ssan.pha Complex or phasor representations of voltage and current

It is common to represent voltage and current in circuits as complex exponentials, especially when they are sinusoidal. Euler's formula is our bridge back-and forth from trigonomentric form ($\cos\theta$ and $\sin\theta$) and exponential form ($e^{j\theta}$):

Euler's formula

Here are a few useful identities implied by Euler's formula.

$$
\boxed{
\begin{aligned}
e^{-j\theta} &= \cos\theta - j\sin\theta && \text{(1a)}\\
\cos\theta &= \mathrm{Re}\left(e^{j\theta}\right) && \text{(1b)}\\
&= \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right) && \text{(1c)}\\
\sin\theta &= \mathrm{Im}\left(e^{j\theta}\right) && \text{(1d)}\\
&= \frac{1}{j2}\left(e^{j\theta} - e^{-j\theta}\right). && \text{(1e)}
\end{aligned}
}
$$

$\theta = \omega t$

$= \frac{1}{2} (e^{j\omega t} \ldots$

These equations can be considered to be describing a vector in the complex plane, which is illustrated in Fig. pha.1. Note that a $e^{j\theta}$ has both a magnitude and a phase.

complex plane

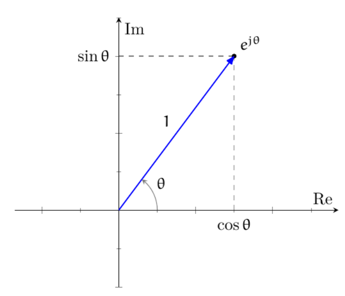

Figure pha.1: Euler's formula interpreted with a vector in the complex plane.

Consider a sinusoidal voltage signal $v(t) = v_0 \cos(\omega t + \phi)$ with amplitude v_0, angular frequency ω, and phase ϕ. We encountered in Lec. can.trss the fact that, for a linear system with a sinusoidal input in steady-state, the output is a sinusoid at the same frequency as the input. The only aspects of the sinusoid that the system changed from input to output were its magnitude (amplitude) and phase. Therefore, these are the two quantities of interest in a steady-state circuit analysis. Our notation simply ignores the frequency ω and represents $v(t)$ as

We call this the complex or phasor form of $v(t)$. This is meant to be shorthand notation and, if interpreted literally, can cause confusion. In fact, mathematically,

phasor form

Technically, we can use this more complicated form in our analysis but we won't because, conveniently, if we just treat the signal as if it was equal to $v_0 e^{j\phi}$, and at the end apply our "implied" $e^{j\omega t}$ term and $\mathrm{Re}()$ to the result, everything just works ... trust me, I'm a doctor ;).

$$
\begin{tikzcd}
v(t) = v_0 \cos(\omega t + \phi) \arrow[d, "\text{phaze it!}"'] & v(t) = v_0' \cos(\omega t + \phi') \\
v(t) = v_0 e^{j\phi} \arrow[r, "\text{circuit operates}"] & v'(t) = v_0' e^{j\phi'} \arrow[u, "\mathrm{Re}(e^{j\omega t}\cdot)"']
\end{tikzcd}
$$

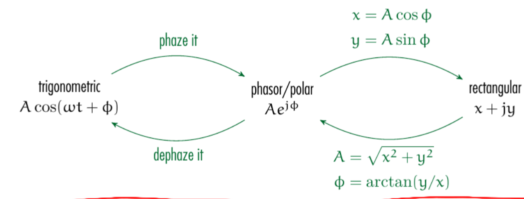

Figure pha.2: showing transformations among trigonometric, phasor or polar, and rectangular forms of representation.

The same process can be used to convert a sinusoidal current to and from phasor form. An alternative notation for a phasor $v_0 e^{j\phi}$ is

Traversing representations

Fig. pha.2 shows transformations one might use to change signal representations. Often we begin with a trigonometric form and convert to phasor/polar form for analysis, which might require switching back and forth between phasor/polar and rectangular, depending on the operation:

- for multiplication or division, phasor/polar form is best and
- for addition or subtraction rectangular form is best.

Finally, it is often desirable to convert the result to trigonometric form, i.e. "dephaze" it.