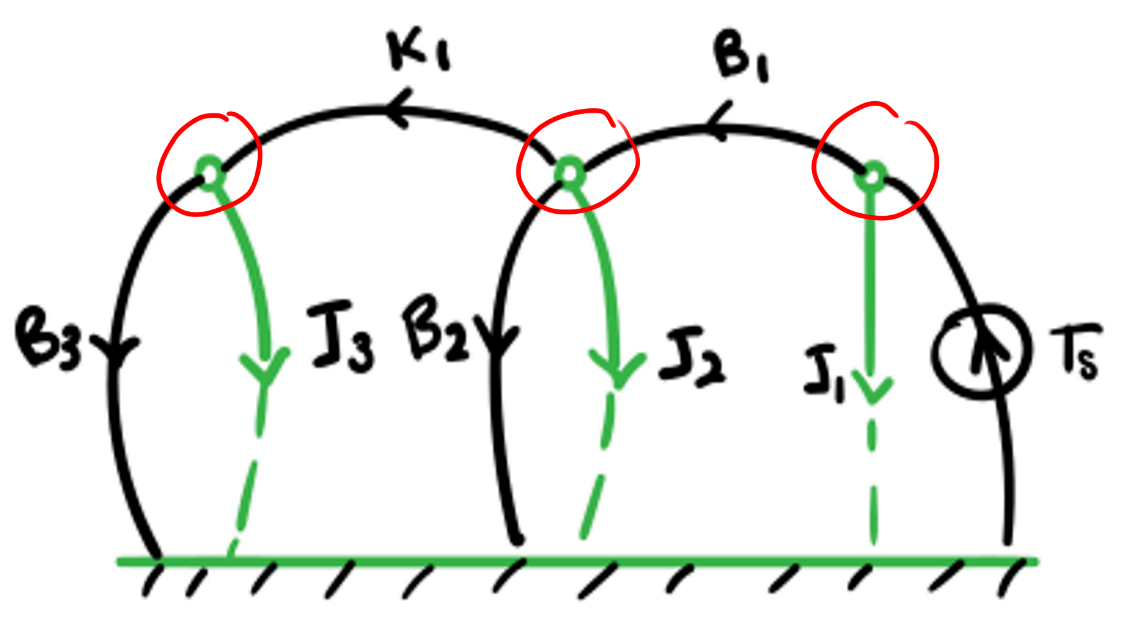
SS. Chunker part 2



$E = 8$
$S_A = 0$
$S_T = 1$
$S = 1$
$N = 4$

1. $2E - S = 16 - 1 = 15$

a. normal tree ✓

b. primary: ω_{J_3} ω_{J_2} ω_{J_1} τ_{B_3} τ_{K_1} τ_{B_1} T_s
secondary: τ_{J_3} τ_{J_2} τ_{J_1} ω_{B_3} ω_{K_1} ω_{B_1} ω_s

c. state: ω_{J_1} ω_{J_2} ω_{J_3} τ_{K_1}

d.
$$x = \begin{bmatrix} \omega_{J_1} \\ \omega_{J_2} \\ \omega_{J_3} \\ \tau_{K_1} \end{bmatrix} \qquad u = \begin{bmatrix} T_s \end{bmatrix} \qquad y = \begin{bmatrix} \omega_{J_1} \\ \omega_{J_2} \\ \omega_{J_3} \\ \tau_{B_3} \end{bmatrix}$$

e. $E - S = 8 - 1 = 7$

$$\tau_{B_3} = B_3 \omega_{B_3}$$
$$\tau_{B_2} = B_2 \omega_{B_2}$$
$$\tau_{B_1} = B_1 \omega_{B_1}$$
$$\frac{d\omega_{J_1}}{dt} = \frac{1}{J_1}\tau_{J_1}$$
$$\frac{d\omega_{J_2}}{dt} = \frac{1}{J_2}\tau_{J_2}$$
$$\frac{d\omega_{J_3}}{dt} = \frac{1}{J_3}\tau_{J_3}$$
$$\frac{d\tau_{K_1}}{dt} = K_1 \omega_{K_1}$$

f. $N - 1 - S_A = 4 - 1 - 0 = 3$

$$\tau_{K_1} = \tau_{J_3} + \tau_{B_3} \Rightarrow \tau_{J_3} = \tau_{K_1} - \tau_{B_3}$$
$$\tau_{B_1} = \tau_{J_2} + \tau_{B_2} + \tau_{K_1} \Rightarrow \tau_{J_2} = \tau_{B_1} - \tau_{B_2} - \tau_{K_1}$$
$$T_s = \tau_{J_1} + \tau_{B_1} \Rightarrow \tau_{J_1} = T_s - \tau_{B_1}$$

g. $E - N + 1 - S_T = 8 - 4 + 1 - 1 = 4$

$$\omega_{B_3} = \omega_{J_3}$$
$$\omega_{K_1} + \omega_{J_3} = \omega_{J_2} \Rightarrow \omega_{K_1} = \omega_{J_2} - \omega_{J_3}$$
$$\omega_{B_2} = \omega_{J_2}$$
$$\omega_{J_2} + \omega_{B_1} = \omega_{J_1} \Rightarrow \omega_{B_1} = \omega_{J_1} - \omega_{J_2}$$

2. a.

$$\tau_{B_3} = B_3 \omega_{J_3}$$
$$\tau_{B_2} = B_2 \omega_{J_2}$$
$$\tau_{B_1} = B_1(\omega_{J_1} - \omega_{J_2})$$
$$\frac{d\omega_{J_1}}{dt} = \frac{T_s - \tau_{B_1}}{J_1}$$
$$\frac{d\omega_{J_2}}{dt} = \frac{\tau_{B_1} - \tau_{B_2} - \tau_{K_1}}{J_2}$$
$$\frac{d\omega_{J_3}}{dt} = \frac{\tau_{K_1} - \tau_{B_3}}{J_3}$$
$$\frac{d\tau_{K_1}}{dt} = K_1(\omega_{J_2} - \omega_{J_3})$$

b.

$$\frac{d\omega_{J_1}}{dt} = \frac{T_s - B_1\omega_{J_1} + B_1\omega_{J_2}}{J_1}$$
$$\frac{d\omega_{J_2}}{dt} = \frac{B_1\omega_{J_1} - B_1\omega_{J_2} - B_2\omega_{J_2} - \tau_{K_1}}{J_2}$$
$$\frac{d\omega_{J_3}}{dt} = \frac{\tau_{K_1} - B_3\omega_{J_3}}{J_3}$$
$$\frac{d\tau_{K_1}}{dt} = K_1(\omega_{J_2} - \omega_{J_3})$$

c. $\dot{x} = Ax + Bu$

$$\underbrace{\frac{d}{dt}\begin{bmatrix} \omega_{J_1} \\ \omega_{J_2} \\ \omega_{J_3} \\ \tau_{K_1} \end{bmatrix}}_{\dot{x}} = \underbrace{\begin{bmatrix} -\frac{B_1}{J_1} & \frac{B_1}{J_1} & 0 & 0 \\ B_1/J_2 & \frac{-B_1 - B_2}{J_2} & 0 & \frac{-1}{J_2} \\ 0 & 0 & -B_3/J_3 & \frac{1}{J_3} \\ 0 & K_1 & -K_1 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \omega_{J_1} \\ \omega_{J_2} \\ \omega_{J_3} \\ \tau_{K_1} \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} 1/J_1 \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} T_s \end{bmatrix}}_{u}$$

$$\underbrace{\begin{bmatrix} \omega_{J_1} \\ \omega_{J_2} \\ \omega_{J_3} \\ \tau_{B_3} \end{bmatrix}}_{y} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & B_3 & 0 \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} \omega_{J_1} \\ \omega_{J_2} \\ \omega_{J_3} \\ \tau_{K_1} \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{D} \underbrace{\begin{bmatrix} T_s \end{bmatrix}}_{u}$$

$$\tau_{B_3} = B_3 \omega_{J_3}$$