Mechanical Engineering
345 – Mechatronics

Midterm Exam 1

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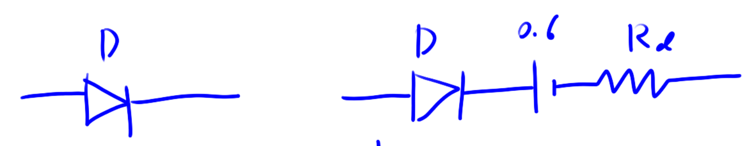
Directions: take-home, all day, open notes, open book. Calculators, MATLAB, etc. allowed. Use your own paper, work neatly, and clearly mark your answers. Partial credit may be given.

Problem bupsrul

Write a one- or two-sentence response to each of the following questions and imperatives. The use of equations is acceptable when they appear in a sentence. Don't quote me (use your own words, other than technical terminology). _____/10 p.

a What is the piecewise linear diode model.
b What are the relationships between input and output voltage and current in a transformer? Why?
c The current through a capacitor becomes zero. What happens to the voltage across the capacitor?
d Explain the how the current from the drain to the source of a MOSFET changes as the gate voltage is varied. Assume the MOSFET is in the saturation region.
e When can we use impedance analysis?



$V_2 = N V_1$ $\quad i_2 = \frac{-1}{N} i_1$ $\quad P_1 = P_2$

$\frac{dv_c}{dt} = \frac{1}{C} i_c$ if $i_c = 0$ $\frac{dv_c}{dt} = 0$ $\quad V_c$ = constant

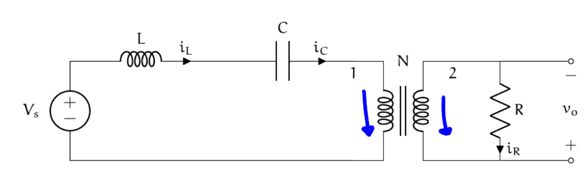
$i_{DS} = K(V_{GS} - V_T)^2$ if V_T is small

$i_{DS} \propto V_{GS}^2$

Problem reorientator

Use the circuit diagram below to answer the following questions and imperatives. Let $V_s = A\sin(\omega t)$. Perform a full circuit analysis, including the transient response to find $v_o(t)$. The initial inductor current is $i_L(0) = 0$ and the initial capacitor voltage $v_C(0) = 0$. _____/40 p.

a Write the elemental, KCL, and KVL equations.
b Write the second-order differential equation for $v_C(t)$ arranged in the standard form.
c Convert the initial condition in i_L to a second initial condition in i_C.
d Let $R = 10\ k\Omega$, $L = 100\ mH$, $C = 100\ \mu F$, $N = 5$, $A = 5\ V$, and $\omega = 500\ \frac{rad}{s}$ and solve for $v_C(t)$.
e Derive an equation to find $v_o(t)$ from $v_C(t)$. This equation will include derivatives of $v_C(t)$. You don't need to add your solution to part **d** into this equation.



a. Elemental Eq's

$V_R = R i_R$

$\frac{dv_c}{dt} = \frac{1}{C} i_c$

$\frac{di_L}{dt} = \frac{1}{L} v_L$

$V_2 = N V_1$

$i_2 = \frac{-1}{N} i_1$

KVL

$V_s = V_L + V_c + V_1$

$V_2 = V_R$

KCL

$i_L = i_c = i_1$

$i_2 = -i_R$

b. $\frac{dv_c}{dt} = \frac{1}{C} i_c = \frac{1}{C} i_L$

$$\frac{d^2 v_c}{dt^2} = \frac{1}{C}\frac{di_L}{dt} = \frac{1}{LC} V_L = \frac{1}{LC}(V_s - V_c - V_1)$$
$$= \frac{1}{LC}\left(V_s - V_c - \frac{1}{N} V_2\right) = \frac{1}{LC}\left(V_s - V_c - \frac{1}{N} V_R\right)$$
$$= \frac{1}{LC}\left(V_s - V_c - \frac{1}{N} R i_R\right)$$
$$= \frac{1}{LC}\left(V_s - V_c + \frac{R}{N} i_2\right) = \frac{1}{LC}\left(V_s - V_c - \frac{R}{N^2} i_1\right)$$
$$= \frac{1}{LC}\left(V_s - V_c - \frac{R}{N^2} i_c\right) = \frac{1}{LC}\left(V_s - V_c - \frac{RC}{N^2}\frac{dv_c}{dt}\right)$$

$$\frac{d^2 v_c}{dt^2} + \frac{R}{LN^2}\frac{dv_c}{dt} + \frac{1}{LC} V_c = \frac{1}{LC} V_s$$

c. $i_L = i_c = C\frac{dv_c}{dt}$

$i_L(0) = 0 \qquad \left.\frac{dv_c}{dt}\right|_{t=0} = 0$

e. $V_o = V_R = V_2 = N V_1 = N(V_s - V_L - V_c)$

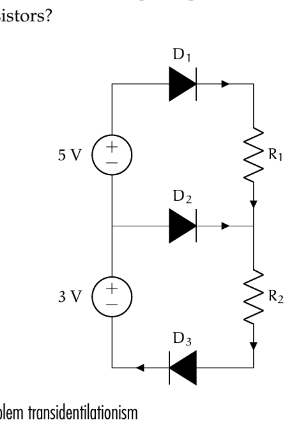
$$= N\left(V_s - L\frac{di_L}{dt} - V_c\right) = N\left(V_s - L\frac{di_c}{dt} - V_c\right)$$

$$\boxed{= N\left(V_s - LC\frac{d^2 v_c}{dt^2} - V_c\right)}$$

Problem unrectangularization

Use the circuit diagram below to answer the following questions. Assume $R_1 = R_2$ and that all diodes are ideal. _____/15 p.

a What state is each diode in?
b What is the voltage drop across each of the resistors?



KCL

$i_{D_1} = i_{R_1}$

$i_{R_1} + i_{D_2} = i_{R_2}$

$i_{R_2} = i_{D_3}$

Elemental Eq's

$V_{R_1} = i_{R_1} R_1$

$V_{R_2} = i_{R_2} R_2$

KVL

$3 + 5 = V_{D_1} + V_{R_1} + V_{R_2} + V_{D_3}$

$3 = V_{D_2} + V_{R_2} + V_{D_3}$

assume D_1 on $\quad D_2$ off $\quad D_3$ on

$V_{D_1} = 0 \quad i_{D_2} = 0 \quad V_{D_3} = 0$

$i_{D_1} = i_{R_1} \qquad i_{R_1} + \cancel{i_{D_2}} = i_{R_2}$

$i_{D_1} = i_{R_1} = i_{R_2} = i_{D_3}$

$8 = \cancel{V_{D_1}} + V_{R_1} + V_{R_2} + \cancel{V_{D_3}}$

$= i_{R_1} R_1 + i_{R_2} R_2 = i_{R_2}(R_1 + R_2) = 2 i_{R_2} R_2$

if $R_2 > 0$ then $i_{R_2} > 0$

$i_{D_1} > 0$ ✓ $\qquad i_{D_3} > 0$ ✓

$3 = V_{D_2} + V_{R_2} + \cancel{V_{D_3}}$

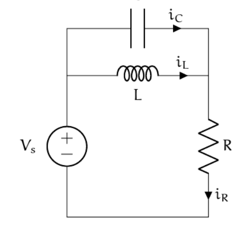
$V_{D_2} = 3 - V_{R_2} = 3 - 4 = -1 < 0$ ✓

$8 = 2 i_{R_2} R_2 = 2 V_{R_2}$

$\boxed{4 = V_{R_2}} \qquad 8 = V_{R_1} + V_{R_2} = V_{R_1} + 4 \qquad V_{R_1} = 8 - 4 = \boxed{4}$

Problem transidentilationism

For the circuit diagram below, perform a circuit analysis to solve for the steady state voltage across the resister R, $v_R(t)$. Assume $V_s = Ae^{j\frac{\pi}{2}}$ in sine phasor form and $A \in \mathbb{R}$. Express your answer in sine phasor form. _____/20 p.



Problem kirfunckle

Consider the circuit below with two constant voltage sources V_1 and V_2. Find the steady state voltage output v_o, assuming $R_1 = R_2 = R_3$. Hint: start solving with the equation $v_o = -v_{R_3}$. _____/15 p.