

Figure ex.1: Circuits for Exercise ssan.

print C

$$Z_{RC} = \frac{1}{\frac{1}{Z_R} + \frac{1}{Z_C}}$$

$$Z_e = Z_{RC} + Z_L = \frac{Z_R Z_C}{Z_R + Z_C} + Z_L$$

$$Z_R = R = \frac{R j\omega C}{R + j\omega C} + j\omega L$$

$$Z_C = \frac{1}{j\omega C} = \frac{R}{j\omega RC + 1} + j\omega L$$

$$Z_L = j\omega L = \frac{R(1 - j\omega RC)}{1 + \omega^2 R^2 C^2} + j\omega L$$

$$Z_e = \frac{R - j\omega R^2 C}{1 + \omega^2 R^2 C^2} + j\omega L$$

$$\frac{R}{1 + \omega^2 R^2 C^2} + j\left(\omega L - \frac{\omega R^2 C}{1 + \omega^2 R^2 C^2}\right)$$

### ssan.exe Exercises for Chapter ssan

#### Exercise ssan.unpopably

Convert the following to trigonometric form:

- $jA(e^{-j\omega t} - e^{j\omega t})$
- $A(e^{j(\omega - 1)t} + e^{-j(\omega - 1)t})$

$$jA(e^{-j\omega t} - e^{j\omega t}) = jA(\cos(\omega t) - j\sin(\omega t) - (\cos(\omega t) + j\sin(\omega t))) = jA(-2)\sin(\omega t) = -2A\sin(\omega t)$$

$$A(e^{j(\omega - 1)t} + e^{-j(\omega - 1)t}) = 2A\cos((\omega - 1)t)$$

#### Exercise ssan.overexplication

Convert the following to phasor form:

- $A \sin(\omega t + \phi)$
- $Ae^{j^2} \sin \omega t$
- $A(\cos \omega t - j \sin \omega t)$

$$A \sin(\omega t + \phi) = \frac{A}{\sqrt{2}} (e^{j(\omega t + \phi)} + e^{-j(\omega t + \phi)})$$

$$Ae^{j^2} \sin \omega t = Ae^{-2\omega t} (e^{j\omega t} + e^{-j\omega t})$$

$$A(\cos \omega t - j \sin \omega t) = \frac{A}{\sqrt{2}} (e^{j\omega t} + e^{-j\omega t} - j(e^{j\omega t} - e^{-j\omega t}))$$

#### Exercise ssan.ampatheticism

Find the combined effective impedance of the circuits shown in Fig. ex.1. Write your answer in rectangular form.

$$A(e^{j\omega t} e^{-j\phi} + e^{-j\omega t} e^{j\phi}) = Ae^{-j\phi} (e^{j\omega t} + e^{-j\omega t}) + Ae^{j\phi} (e^{j\omega t} - e^{-j\omega t})$$

$$= 2Ae^{-j\phi} \cos(\omega t) + j2Ae^{j\phi} \sin(\omega t)$$

$$= A \sqrt{2} (\cos(\omega t) \cos(\phi) + \sin(\omega t) \sin(\phi)) = A \sqrt{2} \cos(\omega t - \phi)$$

$$= \frac{A}{\sqrt{2}} (e^{j(\omega t - \phi)} + e^{-j(\omega t - \phi)})$$

$$= Ae^{-j\phi} e^{j\omega t} + Ae^{j\phi} e^{-j\omega t}$$

Point d

$$Z_e = \frac{1}{\frac{1}{Z_R} + \frac{1}{Z_C} + \frac{1}{Z_L}} = \frac{1}{\frac{1}{R} + j\omega C + \frac{1}{j\omega L}}$$

$$= \frac{1}{\frac{1 + j\omega RC + \frac{R}{j\omega L}}{R}} = \frac{R}{1 + j\omega RC - \frac{R}{\omega L}}$$

$$= \frac{R}{\frac{j\omega RL - \omega^2 R^2 C + R}{\omega L}} = \frac{R \omega L}{j\omega RL - \omega^2 R^2 C + R}$$

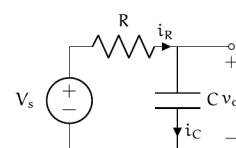
$$= \frac{\omega L^2 + (R - \omega^2 R^2 C)^2}{\omega L^2 + (R - \omega^2 R^2 C)^2}$$

#### Exercise ssan.nap

For the RC circuit diagram below, perform a circuit analysis to solve for the steady state voltage  $v_o(t)$  if  $V_s(t) = A \sin \omega t$ , where  $A \in \mathbb{R}$  is a given amplitude and  $\omega \in \mathbb{R}$  is a given angular frequency. Use a sine phasor in the problem. Write your answer as a single sine phasor in polar form. Evaluate your answer for the following two sets of parameters.

- $A = 2.5 \text{ V}$   
 $\omega = 10 \times 10^3, 20 \times 10^3 \text{ rad/s}$   
 $R = 100, 1000 \Omega$   
 $C = 100, 10 \text{ nF}$

The first set should yield  $v_o = 1.99e^{-10.0997}$ .

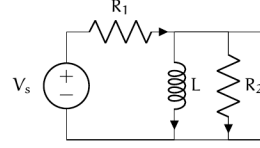


#### Exercise ssan.vestmentid

For the circuit diagram below, perform a complete circuit analysis to solve for the steady state voltage  $v_o(t)$  if  $V_s(t) = A \sin \omega t$ , where  $A \in \mathbb{R}$  is a given amplitude and  $\omega \in \mathbb{R}$  is a given angular frequency. Use a sine phasor in the problem. Write your answer as a single sine phasor in polar form. Evaluate your answer for the following two sets of parameters.

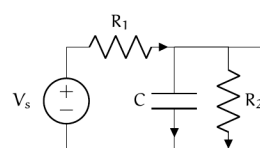
- $A = 3.8 \text{ V}$   
 $\omega = 30 \times 10^3, 60 \times 10^3 \text{ rad/s}$   
 $R_1 = 100, 1000 \Omega$   
 $R_2 = 1000, 100 \Omega$   
 $L = 10, 100 \text{ mH}$

The first set should yield  $v_o = 2.61e^{10.294}$ .



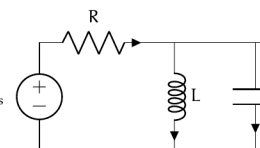
#### Exercise ssan.belago

For the circuit diagram below, solve for the steady state voltage  $v_o(t)$  if  $V_s(t) = Ae^{j\omega t}$ , where  $A \in \mathbb{R}$  is a given input amplitude and  $\phi \in \mathbb{R}$  is a given input phase. Write your answer as a single phasor in polar form (you may use intermediate variables in this final form as long as they're clearly stated).



#### Exercise ssan.oveparticular

For the circuit diagram below, solve for the steady state output voltage  $v_o(t)$  if  $V_s(t) = A \cos(\omega t)$ . Do write  $V_s$  and the impedance of each element in phasor/polar form. Do not substitute  $V_s$  or the impedance of each element into your expression for  $v_o(t)$ . Recommendation: use a divider rule.

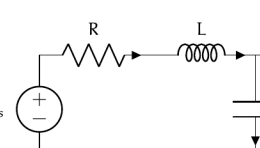


#### Exercise ssan.radionizometer

For the circuit diagram below, solve for the steady state output voltage  $v_o(t)$  if  $V_s(t) = 3 \sin(10t)$ . Use a sine phasor in the problem. Write your answer as a single sine phasor in polar form. Evaluate your answer for the following two sets of parameters.

- $R = 10, 10^3 \Omega$   
 $L = 500, 50 \text{ mH}$   
 $C = 100, 10 \mu\text{F}$

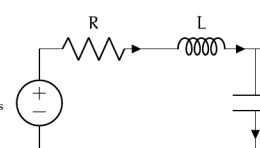
The first set should yield  $v_o = 3.01e^{-10.0100}$ .



#### Exercise ssan.melicid

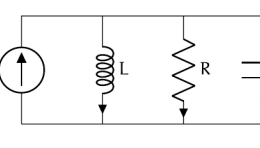
For the circuit diagram below, solve for  $v_o(t)$  if  $V_s(t) = A \sin \omega t$ , where  $A = 2 \text{ V}$  is the given amplitude and  $\omega \in \mathbb{R}$  is a given angular frequency. Let  $R = 50 \Omega, L = 50 \text{ mH}$ , and  $C = 200 \text{ nF}$ . Find the steady-state ratio of the output amplitude to the input amplitude  $A$  for  $\omega = \{5000, 10000, 50000\} \text{ rad/s}$ . Plot the steady-state ratio as a function of  $\omega$  in MATLAB, Python, or Mathematica. This circuit is called a low-pass filter—explain why this makes sense. Note that using impedance methods for steady state analysis makes this problem much easier than the transient analysis of this circuit in Exercise can.

low-pass filter



#### Exercise ssan.entirefully

For the circuit diagram below, perform a circuit analysis to solve for the steady state voltage  $v_o(t)$  if  $I_s = Ae^{j\omega t}$ , where  $A > 0$  is a given amplitude. Identify all impedance values in the circuit, but express your answer in terms of impedances (i.e. don't substitute for them in your final expression).



20 p.

nlmml

## Nonlinear and multiport circuit elements

Thus far, we have considered only one-port, linear circuit elements. One-port elements have two terminals. Linear elements have voltage-current relationships that can be described by linear algebraic or differential equations.

Multi-port elements are those that have more than one port. In this chapter, we will consider several multi-port elements: transformers (two-port), transistors (two-port), and opamps (four-port).

Nonlinear elements have voltage-current relationships that cannot be described by a linear algebraic or differential equations. The convenient impedance methods of Chapter ssan apply only to linear circuits, so we must return to the differential equation-based analysis of Chapter can. In this chapter, we will consider several nonlinear circuits containing three different classes of nonlinear elements: diodes, transistors, and opamps.

A great number of the most useful circuits today include multi-port and nonlinear elements. Tasks such as ac-dc conversion, switching, amplification, and isolation require these elements.

We explore only the fundamentals of each element considered and present basic analytic techniques, but further exploration in Horowitz and Hill,<sup>1</sup> Agarwal and Lang,<sup>2</sup> and Ulaby, Maharbiz and Farsaei<sup>3</sup> is encouraged.

multi-port

nonlinear element

1. Horowitz and Hill, The Art of Electronics.  
2. A. Agarwal and J. Lang, Fundamentals of Analog and Digital Electronic Circuits. The Morgan Kaufmann Series in Computer Architecture and Design. Elsevier Science, 2005. ISBN: 9780080508114.  
3. Fawwaz T. Ulaby, Michel M. Maharbiz and Cynthia M. Furse, Circuit Analysis and Design. ISBN 978-1-60795-484-5. Michigan Publishing, 2018.