix} V_{m_1} \\ V_{m_2} \end{bmatrix}$$

a Determine the normal tree, state variables, system order, state vector, input vector, and output vector.
b Write the required elemental, continuity, and compatibility equations.
c Solve for the state equation in standard form.
d Solve for the output equation in standard form.

Primary: V_{m_1}, V_{m_2}, F_s, F_B, F_K

Secondary: F_{m_1}, F_{m_2}, V_s, V_B, V_K

elemental:

$$\frac{dV_{m_1}}{dt} = \frac{1}{m_1} F_{m_1} = \frac{1}{m_1}(F_s - F_K)$$

$$\frac{dV_{m_2}}{dt} = \frac{1}{m_2} F_{m_2} = \frac{1}{m_2}(F_K - F_B) = \frac{1}{m_2}(F_K - BV_{m_2})$$

$$\frac{dF_K}{dt} = K V_K = K(V_{m_1} - V_{m_2})$$

$$F_B = BV_B = BV_{m_2}$$

Continuity:

$$F_{m_1} = F_s - F_K$$
$$F_{m_2} = F_K - F_B$$

Compatability:

$$V_K = V_{m_1} - V_{m_2}$$
$$V_B = V_{m_2}$$

$$\begin{bmatrix} \dot V_{m_1} \\ \dot V_{m_2} \\ \dot F_K \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1/m_1 \\ 0 & -B/m_2 & 1/m_2 \\ K & -K & 0 \end{bmatrix} \begin{bmatrix} V_{m_1} \\ V_{m_2} \\ F_K \end{bmatrix} + \begin{bmatrix} 1/m_1 \\ 0 \\ 0 \end{bmatrix} [F_s]$$

$$\begin{bmatrix} V_{m_1} \\ V_{m_2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} V_{m_1} \\ V_{m_2} \\ F_K \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} [F_s]$$

$$[V_{m_1} + V_{m_2}] = [1 \;\; 1 \;\; 0] \begin{bmatrix} V_{m_1} \\ V_{m_2} \\ F_K \end{bmatrix} + [0][F_s]$$