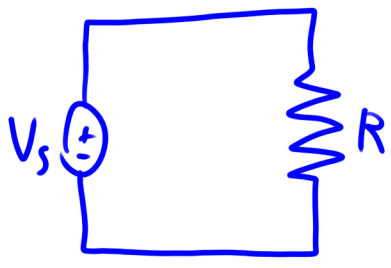
3.7. Consider an electric home heating system. A resistive element R is connected to the 110-volt, 60-Hz electric supply.

(a) Determine the current flow through the element as a function of time.

(b) Determine the average thermal power generated per cycle.

(c) If the heater is rated at 1000 watts, what is the value of the heater resistance R?

a)



$$V_s = \frac{110}{2}\sin(60(2\pi)t)$$

$$V_R = R\,i_R \qquad V_R = V_s$$

$$i_R = \frac{V_R}{R} = \boxed{\frac{V_s}{R}}$$

b) $$P(t) = V_R\, i_R = V_s \frac{V_s}{R} = \frac{V_s^2}{R}$$

$$P = \frac{1}{t}\int_0^t \frac{V_s^2}{R}\,dt = \frac{60}{R}\int_0^{1/60} V_s^2\,dt$$

$$= \frac{60}{R}\,\frac{110^2}{4}\int_0^{1/60} \sin^2(120\pi t)\,dt$$

$$= \frac{181500}{R}\int_0^{1/60} \frac{1-\cos(120\pi t)}{2}\,dt$$

$$= \frac{9075}{R}\left(t - \frac{\sin(120\pi t)}{120\pi}\right)\Bigg|_0^{1/60}$$

$$= \frac{9075}{R}\,t\Big|_0^{1/60} = \frac{9075}{60R} = \boxed{\frac{1512}{R} = P}$$

c) $$1000 = \frac{1512}{R} \implies R = \frac{1512}{1000} = \boxed{1.51\,\Omega}$$