Mechanical Engineering
345 – Mechatronics

Midterm Exam 1

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Directions: take-home, all day, open notes, open book. Calculators, MATLAB, etc. allowed. Use your own paper, work neatly, and clearly mark your answers. Partial credit may be given.

Problem bupsrul

_____/10 p.

Write a one- or two-sentence response to each of the following questions and imperatives. The use of equations is acceptable when they appear in a sentence. Don't quote me (use your own words, other than technical terminology).

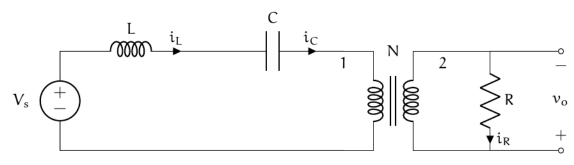
a What is the piecewise linear diode model.
b What are the relationships between input and output voltage and current in a transformer? Why?
c The current through a capacitor becomes zero. What happens to the voltage across the capacitor?
d Explain the how the current from the drain to the source of a MOSFET changes as the gate voltage is varied. Assume the MOSFET is in the saturation region.
e When can we use impedance analysis?

Problem reorientator

_____/40 p.

Use the circuit diagram below to answer the following questions and imperatives. Let $V_s = A\sin(\omega t)$. Perform a full circuit analysis, including the transient response to find $v_o(t)$. The initial inductor current is $i_L(0) = 0$ and the initial capacitor voltage $v_C(0) = 0$.

a Write the elemental, KCL, and KVL equations.
b Write the second-order differential equation for $v_C(t)$ arranged in the standard form.
c Convert the initial condition in i_L to a second initial condition in i_C.
d Let $R = 10\ \text{k}\Omega$, $L = 100$ mH, $C = 100\ \mu$F, $N = 5$, $A = 5$ V, and $\omega = 500\ \frac{\text{rad}}{\text{s}}$ and solve for $v_C(t)$.
e Derive an equation to find $v_o(t)$ from $v_C(t)$. This equation will include derivatives of $v_C(t)$. You don't need to add your solution to part **d** into this equation.

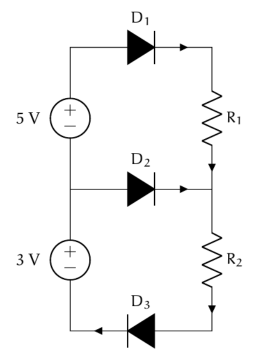


Problem unrectangularization

_____/15 p.

Use the circuit diagram below to answer the following questions. Assume $R_1 = R_2$ and that all diodes are ideal.

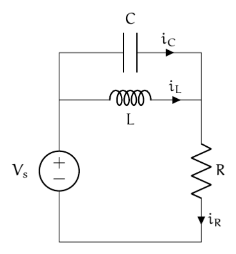
a What state is each diode in?
b What is the voltage drop across each of the resistors?



Problem transidentilationism

_____/20 p.

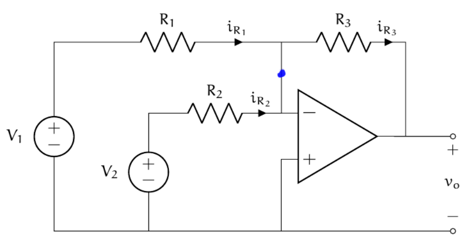
For the circuit diagram below, perform a circuit analysis to solve for the steady state voltage across the resister R, $v_R(t)$. Assume $V_s = Ae^{j\frac{\pi}{2}}$ in sine phasor form and $A \in \mathbb{R}$. Express your answer in sine phasor form.



Problem kirfunckle

_____/15 p.

Consider the circuit below with two constant voltage sources V_1 and V_2. Find the steady state voltage output v_o, assuming $R_1 = R_2 = R_3$. Hint: start solving with the equation $v_o = -v_{R_3}$.



KCL

$i_{R_1} + i_{R_2} = i_{R_3}$

KVL

$V_1 = V_{R_1}$

$V_2 = V_{R_2}$

$V_1 = V_{R_1} + V_{R_3} + V_o$

$V_2 = V_{R_2} + V_{R_3} + V_o$

$0 = V_{R_3} + V_o$

Elemental Eq's

$V_{R_1} = R_1 i_{R_1}$

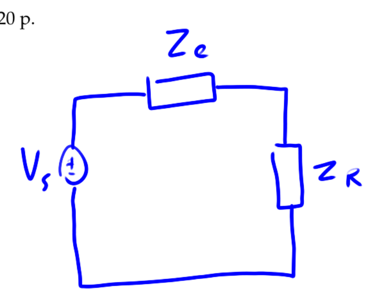
$V_{R_2} = R_2 i_{R_2}$

$V_{R_3} = R_3 i_{R_3}$

$V_+ = V_-$

$i_+ = i_- = 0$

$$V_o = -V_{R_3} = -R_3 i_{R_3}$$
$$= -R_3(i_{R_1} + i_{R_2})$$
$$= -R_3\left(\frac{V_{R_1}}{R_1} + \frac{V_{R_2}}{R_2}\right)$$
$$= -R_3\left(\frac{V_1}{R_1} + \frac{V_2}{R_2}\right)$$
$$= -(V_1 + V_2)$$



$Z_R = R$

$Z_C = \frac{1}{j\omega C}$

$Z_L = j\omega L$

$$V_R = V_s \frac{Z_R}{Z_e + Z_R}$$

$$Z_e = \frac{1}{\frac{1}{Z_C} + \frac{1}{Z_L}} = \frac{1}{j\omega C + \frac{1}{j\omega L}} = \frac{j\omega L}{1 - \omega^2 LC}$$

$$V_R = V_s \frac{R}{\frac{j\omega L}{1-\omega^2 LC} + R} \frac{1-\omega^2 LC}{1-\omega^2 LC}$$

$$= V_s \frac{R - \omega^2 RLC}{j\omega L + R - \omega^2 RLC} \frac{R - \omega^2 RLC - j\omega L}{R - \omega^2 RLC - j\omega L}$$

$$= V_s \frac{R^2 - \omega^2 R^2 LC - j\omega RL - \omega^2 R^2 LC + \omega^4 L^2 R^2 C^2 + j\omega^3 L^2 RC}{R^2 - 2\omega^2 R^2 LC + \omega^4 L^2 R^2 C^2 + \omega^2 L^2}$$

$$= V_s Z$$

$$\text{Re}(Z) = \frac{R^2 - 2\omega^2 R^2 LC + \omega^4 L^2 R^2 C^2}{R^2 - 2\omega^2 R^2 LC + \omega^4 L^2 R^2 C^2 + \omega^2 L^2}$$

$$\text{Im}(Z) = \frac{\omega^3 L^2 RC - \omega RL}{R^2 - 2\omega^2 R^2 LC + \omega^4 L^2 R^2 C^2 + \omega^2 L^2}$$

$$B = |Z| = \sqrt{\text{Re}(Z)^2 + \text{Im}(Z)^2}$$

$$= \boxed{\frac{\sqrt{(R^2 - 2\omega^2 R^2 LC + \omega^4 L^2 R^2 C^2)^2 + (\omega^3 L^2 RC - \omega RL)^2}}{R^2 - 2\omega^2 R^2 LC + \omega^4 L^2 R^2 C^2 + \omega^2 L^2}}$$

$$\phi = \tan^{-1}\left(\frac{\text{Im}(Z)}{\text{Re}(Z)}\right)$$

$$= \boxed{\tan^{-1}\left(\frac{\omega^3 L^2 RC - \omega RL}{R^2 - 2\omega^2 R^2 LC + \omega^4 L^2 R^2 C^2}\right)}$$

$$V_R = V_s Z = Ae^{j\pi/2} Be^{j\phi}$$
$$= \boxed{ABe^{j(\pi/2 + \phi)}}$$

for lab

$V_o = V_s Z(\omega)$

$\frac{V_o}{V_s} = |Z(\omega)|$